What's the Matter with the CMB

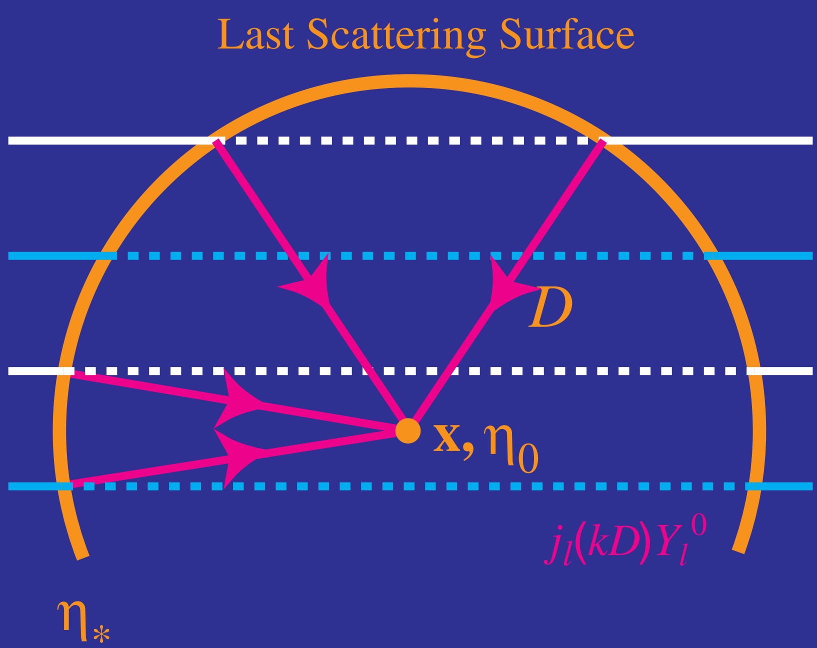
COBE Normalization

COBE Normalization

- Sachs-Wolfe Effect relates the COBE detection to the gravitational potential on the last scattering surface

$$[\Theta + \Psi](\hat{\mathbf{n}}, \mathbf{x}) = \frac{1}{3}\Psi(\mathbf{x} + D\hat{\mathbf{n}}, \eta_*)$$

$$D = \eta_0 - \eta$$



- Decompose the angular and spatial information into normal modes: spherical harmonics for angular, plane waves for spatial

$$G_\ell^m(\hat{\mathbf{n}}, \mathbf{x}, \mathbf{k}) \quad = \quad (-i)^\ell \sqrt{\frac{4\pi}{2\ell + 1}} Y_\ell^m(\hat{\mathbf{n}}) e^{i\mathbf{k}\cdot\mathbf{x}} \,.$$

COBE Normalization *cont.*

- Multipole moment decomposition for each **k**

$$\Theta(\mathbf{n}, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \sum_{\ell m} \Theta_\ell^{(m)}(k) G_\ell^m(\mathbf{x}, \mathbf{k}, \mathbf{n})$$

- Power spectrum is the integral over **k** modes

$$C_\ell = 4\pi \int \frac{d^3k}{(2\pi)^3} \sum_m \frac{\left\langle \Theta_\ell^{(m)*} \Theta_\ell^{(m)} \right\rangle}{(2\ell+1)^2}$$

- Fourier transform Sachs-Wolfe source

$$[\Theta + \Psi](\hat{\mathbf{n}}, \mathbf{x}) = \frac{1}{3} \int \frac{d^3k}{(2\pi)^3} \Psi(k, \eta_*) e^{i\mathbf{k}\cdot(D\hat{\mathbf{n}}+\mathbf{x})}$$

- Decompose plane wave

$$\exp(i\mathbf{k}D \cdot \hat{\mathbf{n}}) = \sum_\ell (-i)^\ell \sqrt{4\pi(2\ell+1)} j_\ell(kD) Y_\ell^0(\mathbf{n}),$$

COBE Normalization *cont.*

- Extract multipole moment, assume a constant potential

$$\frac{\Theta_\ell^{(0)}}{2\ell+1} = \frac{1}{3}\Psi(k,\eta_*)j_\ell(kD)$$
$$= \frac{1}{3}\Psi(k,\eta_0)j_\ell(kD)$$

- Construct angular power spectrum

$$C_\ell = 4\pi \int \frac{dk}{k} j_\ell^2(kD) \frac{1}{9}\Delta_\Psi^2$$

- For scale invariant potential (*n*=1), integral reduces to

$$\int_0^\infty \frac{dx}{x} j_\ell^2(x) = \frac{1}{2\ell(\ell+1)}$$

- Log power spectrum = Log potential spectrum / 9

$$\frac{\ell(\ell+1)}{2\pi} C_\ell = \frac{1}{9}\Delta_\Psi^2 \quad (n=1)$$

COBE Normalization *cont.*

- Relate to density fluctuations: Poisson equation and Friedmann eqn.

$$\begin{aligned} k^2\Psi &= -4\pi G a^2 \delta\rho \\ &= -\frac{3}{2} H_0^2 \Omega_m^2 \delta \end{aligned}$$

- Power spectra relation

$$\Delta_\Psi^2 = \frac{9}{4}\left(\frac{H_0}{k}\right)^4 \Omega_m^2 \Delta_\delta^2$$

- In terms of density fluctuation at horizon and transfer function

$$\Delta_\delta^2 \equiv \delta_H^2 \left(\frac{k}{H_0}\right)^{n+3} T^2(k)$$

- For scale invariant potential

$$\frac{\ell(\ell+1)}{2\pi} C_\ell = \frac{1}{4}\Omega_m^2 \delta_H^2 \quad (n=1)$$

COBE Normalization *cont.*

- Some numbers

$$\frac{\ell(\ell+1)}{2\pi}C_\ell = \frac{1}{4}\Omega_m^2\delta_H^2 \quad (n=1)$$

$$= \left(\frac{28\mu\mathrm{K}}{2.726\times10^6\mu\mathrm{K}}\right)^2 \approx 10^{-10}$$

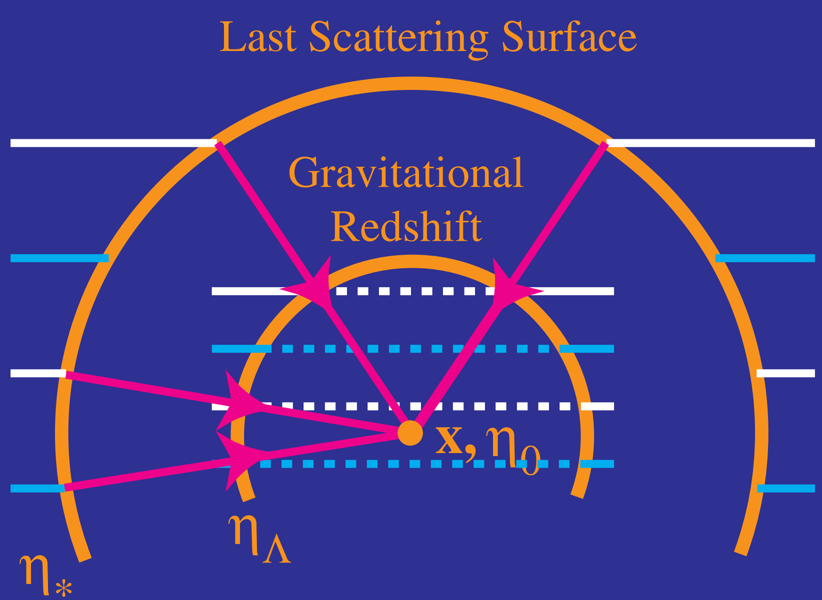
$$\delta_H \approx (2\times10^{-5})\Omega_m^{-1}$$

- Detailed Calculation from Bunn & White (1997) including decay of potential in low density universe and tilt

$$\delta_H = 1.94\times10^{-5}\Omega_m^{-0.785-0.05\ln\Omega_m}e^{-0.95(n-1)-0.169(n-1)^2}$$

Normalization Caveats

- Why aren't models like cosmological defects, which have large scale power at last scattering, automatically ruled out by COBE?

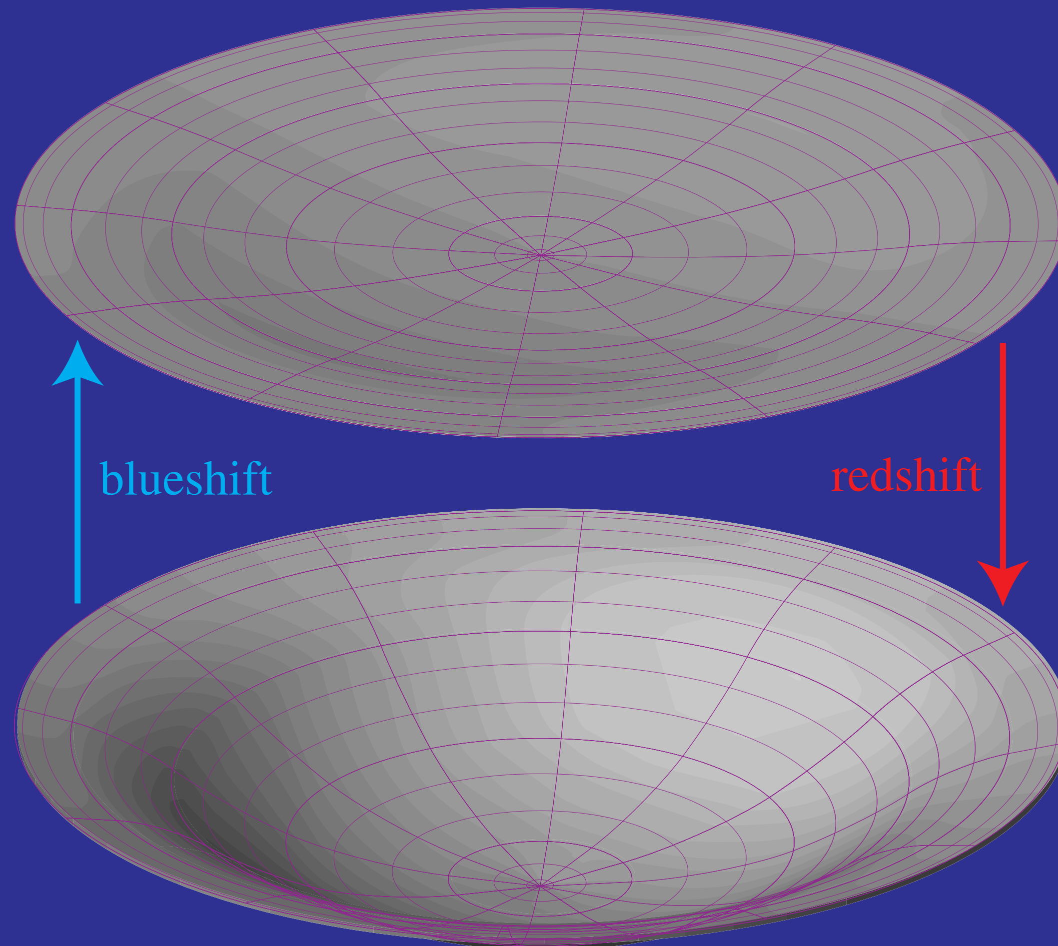


- As the photons propagate through the large-scale structure of the universe, gravitational redshifts from time-varying potentials can generate large angle fluctuations

- Dark energy domination implies potential decay, linear effect is called the Integrated Sachs Wolfe (ISW) effect

Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00}=-(1+\Psi)^2\ \delta_{ij}$



Kofman & Starobinskii (1985)

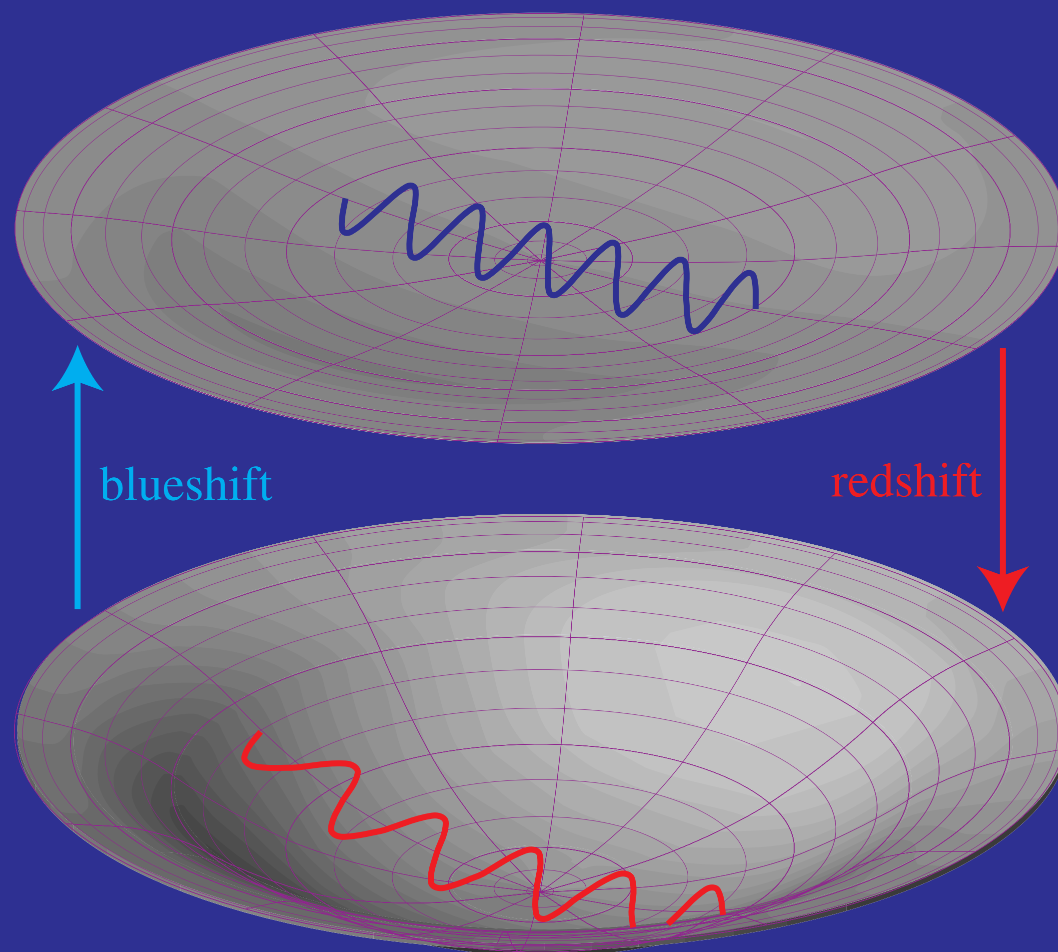
Hu & Sugiyama (1994)

Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00}=-(1+\Psi)^2\ \delta_{ij}$
- Perturbed cosmological redshift

  $g_{ij}=a^2(1+\Psi)^2\ \delta_{ij}$

  $\delta T/T = -\delta a/a = \Psi$

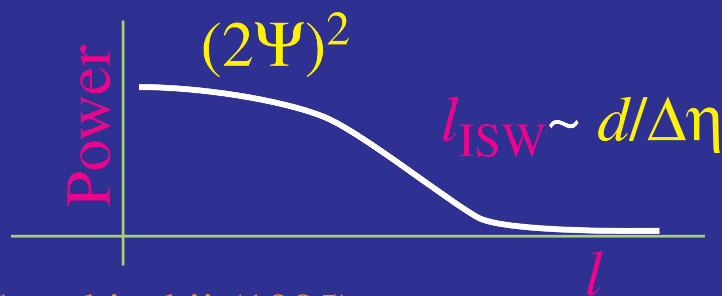


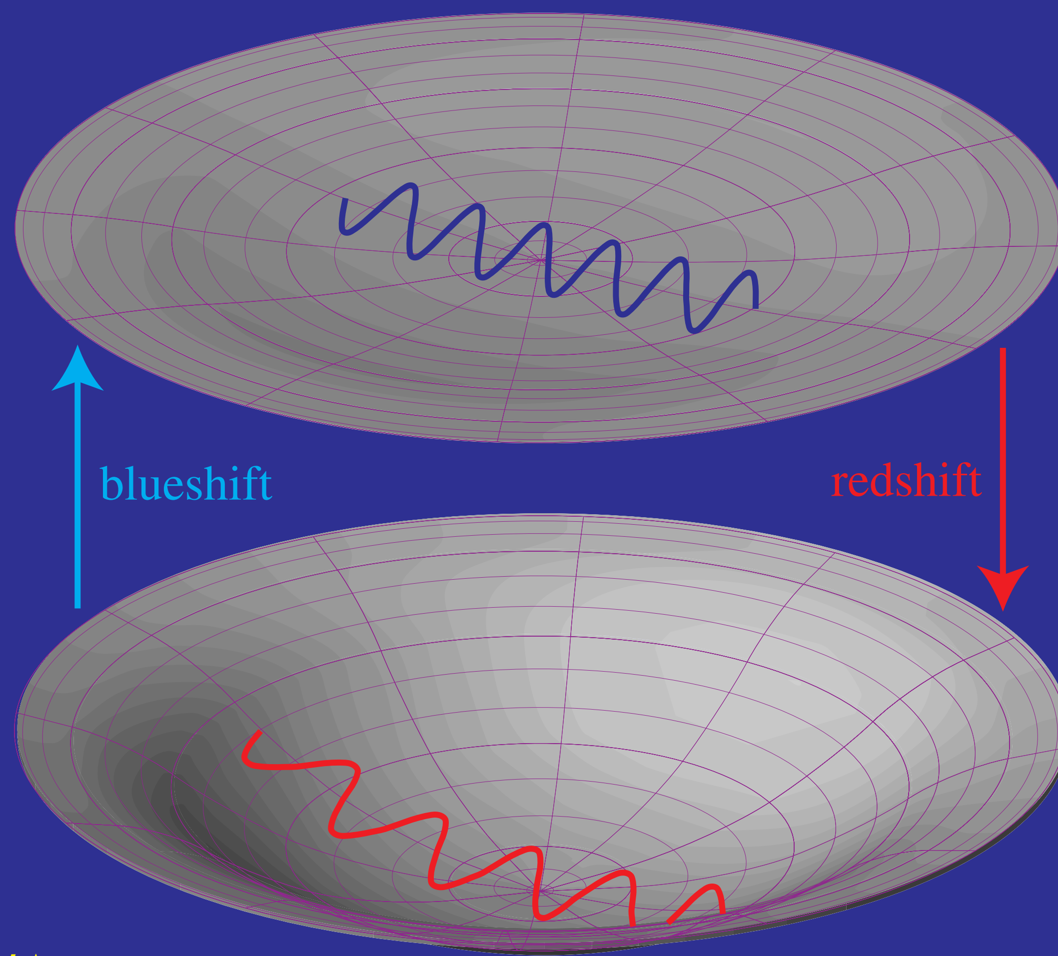
Kofman & Starobinskii (1985)

Hu & Sugiyama (1994)

Integrated Sachs–Wolfe Effect

- Potential redshift: g_{00}=–$(1+\Psi)^2\ \delta_{ij}$
- Perturbed cosmological redshift
  $g_{ij}=a^2(1+\Psi)^2\ \delta_{ij}$
  $\delta T/T = -\delta a/a = \Psi$
- Time–varying potential
  Rapid compared with λ/c
  $\delta T/T = -2\Delta\Psi$
  Slow compared with λ/c
  redshift–blueshift cancel
- Imprint characteristic time scale of decay in angular spectrum





Kofman & Starobinskii (1985)

Hu & Sugiyama (1994)

Calculation of Secondary Anisotropies

- Addition of angular momentum gives

$$\begin{pmatrix}\text{multipole}\\\text{moment}\end{pmatrix} = \int \begin{pmatrix}\text{clebsch}\\\text{gordan}\end{pmatrix}\begin{pmatrix}\text{bessel}\\\text{function}\end{pmatrix} \text{Source}\ d\begin{pmatrix}\text{line of}\\\text{sight}\end{pmatrix}$$

- Primary anisotropies: source sharply peaked at last scattering
  Tight Coupling Approximation:

$$\begin{pmatrix}\text{multipole}\\\text{moment}\end{pmatrix} \sim \begin{pmatrix}\text{clebsch}\\\text{gordan}\end{pmatrix}\begin{pmatrix}\text{bessel}\\\text{function}\end{pmatrix} \int \text{Source}\ d\begin{pmatrix}\text{line of}\\\text{sight}\end{pmatrix}$$

- Secondary anisotropies: source slowly–varying in time
  Weak Coupling Approximation:

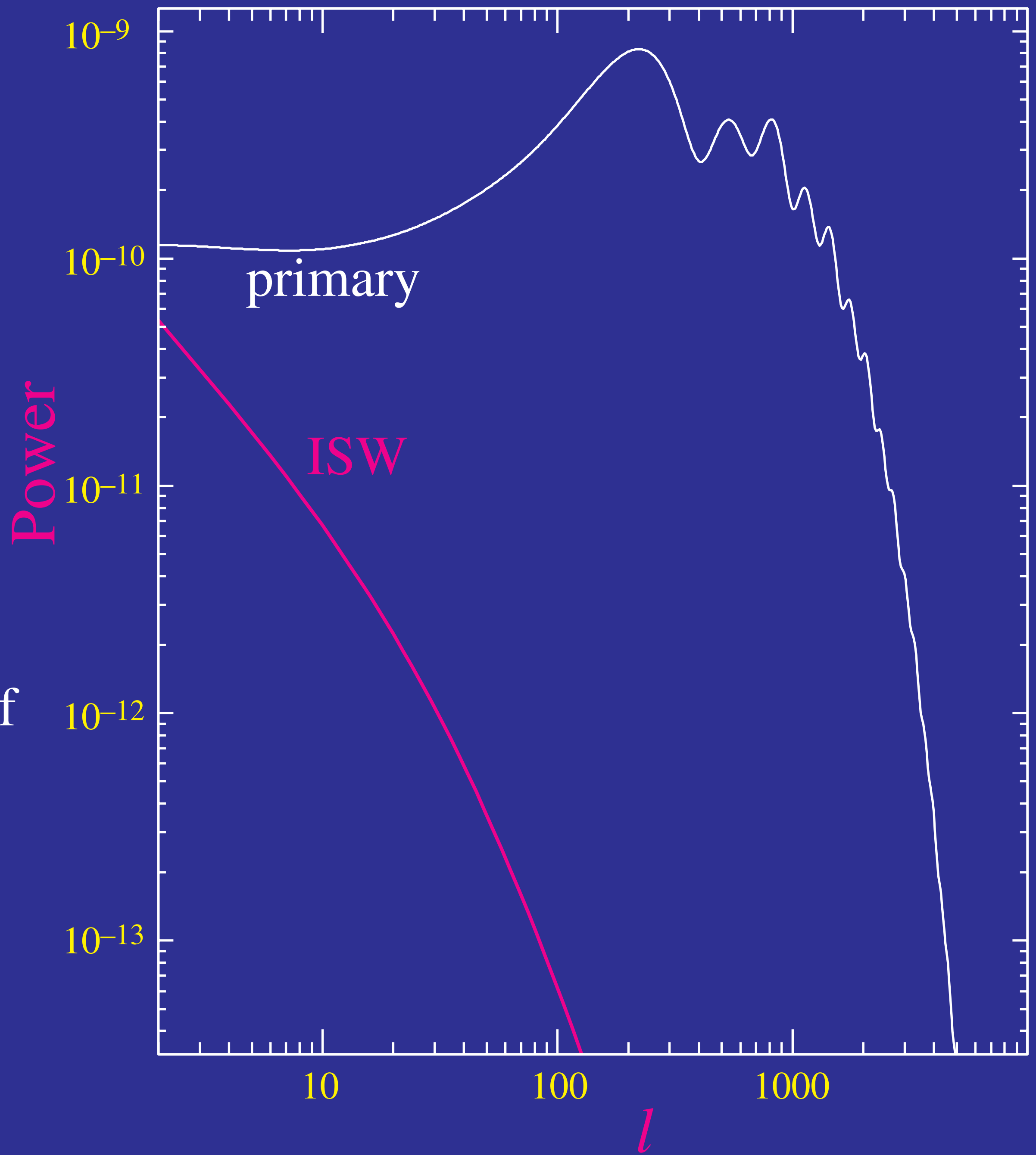
$$\begin{pmatrix}\text{multipole}\\\text{moment}\end{pmatrix} \sim \text{Source} \begin{pmatrix}\text{clebsch}\\\text{gordan}\end{pmatrix} \int \begin{pmatrix}\text{bessel}\\\text{function}\end{pmatrix} d\begin{pmatrix}\text{line of}\\\text{sight}\end{pmatrix}$$

- Log power spectrum of CMB ~ (cg)*Log power spectrum of source / l
- Scalar source and scalar field on sky: weak coupling = limber approx.

Hu & White (1996); Hu (2000)

ISW Effect in the Power Spectrum

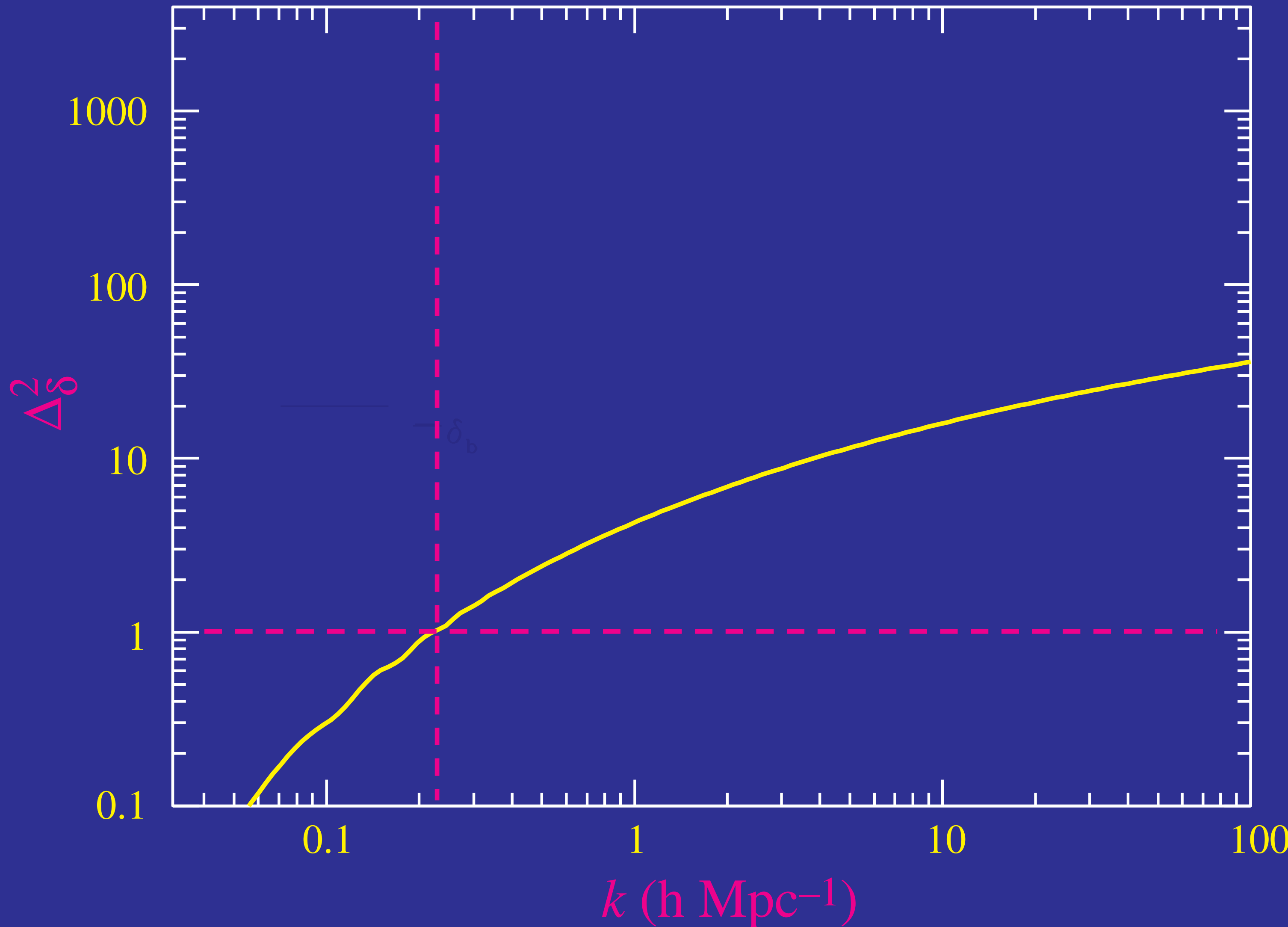
- ISW effect cancelled on small scales
- Barely affects the COBE normalization
- Cosmic variance limited in detectability
- But... a unique probe of dark energy
- Cross correlation and Higher order statistics



Into the
Non-Linear Regime

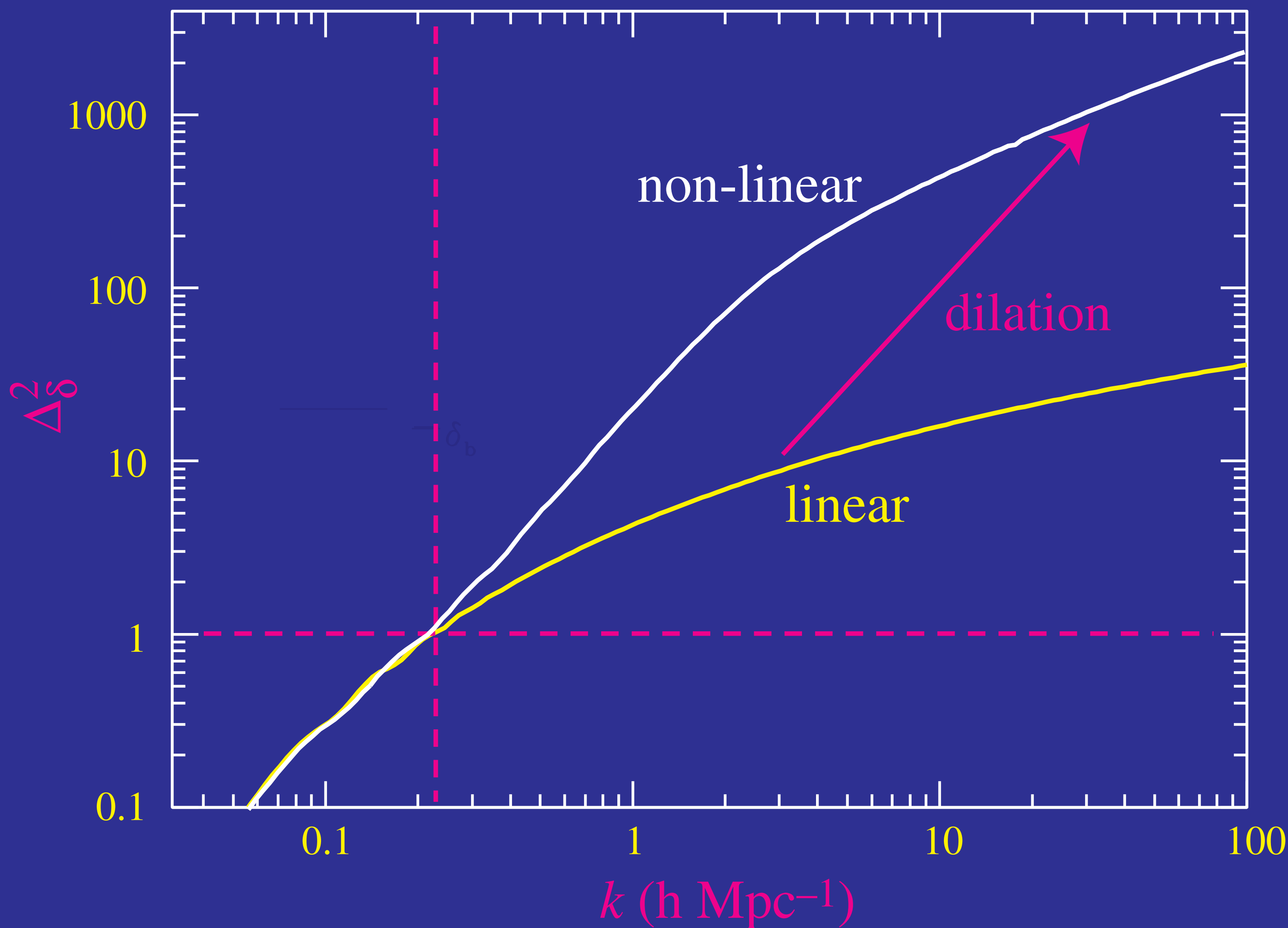
COBE Normalized Power Spectrum

- Non-linear scale at k ~ 0.2 *h*/Mpc



COBE Normalized Power Spectrum

- Fully non-linear power spectrum dilates scale and increases amplitude



HKLM / PD Scaling Relation

- Gravitational collapse implies that the density fluctuation at a given non-linear scale comes from a much larger region originally
- Particle number conserved so density enhancement must come from a change in volume:

$$k = [1+\Delta^2_\delta]^{1/3} k_{\rm lin}$$

- Ansatz: there is a universal mapping between the linear spectrum and non-linear spectrum

$$\Delta^2_\delta(k) = f_{\rm nl}[\Delta^2_\delta(k_{\rm lin})]$$

- Linear limit

$$f_{\rm nl}[x \ll 1] = x$$

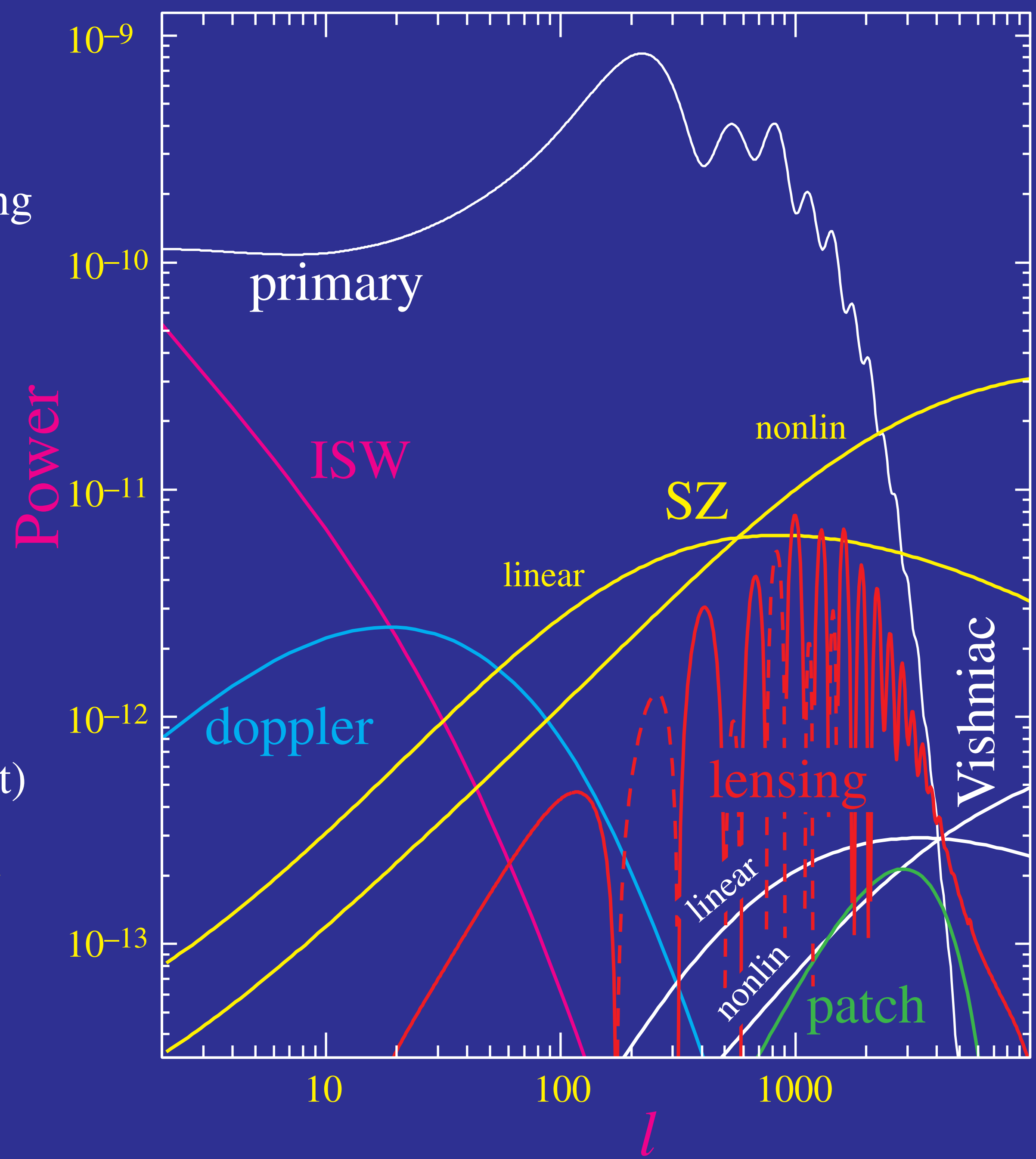
- Stable clustering limit in a flat matter dominated universe

$$f_{\rm nl}[x \gg 1] = x^{3/2}$$

if clustering is fixed in physical coordinates power scales with a^3

Secondary Anisotropies: Power Spectra

- Gravitational Effects
  - ISW Effect
    (redshift from decaying potentials)
  - Weak Lensing
    (smooths peaks and generates power <1')
- Scattering Effects
  - Doppler Effect
  - Vishniac Effect
    (LSS kinetic SZ effect)
  - Patchy Reionization
  - SZ effect
    (LSS thermal)



Baryon Suppression

Small-Scale CDM Perturbations

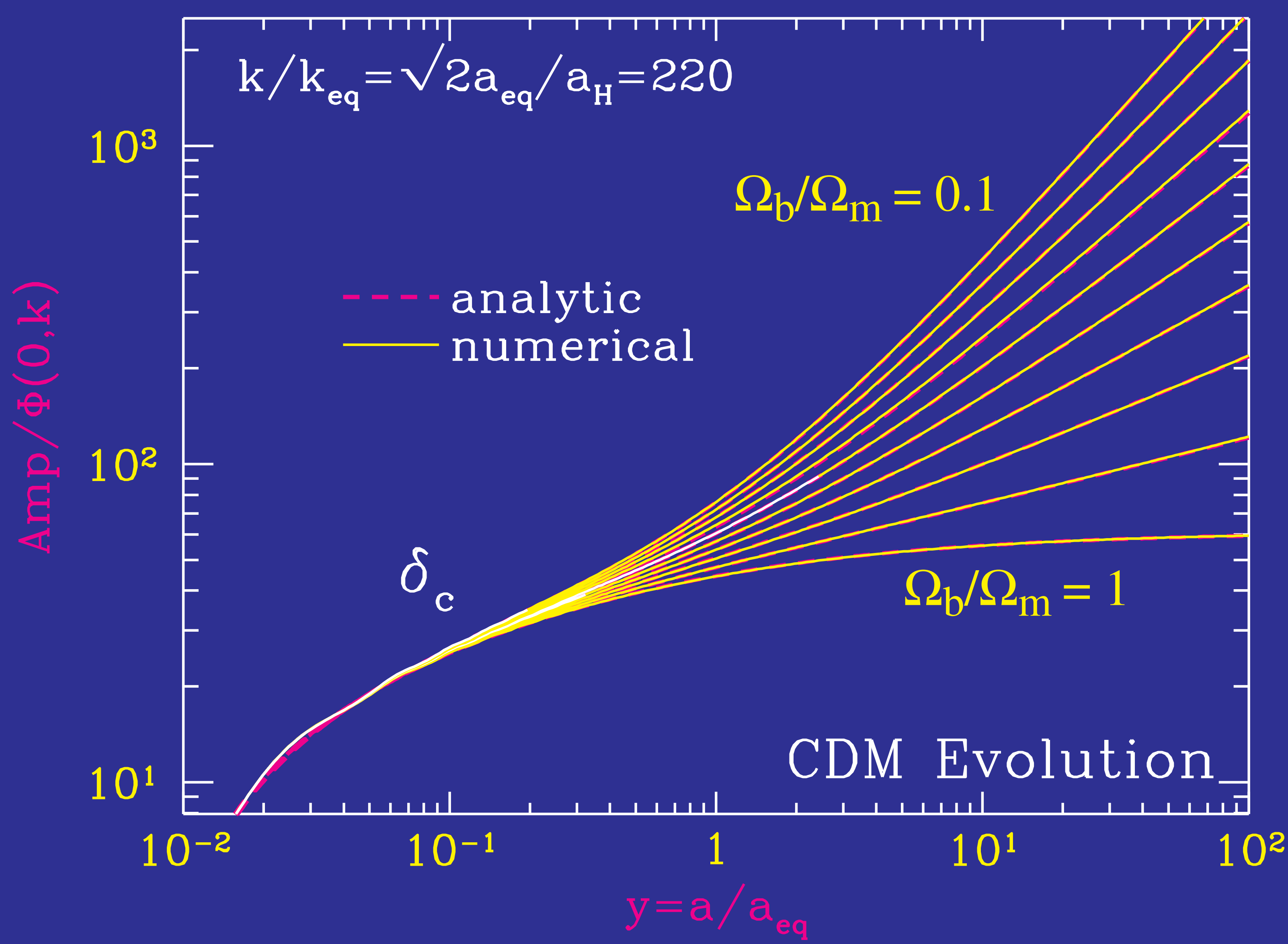
- Modes which enter the horizon during radiation domination
- CDM perturbations get boosted at horizon crossing by the decay of the gravitational potentials associated with radiation
- Enter into logarithmically growing mode due to the presence of a dominant smooth radiation background
- If baryons are dynamically neligible, linear growth begins when the universe becomes matter dominated
- If baryons are a substantial fraction of total matter, they act as a smooth matter background and suppress growth to

$$a^p, \qquad p = 1 - 3\Omega_b/5\Omega_m$$

- Likewise if there is a component of massive neutrinos

Growth Suppression from Baryons

- Before the end of the drag epoch, the smooth baryons suppress growth



Hu & Sugiyama (1996)

Compton Drag

- Momentum conservation in scattering causes a drag force on the baryons

- Relative momentum density $R=3\rho_b/4\rho_\gamma$ defines a drag rate related to the scattering rate by

$$\dot{\tau}_d = \dot{\tau}/R$$

- Compton drag epoch ends when

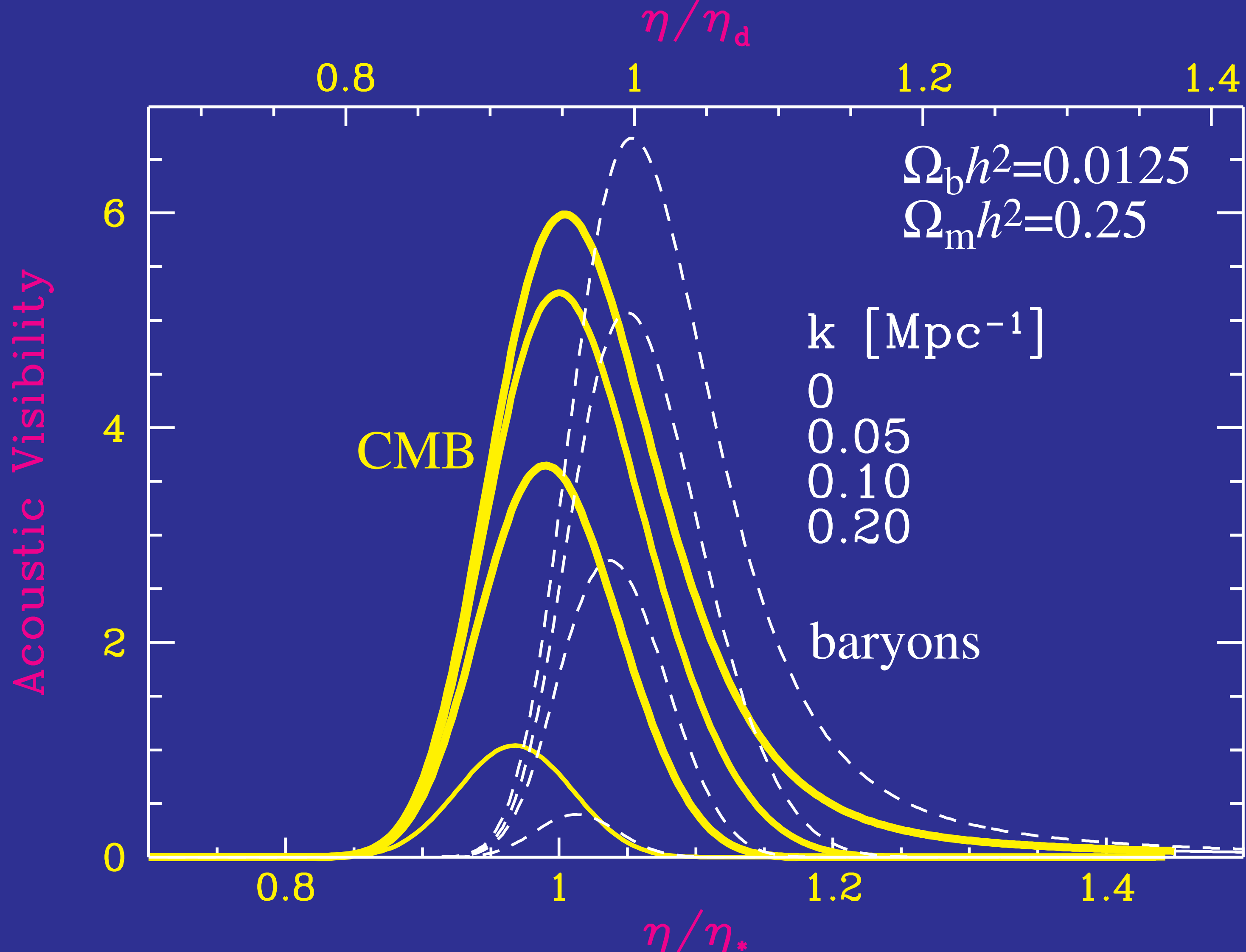
$$\tau_d(z_d) = 1$$

- "Visibility function" for the baryons

$$\dot{\tau}_d \, e^{-\tau_d}$$

Acoustic Visibility

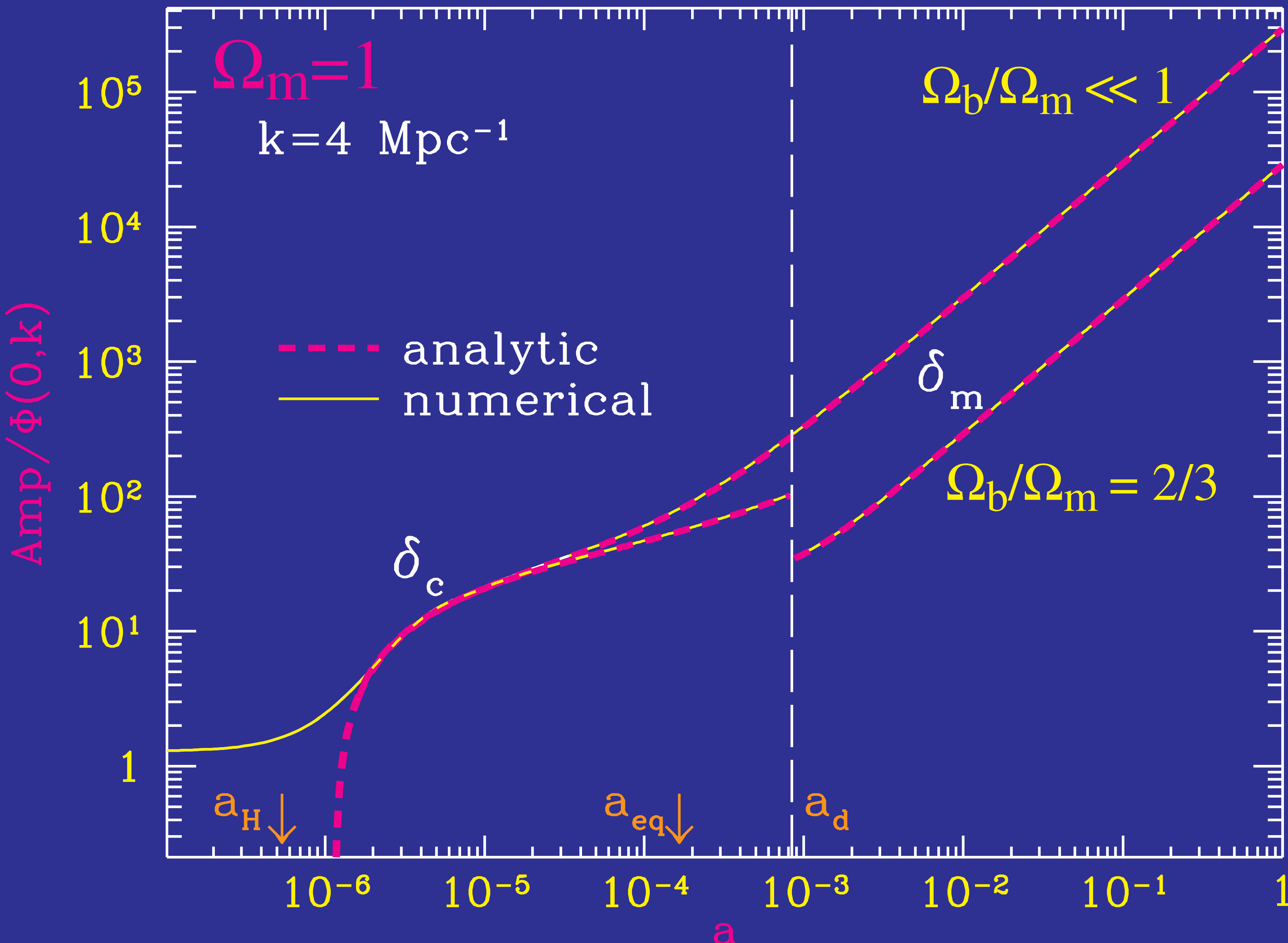
- Effective visibility for CMB and baryons accouting for damping



Hu & Sugiyama (1996)

Net Suppression from Baryons

- At the end of the drag epoch, match both onto linearly growing mode



Hu & Sugiyama (1996)

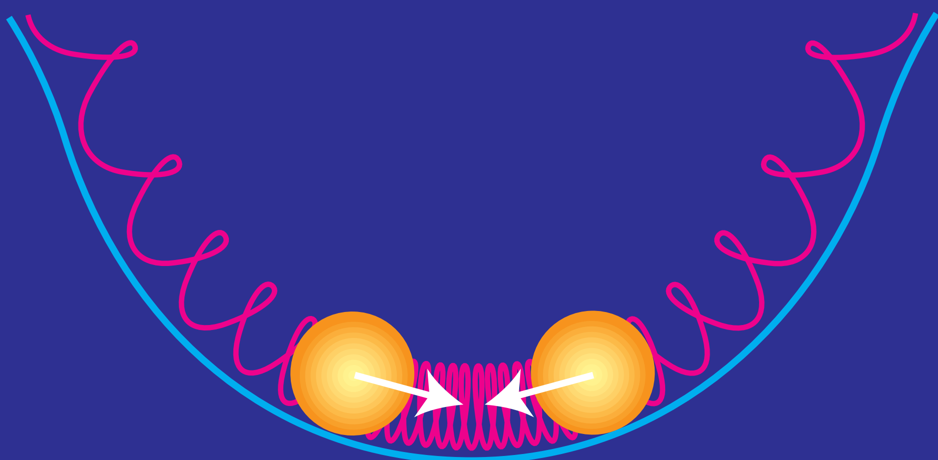
Baryons in the Transfer Function

- Substantial suppression of small scale power
- Appearance of oscillations at high baryon fractions

Baryon Wiggles

Acoustic Peaks in the Matter

- Baryon density & velocity oscillates with CMB
- Baryons decouple at $\tau / R \sim 1$, the end of Compton drag epoch
- Decoupling: δ_b(drag) $\sim V_b$(drag), but not frozen

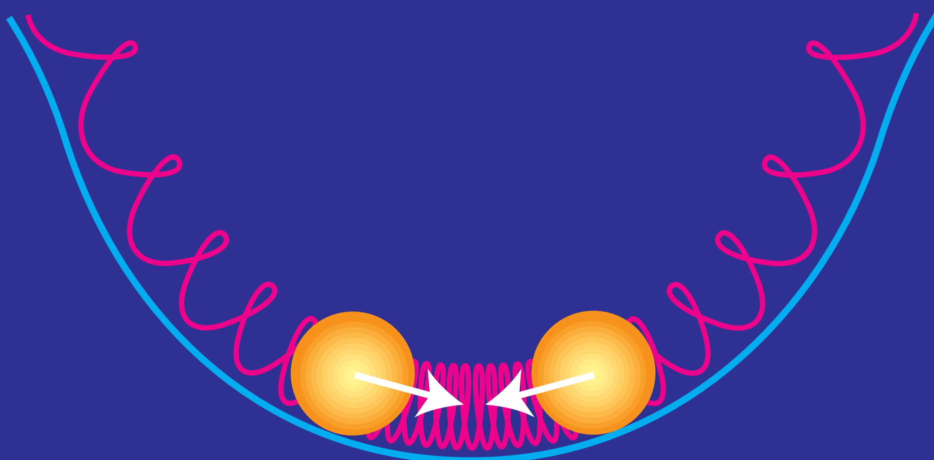


End of Drag Epoch

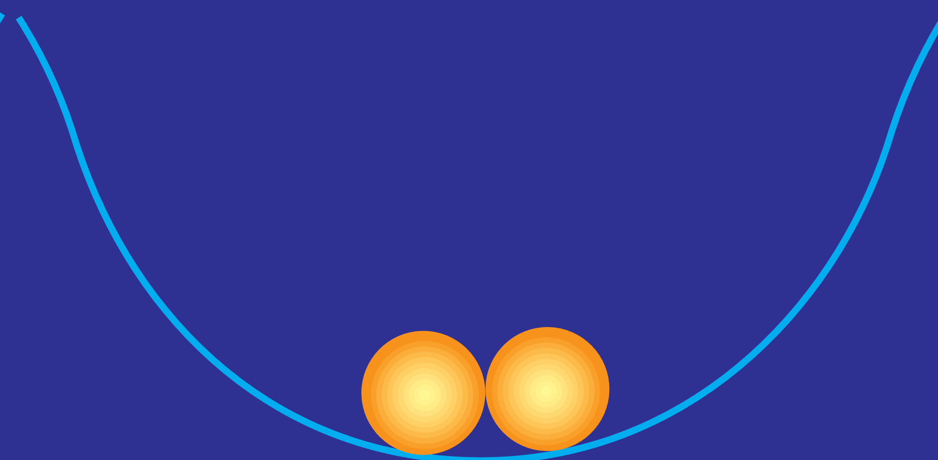
Hu & Sugiyama (1996)

Acoustic Peaks in the Matter

- Baryon density & velocity oscillates with CMB
- Baryons decouple at $\tau / R \sim 1$, the end of Compton drag epoch
- Decoupling: $\delta_b(\text{drag}) \sim V_b(\text{drag})$, but not frozen
- Continuity: $\dot{\delta}_b = -kV_b$
- Velocity Overshoot Dominates: $\delta_b \sim V_b(\text{drag})\, k\eta \gg \delta_b(\text{drag})$
- Oscillations $\pi/2$ out of phase with CMB
- Infall into potential wells (DC component)



End of Drag Epoch

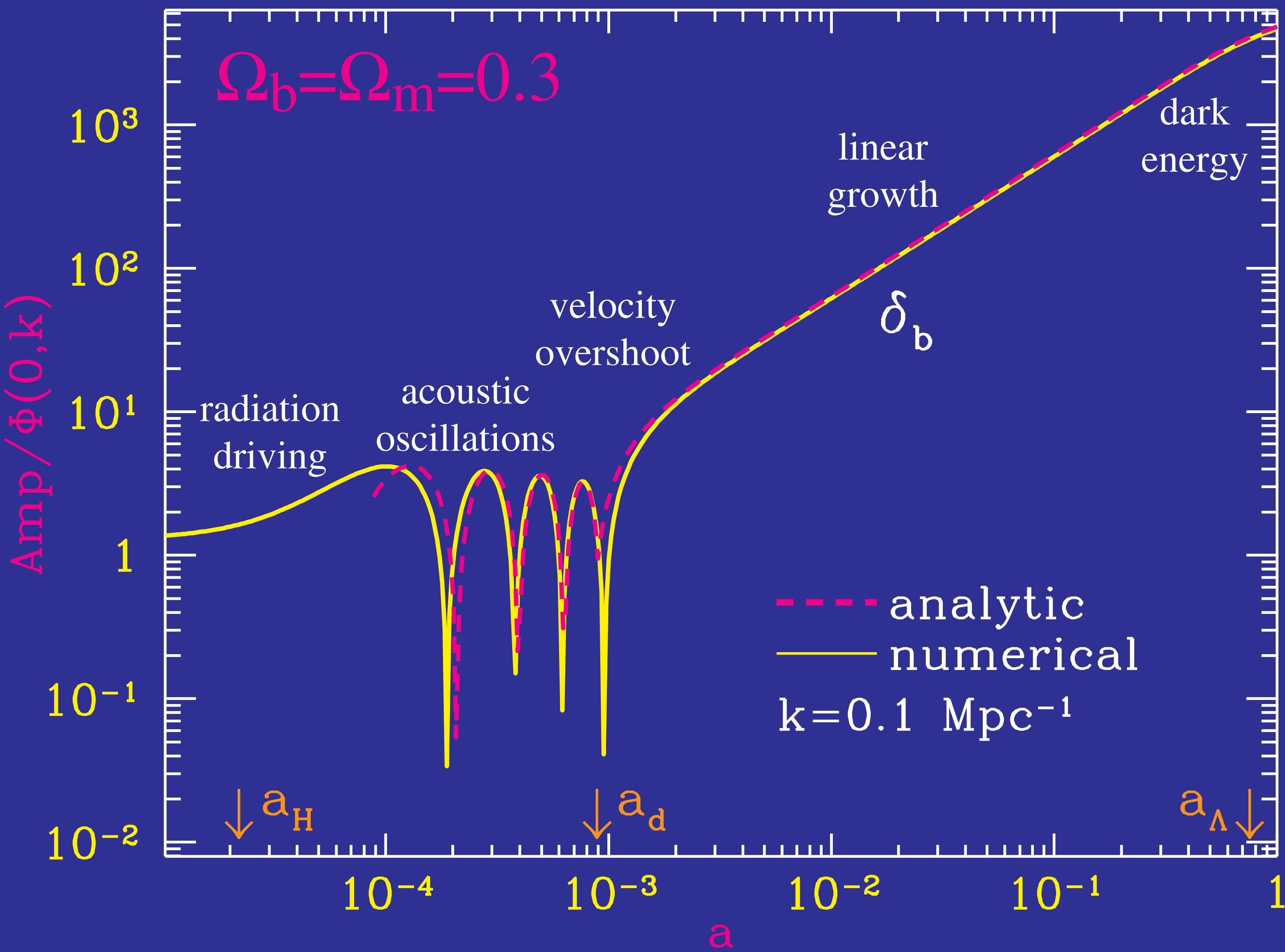


Velocity Overshoot + Infall

Hu & Sugiyama (1996)

Velocity Overshoot

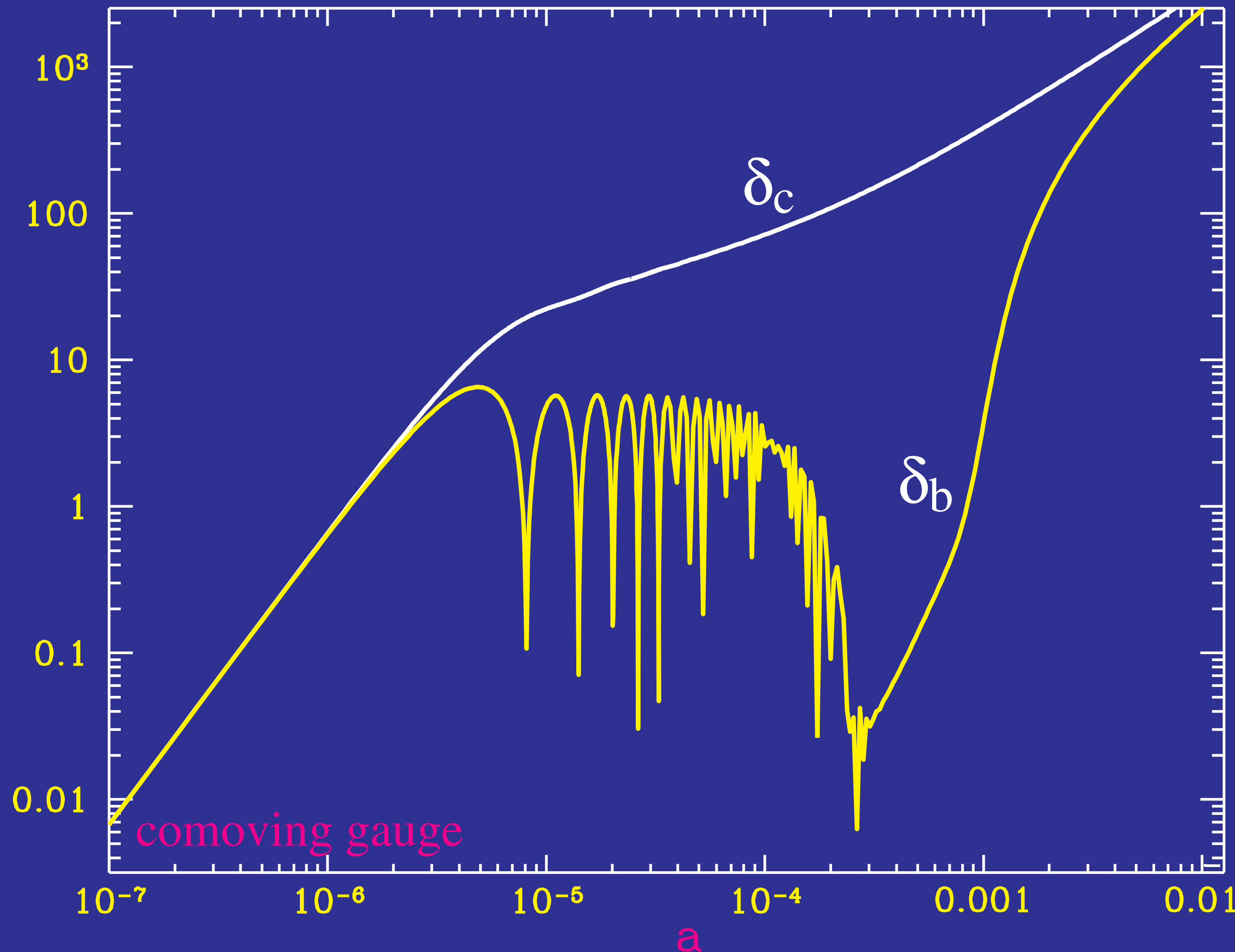
- Time evolution for baryon only universe



Hu & Sugiyama (1996)

Infall into CDM Wells

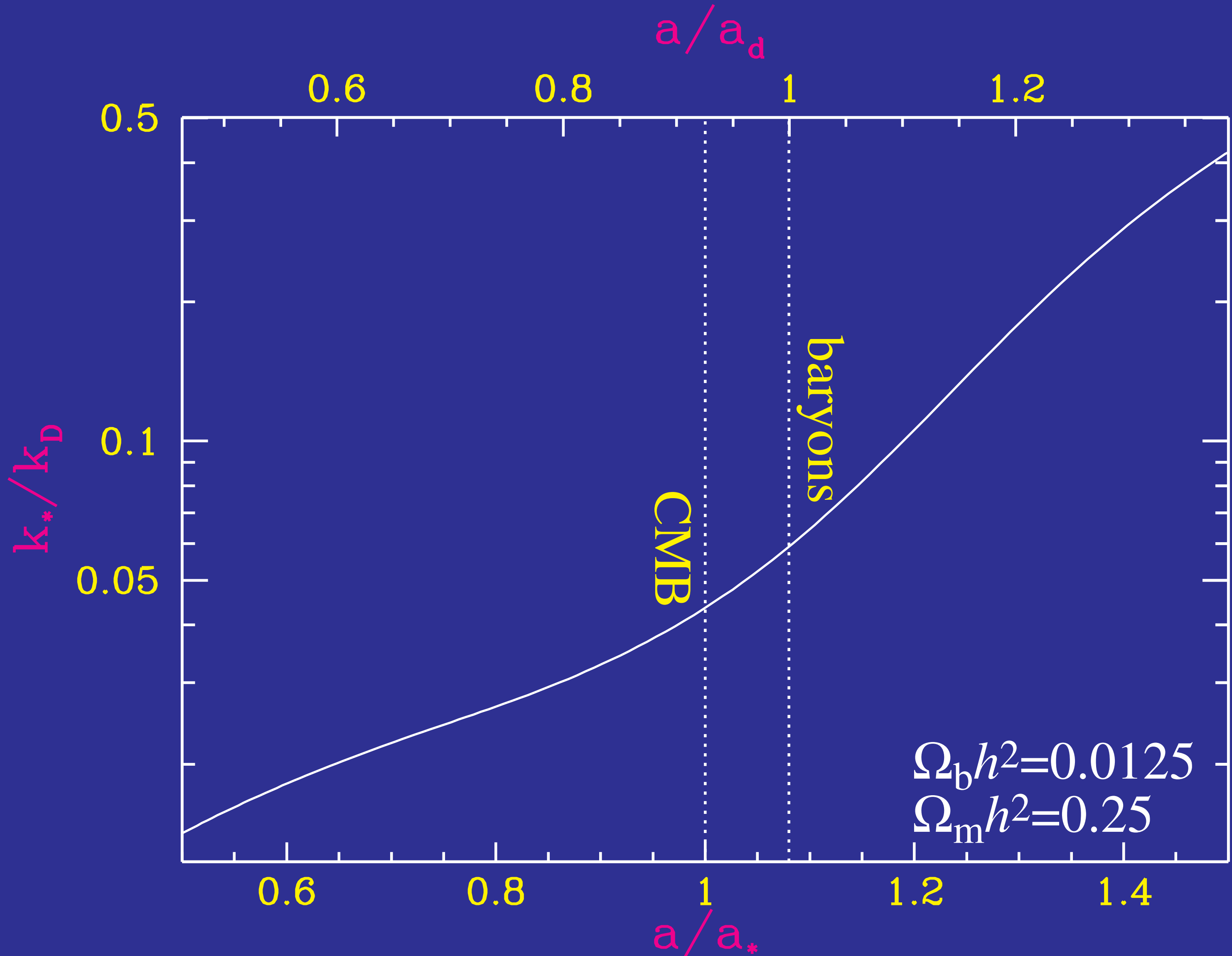
- Infall into CDM potential wells after the drag epoch



Hu & Sugiyama (1996)

Time Evolution

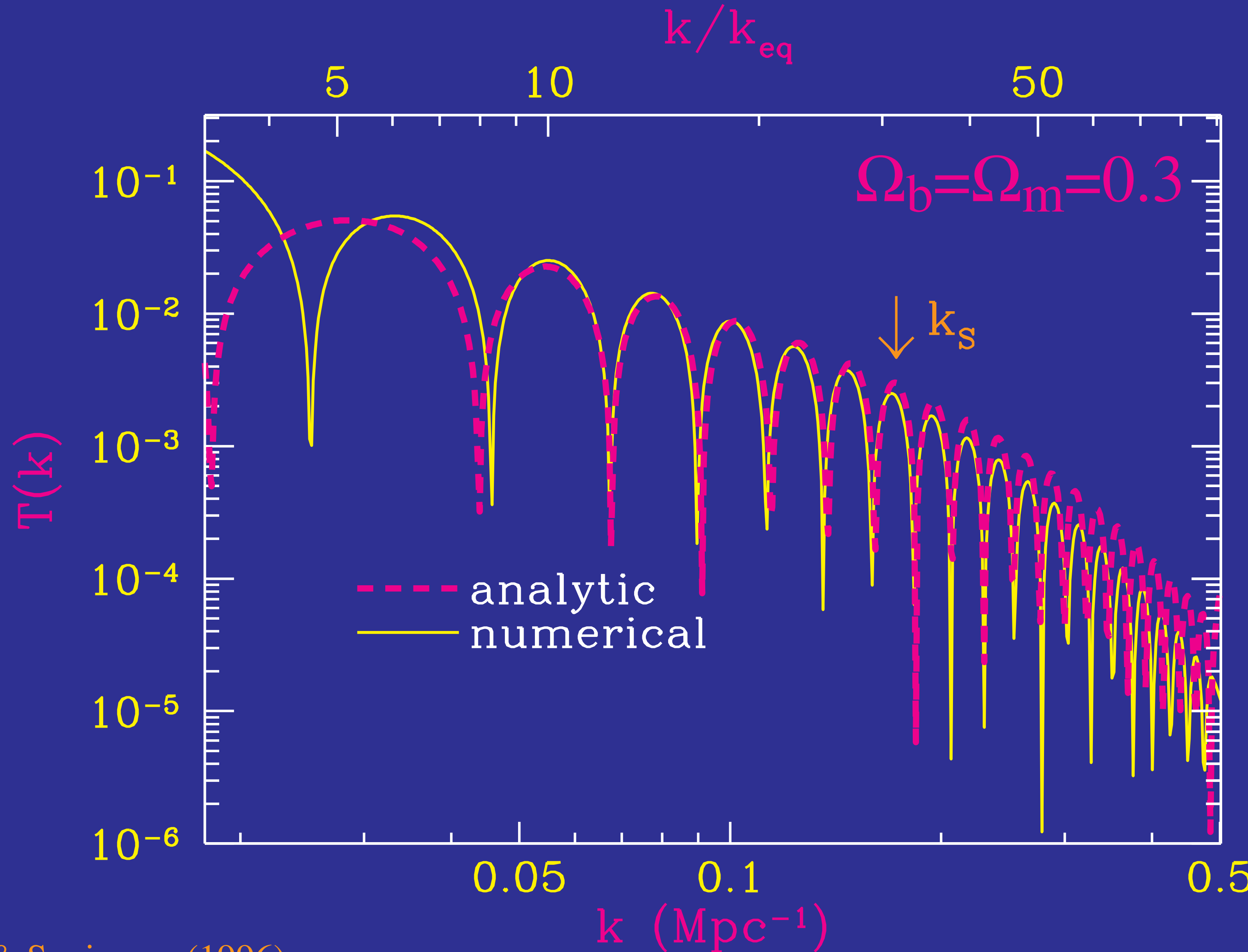
- Diffusion length compared with horizon at recombination



Hu & Sugiyama (1996)

Oscillations in the Transfer Function

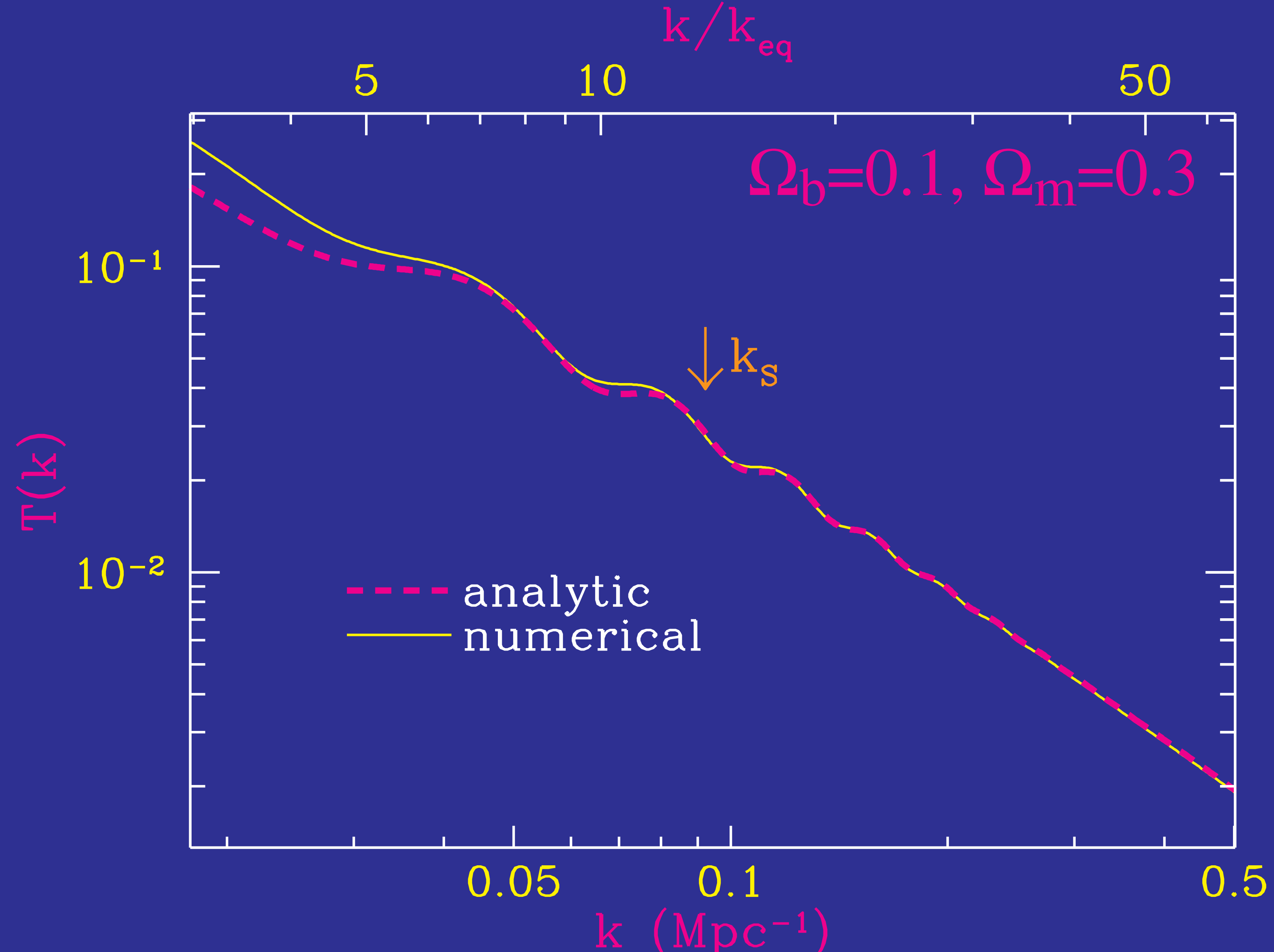
- Transfer function in a baryon only universe



Hu & Sugiyama (1996)

Wiggles in the Transfer Function

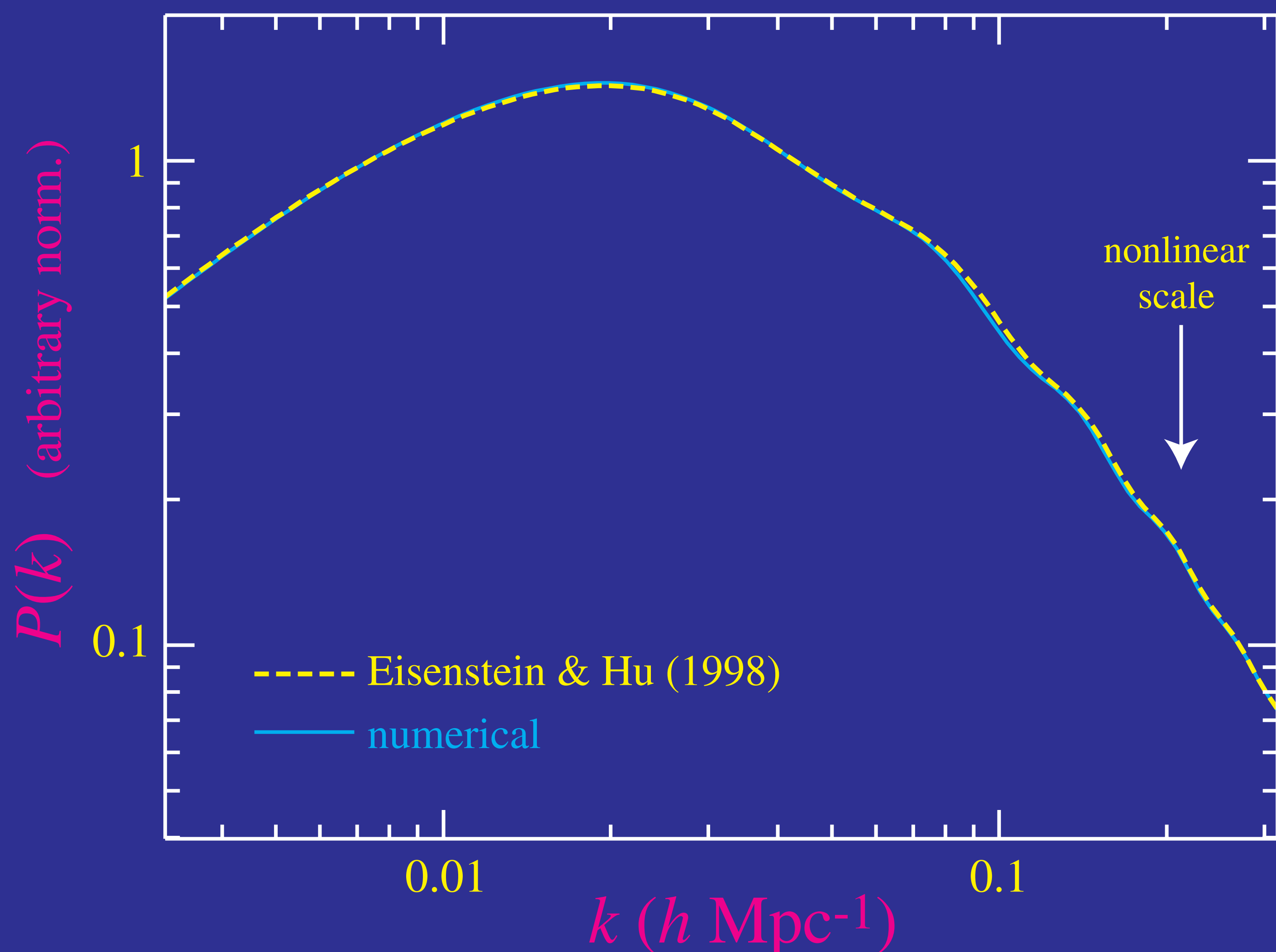
- Transfer function in a CDM dominated universe (f_b=1/3)



Hu & Sugiyama (1996)

Features in the Power Spectrum

- Features in the linear power spectrum
- Break at sound horizon
- Oscillations at small scales; washed out by nonlinearities



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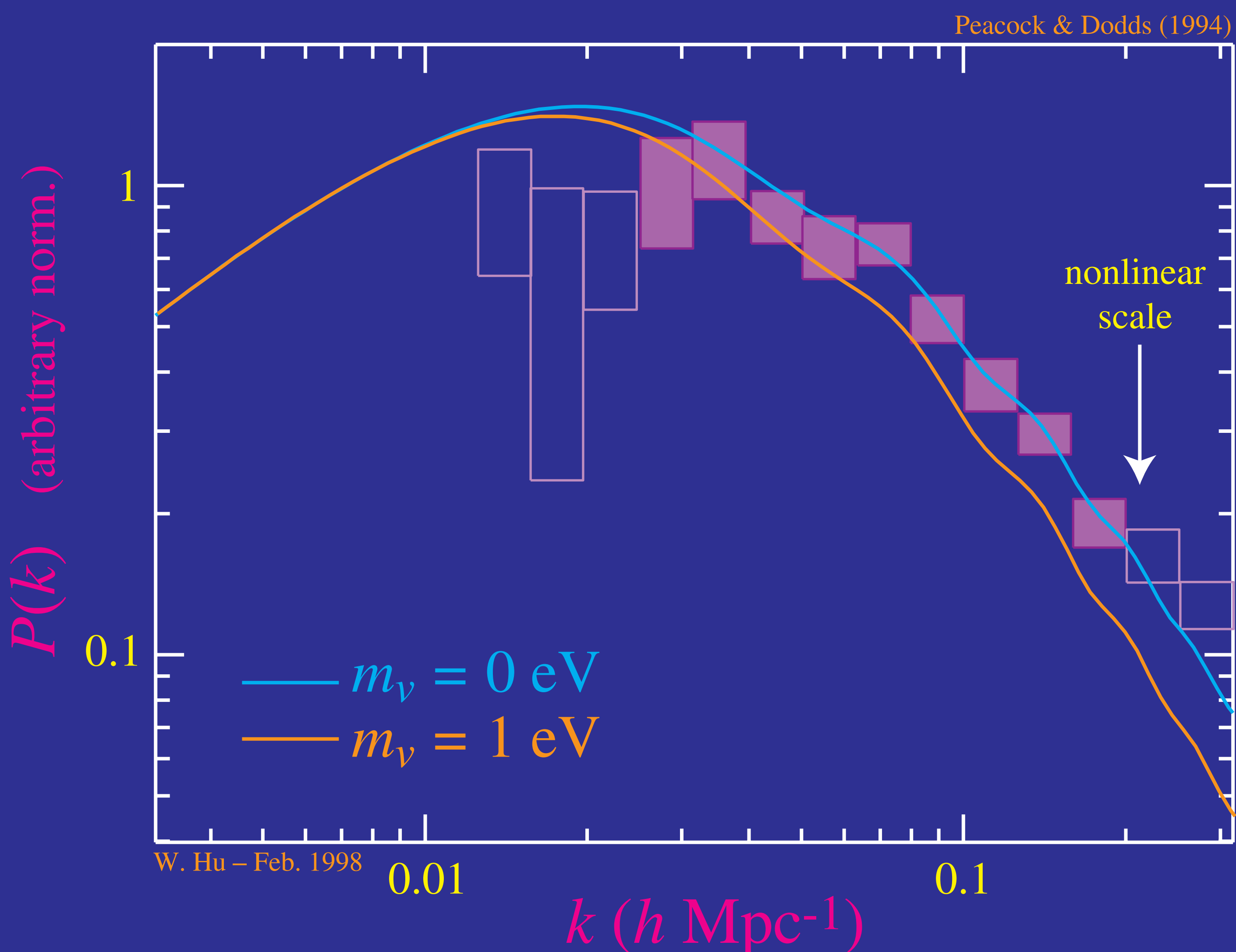

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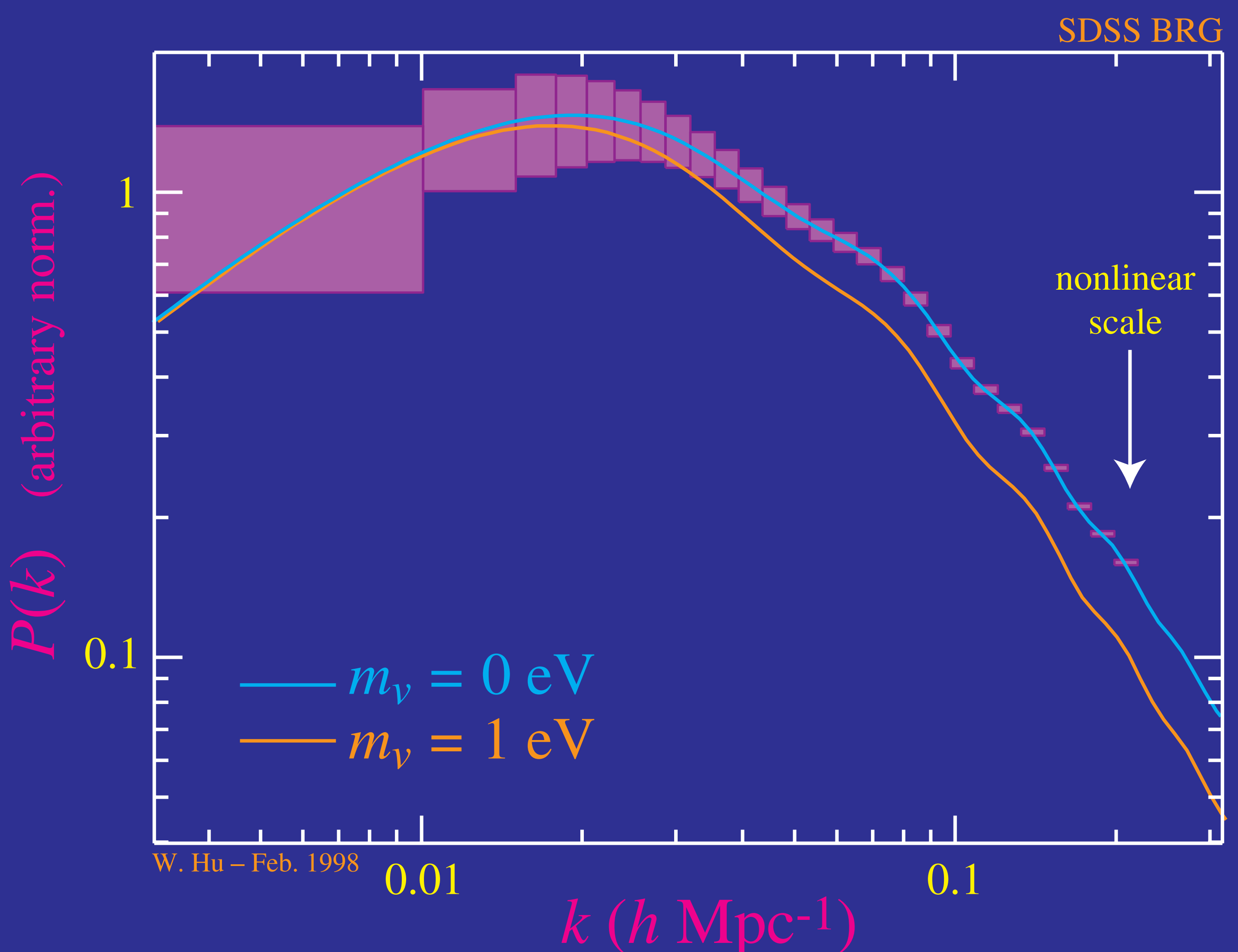
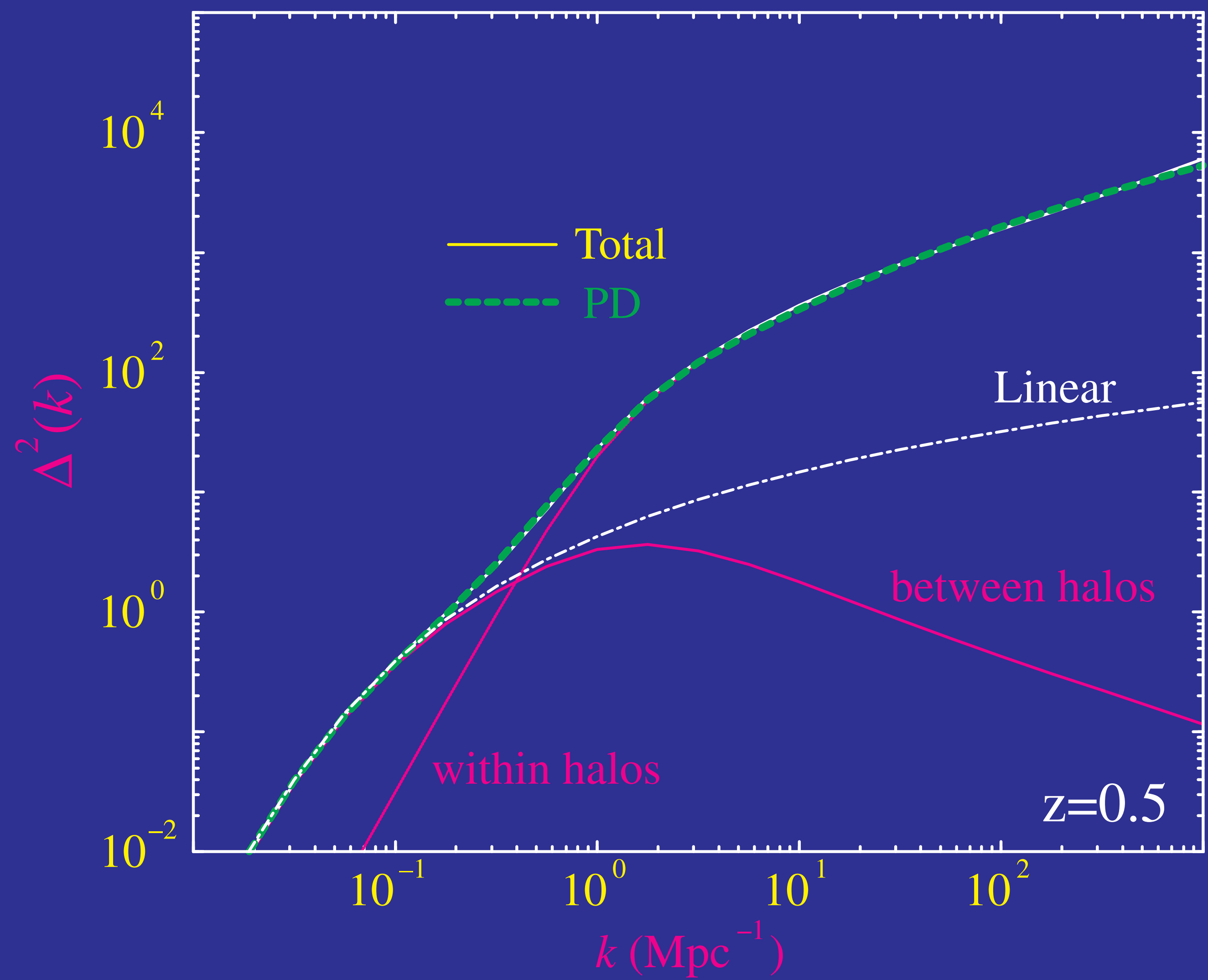


Baryon Wiggles in Non-Linear Regime

- Mode coupling washes out features in the initial power spectrum
- (HKLM/PD mapping fails to describe this effect!)
- Relationship between dark matter and galaxies ("bias") non-linear
- Better: think of the dark matter as being comprised of discrete virialized halos: the halo model
- DM power spectrum = correlations within halos + correlations between halos
- Ingredients: halo number density (Press-Schechter)
  halo profiles (NFW)
  halo bias (Mo & White)
  linear power spectrum (cosmology)
- Galaxy power spectrum modeled by assigning galaxies to halos

Halo Model of the Power Spectrum

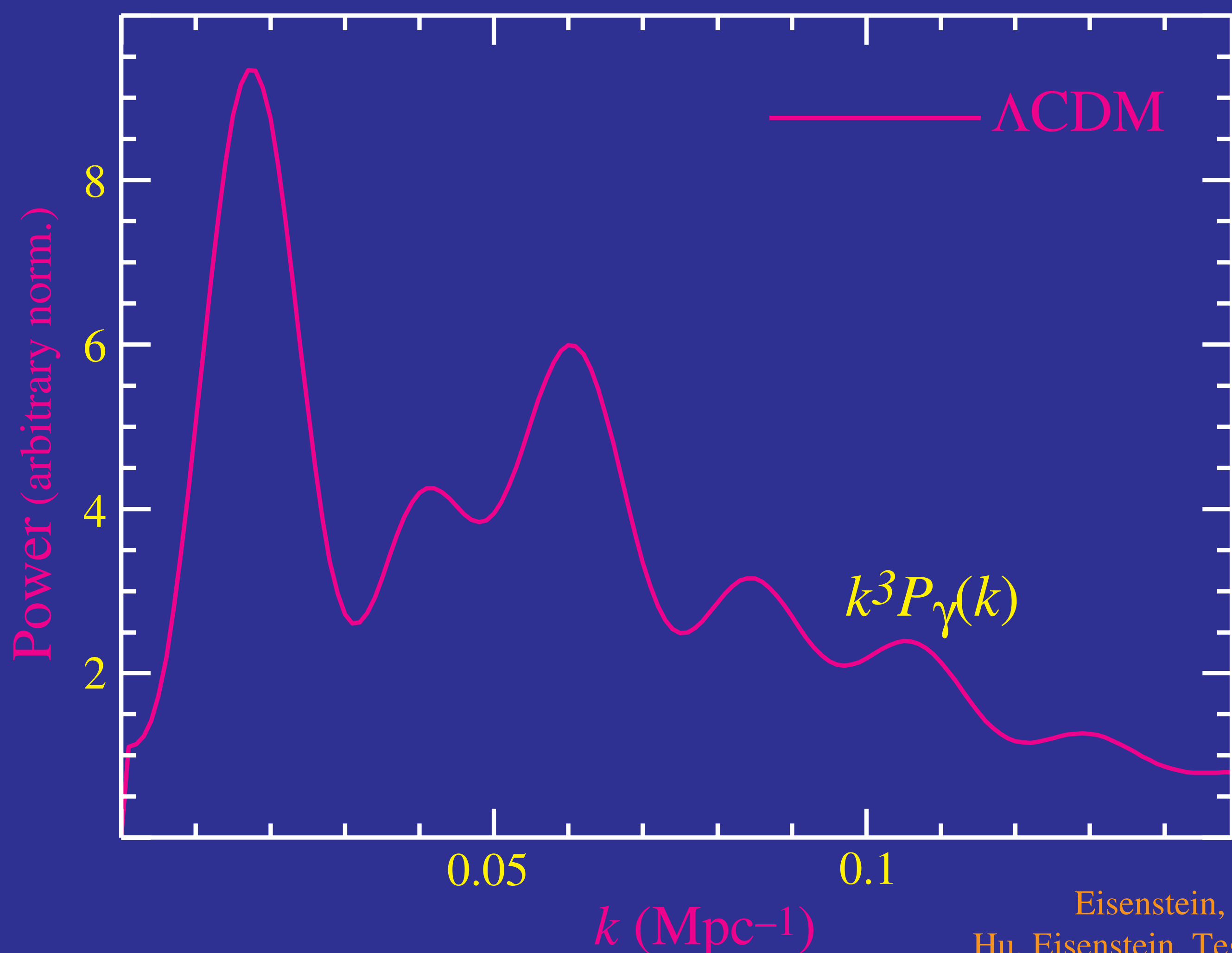


Seljak (2000); figure from Cooray & Hu (2000)

Complementarity

Combining Features in LSS + CMB

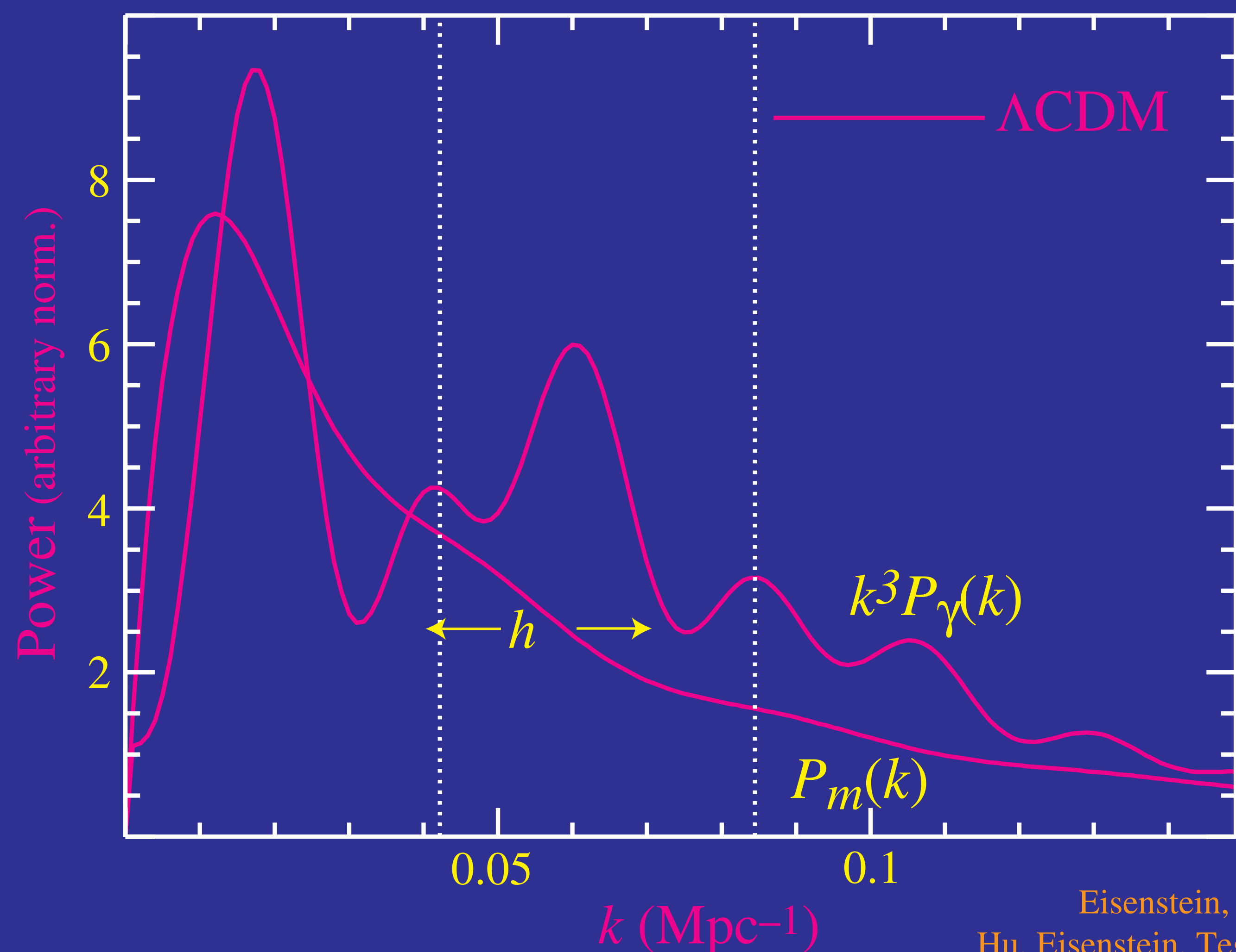
- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale $l_{\rm peak}$(CMB) → $k_{\rm peak}$(LSS) in $\rm Mpc^{-1}$



Eisenstein, Hu & Tegmark (1998)
Hu, Eisenstein, Tegmark & White (1998)

Combining Features in LSS + CMB

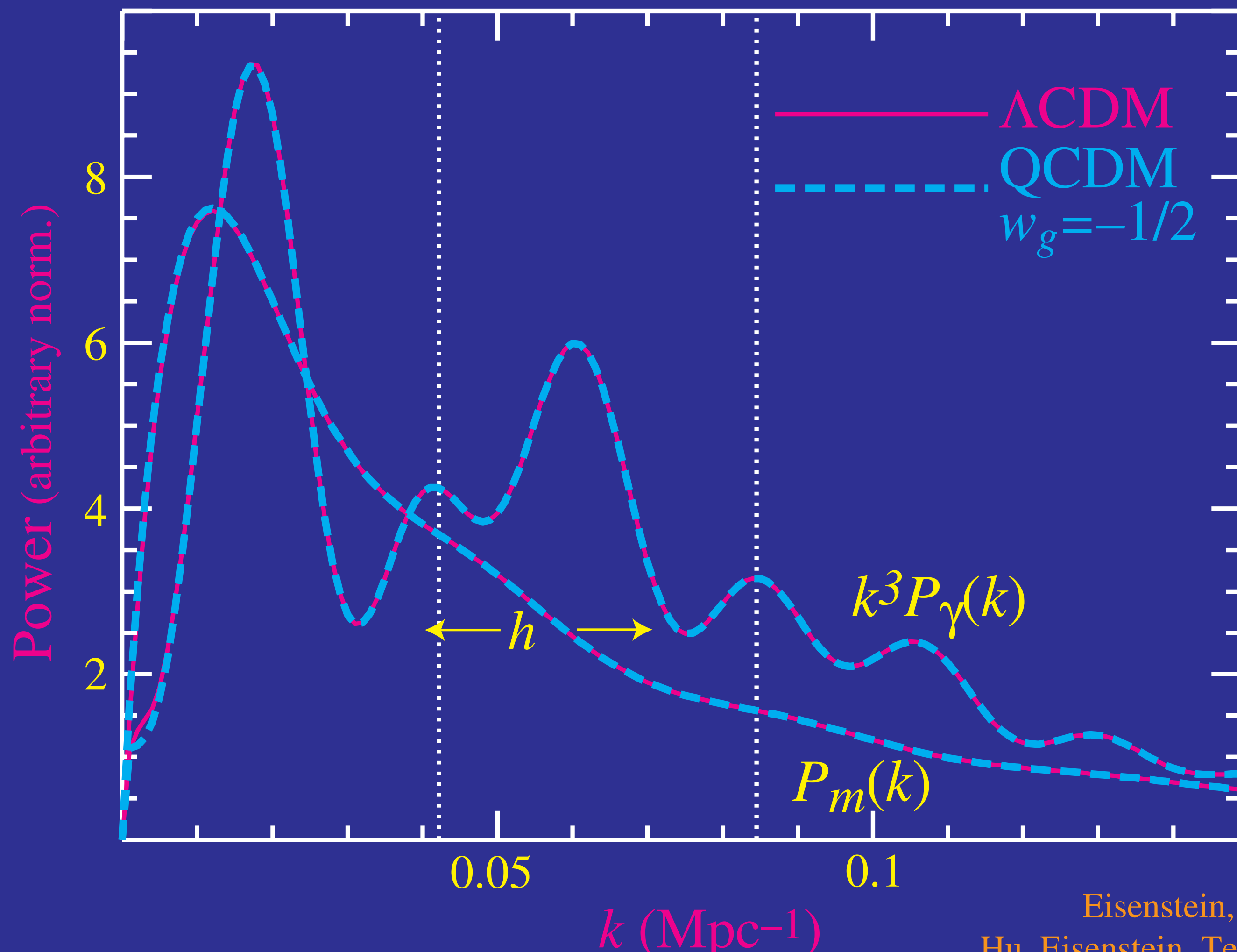
- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale $l_{\rm peak}$(CMB) $\rightarrow$ $k_{\rm peak}$(LSS) in $\rm Mpc^{-1}$
- Measure in redshift survey $k_{\rm peak}$(LSS) in $h\ \rm Mpc^{-1}$ $\rightarrow$ h



Eisenstein, Hu & Tegmark (1998)
Hu, Eisenstein, Tegmark & White (1998)

Combining Features in LSS + CMB

- Consistency check on thermal history and photon–baryon ratio
- Infer physical scale l_{peak}(CMB) → k_{peak}(LSS) in Mpc^{-1}
- Measure in redshift survey k_{peak}(LSS) in $h\ \mathrm{Mpc}^{-1}$ → h
- Robust to low redshift physics (e.g. quintessence, GDM)



Eisenstein, Hu & Tegmark (1998)
Hu, Eisenstein, Tegmark & White (1998)

Classical Cosmology

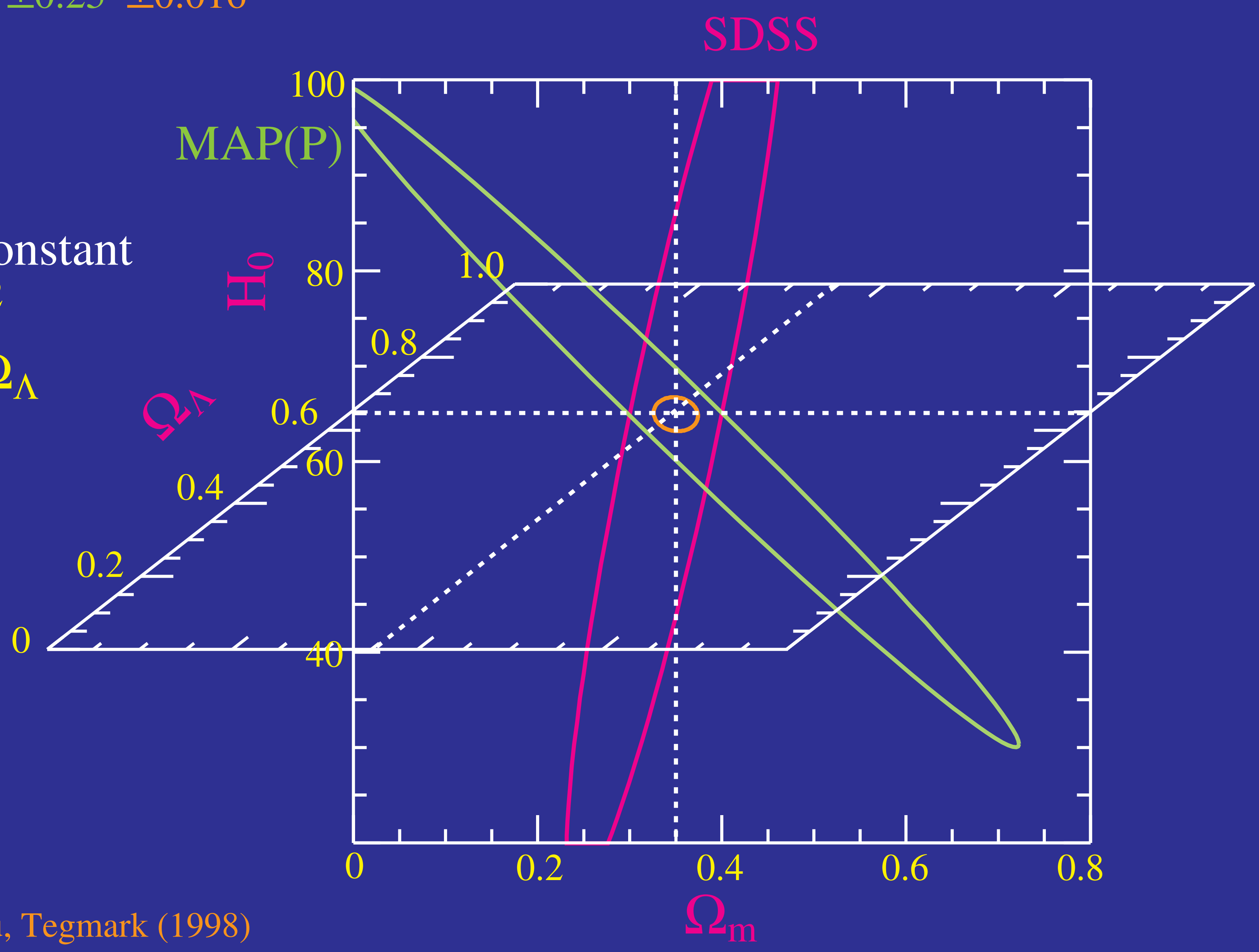
	MAP	+P	+SDSS
H_0	±130	±23	±1.2
Ω_m	±1.4	±0.25	±0.016

CMB:

~line of constant

$\Omega_m H_0^2$

$\Omega_m + \Omega_\Lambda$

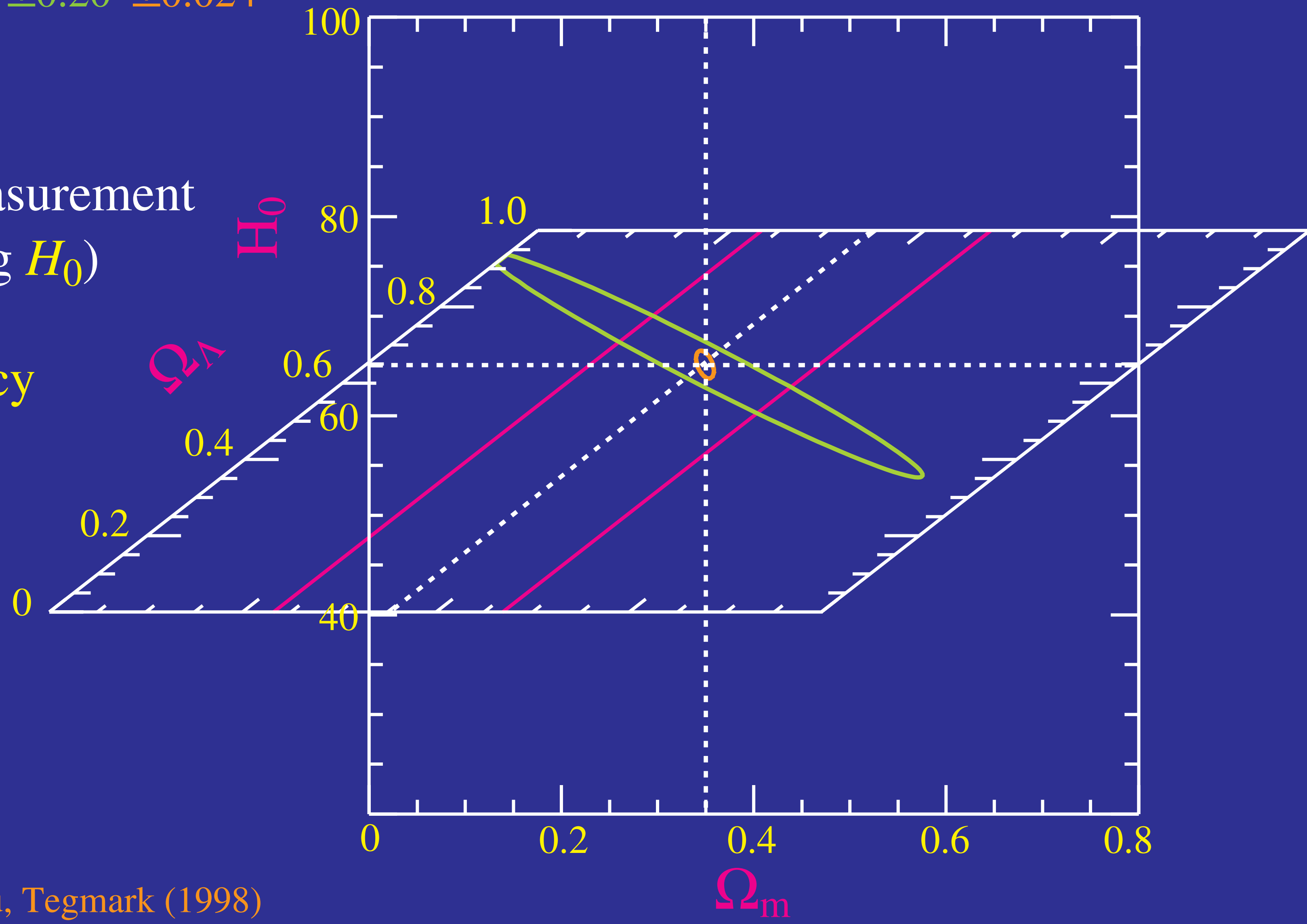


Eisenstein, Hu, Tegmark (1998)

Classical Cosmology

	MAP	+P	+SDSS
H_0	±130	±23	±1.2
Ω_m	±1.4	±0.25	±0.016
Ω_Λ	±1.1	±0.20	±0.024

Any
other measurement
(including H_0)
breaks
degeneracy

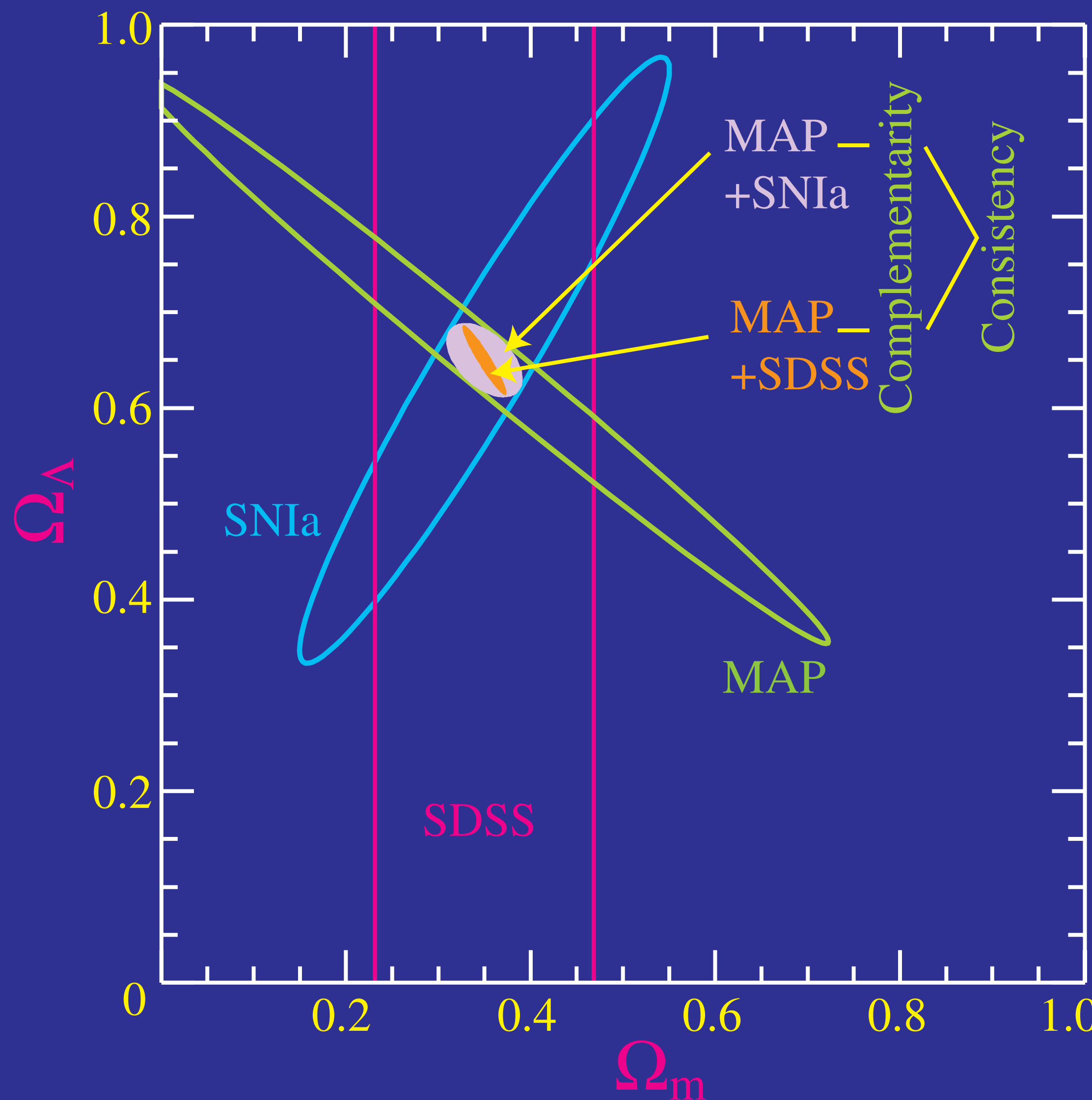


Eisenstein, Hu, Tegmark (1998)

	MAP	+P	+SDSS
H_0	±130	±23	±1.2
Ω_m	±1.4	±0.25	±0.016
Ω_Λ	±1.1	±0.20	±0.024

Classical Cosmology

Many opportunities
for consistency checks!
(e.g. high-z SNIa)



Eisenstein, Hu, Tegmark (1998)