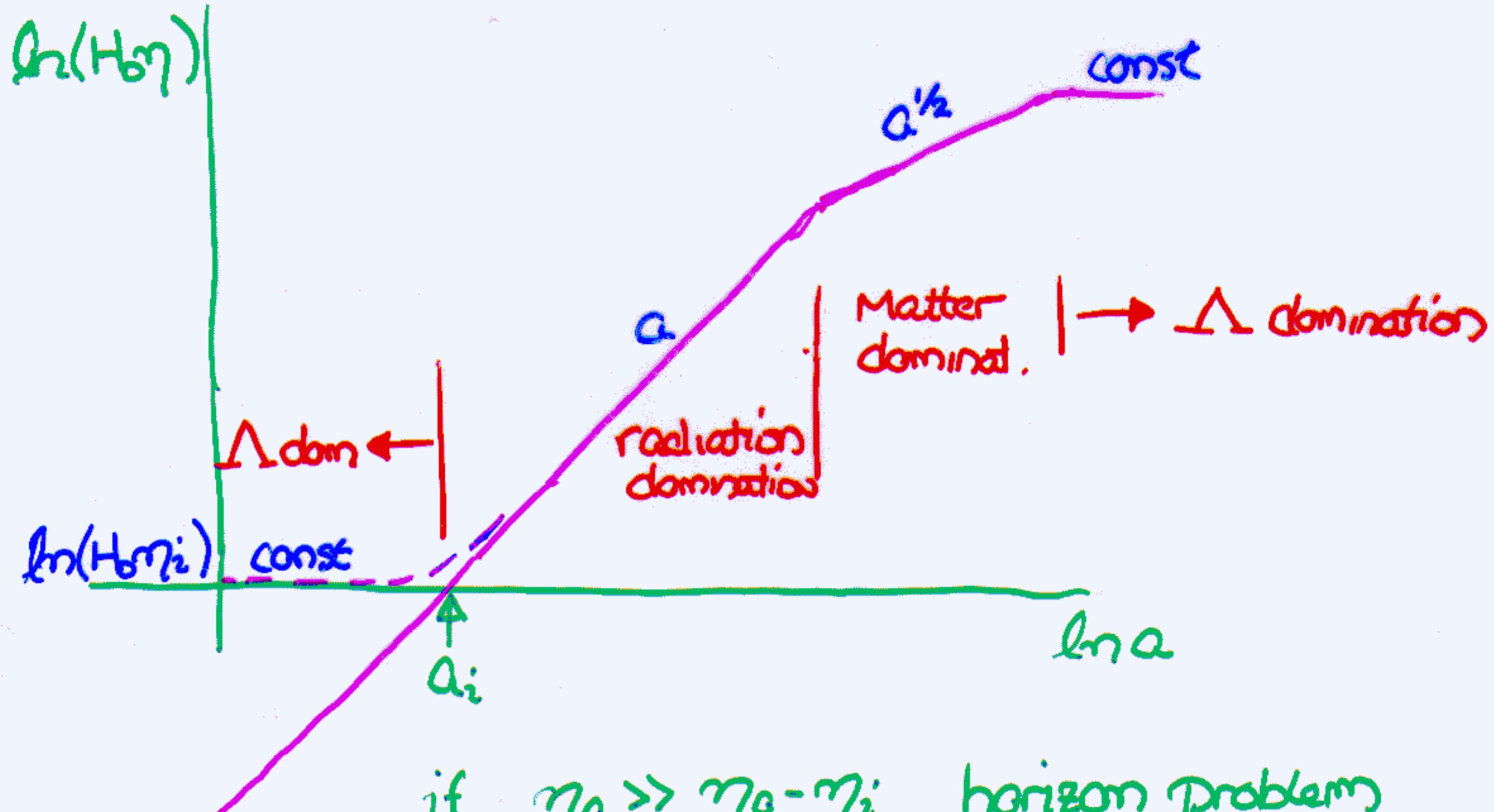


Inflation

- Horizon Problem: why is the universe so smooth on scales larger than the distance light can travel in a matter or radiation dominated universe?
if smooth by fiat why are the tiny 10^{-5} CMB fluctuations correlated on these large scales
- Flatness Problem: why is the radius of curvature of the universe larger than the observable universe?
- Relic Problem: why don't relics (e.g. monopoles) dominate the energy density today

Horizon Problem (comoving coordinates)

$$\eta = \int \frac{dt}{a}$$



if $\eta_0 \gg \eta_0 - \eta_i$ horizon problem solved

\Rightarrow distance light travels $\Delta < a_i$
 \gg distance light travels after $\Delta > a_i$

Proved in PS#1 that $\eta \propto a^P$ $P > 0$ if $\omega > -\frac{1}{3}$
 so that $\eta_0 - \eta_i \sim \eta_0(1 - a_i^P) \sim \eta_0$

$\Rightarrow \omega < -\frac{1}{3}$ to solve horizon problem

\Rightarrow accelerating universe makes observable universe before a_i much larger than after a_i

- Flatness & Relic Problems also Solved

- Comoving radius of curvature $|K|^{-\frac{1}{2}}$

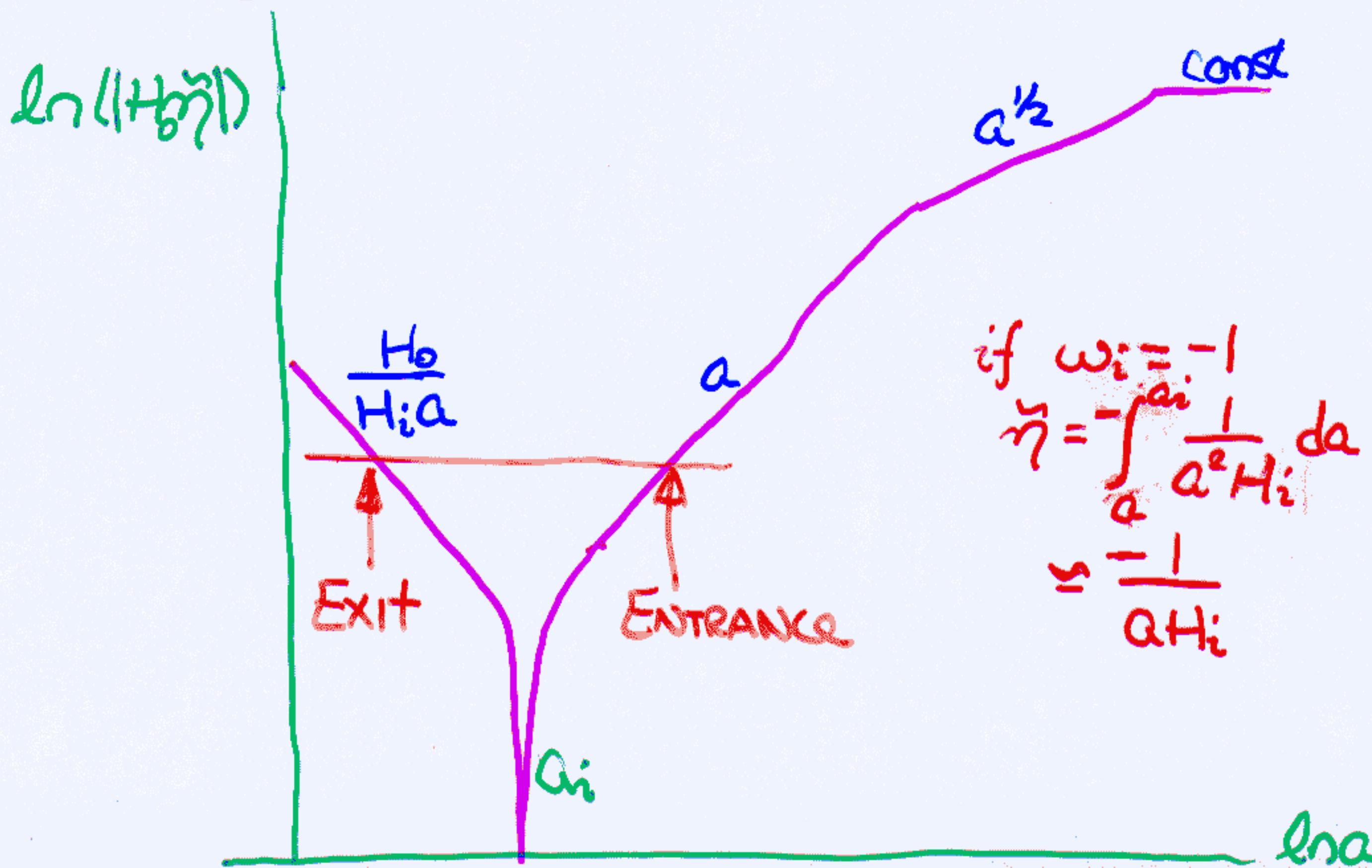
$$|K|^{-\frac{1}{2}} \sim \eta_0 \gg \eta_0 - \eta_i$$

- Alternatively $\rho \propto a^{3(1+w)}$

so all contributions to Friedmann eqn with $w \geq -\frac{1}{3}$ rapidly redshift away with expansion

$$\omega_K = -\frac{1}{3} \quad \omega_{rad} = \frac{1}{3} \quad \omega_{matter} = 0$$

- "Entering" and "Exiting" Horizon $\tilde{\eta} = \eta - \eta_i$



• Minimal Amount of Inflation

- Current horizon scale must have exited during inflation

$$\frac{H_0}{H_i a_{\text{start}}} \gg 1$$

$$\frac{a_{\text{start}}}{a_i} \ll \frac{H_0}{H_i a_i}$$

$$H_0 = 2 \times 10^{-42} \text{ h GeV}$$

$$H_i = \sqrt{\frac{8\pi G}{3}} \rho_i$$

$$a_i \propto \left(\frac{T_i}{T_{\text{CMB}}}\right)^{-1} \quad (\text{correct for } a_{\text{reheat}}/a_i)$$

- For typical $T_i (= 10^{10} \text{ GeV})$ $\rho_i^{1/4} (= 10^{14} \text{ GeV})$

$$N \equiv \ln\left(\frac{a_{\text{start}}}{a_i}\right) \gg 60$$

or "horizon scale today exited horizon ≈ 60 e-folds before end of inflation"

• Quantum Fluctuations of Inflaton

- Inflation satisfies perfect cosmological principle: time translationally invariant $\rho_i = \text{const}$

- Fluctuations must also be scale invariant

$$\Delta_x^2 \equiv \frac{k^3}{2\pi^2} P_x = \text{constant} \quad \text{for fluctuations in quantity } X$$

- Only scale in problem: H_i
dimensionless quantities, e.g. curvature

$$\sim H_i/m_p$$

$$\Delta_{\text{curvature}}^2 \simeq (H_i/m_p)^2$$

$$= \frac{2}{3\pi(1+w_i)} \left(\frac{H_i}{m_p}\right)^2$$

scalar field inflection

from calc
below.

- In PS #2, showed that a classical scalar field can drive inflation ($w_i = w_\phi < -\frac{1}{3}$)

⇒ Quantum fluctuations in ϕ

yield curvature fluctuations
that are scale invariant

Slow-Roll Conditions

- Vacuum Dominated

$$w_\phi \approx -1$$

$$\epsilon \equiv \frac{3}{2}(1+w_\phi) \ll 1$$

- Overdamped Oscillator (expansion \Rightarrow friction)

$$\ddot{\phi}_0 + 2\frac{\dot{a}}{a}\dot{\phi}_0 = -a^2 V'$$

$$\dot{\phi}_0 = \frac{d\phi_0}{dm} = a \frac{d\phi_0}{dt}$$

$$\ddot{\phi}_0 = \frac{d\dot{\phi}_0}{dm} = a \frac{d}{dt} a \frac{d\phi_0}{dt} = a^2 \frac{d^2 \phi_0}{dt^2} + a^2 \left(\frac{1}{a} \frac{da}{dt} \right) \frac{d\phi_0}{dt}$$

$$\frac{d^2 \phi_0}{dt^2} + 3H \frac{d\phi_0}{dt} + V' = 0$$

slow roll: $\frac{d^2 \phi_0}{dt^2} = 0$ or $\ddot{\phi}_0 = \frac{\dot{a}}{a} \dot{\phi}_0$

$$\delta \equiv \frac{\dot{\phi}_0}{\dot{a}} \left(\frac{\dot{a}}{a} \right)^{-1} - 1$$

also

$$\delta = -3 - \frac{a^2 V'}{\dot{\phi}_0} \left(\frac{\dot{a}}{a} \right)^{-1}$$

• Relation to $V(\phi)$

$$\left. \begin{array}{l} P_\phi = \frac{1}{2} a^{-2} \dot{\phi}_0^2 - V \\ Q_\phi = \frac{1}{2} a^{-2} \dot{\phi}_0^2 + V \end{array} \right\} \quad \omega_\phi = \left(\frac{1 - \frac{1}{2} \dot{\phi}_0^2/aV}{1 + \frac{1}{2} \dot{\phi}_0^2/aV} \right) \approx -1 \left(1 - \dot{\phi}^2/a^2 V \right)$$

$$\therefore \epsilon = \frac{3}{2} \frac{\dot{\phi}_0^2}{a^2 V} \quad \boxed{3 \dot{\phi}_0 \frac{\dot{a}}{a} = -Q^2 V'}$$

$$= \frac{3}{2} \frac{1}{a^2 V} \frac{a^4 V'^2}{9 (\dot{a}/a)^2}$$

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G a^2}{3} V$$

$$= \frac{1}{6} \frac{3}{8\pi G} \left(\frac{V'}{V} \right)^2$$

$$\boxed{\epsilon = \frac{1}{16\pi G} \left(\frac{V'}{V} \right)^2}$$

$$\dot{\phi}_0 \approx -a^2 \left(\frac{\dot{a}}{a} \right)^{-1} \frac{V'}{3}$$

$$\boxed{\frac{d}{d\eta} \left(\frac{\dot{a}}{a} \right) = -\frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 (1+3w_\phi)} \\ = \left(\frac{\dot{a}}{a} \right)^2 (1-\epsilon)$$

$$\ddot{\phi}_0 = -2a \frac{\dot{a}^2 V'}{3} + \frac{a^2 V'}{3} (1-\epsilon) - a^2 \left(\frac{\dot{a}}{a} \right)^{-1} \frac{V''}{3} \dot{\phi}_0$$

$$= -\frac{a^2 V'}{3} (1+\epsilon) + a^4 \left(\frac{\dot{a}}{a} \right)^{-2} \frac{V' V''}{9}$$

$$\boxed{\delta = -\frac{a^2 V'}{3} (1+\epsilon) + \frac{a^2}{9} \frac{3}{8\pi G} \frac{V' V''}{V} - 1}$$

$$\boxed{\delta = \epsilon - \frac{1}{8\pi G} \frac{V''}{V}}$$