Astro 448: Problem Set 3

Due Feb. 7

1. Slow Roll Relations

Defining the ϵ slow-roll parameter

$$\epsilon = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 \tag{1}$$

show that

$$\frac{\dot{\epsilon}}{\epsilon} = 2\frac{\dot{a}}{a}(\epsilon + \delta) \tag{2}$$

where the δ slow-roll parameter is

$$\delta = \epsilon - \frac{1}{8\pi G}\frac{V''}{V}\,. \tag{3}$$

We used this relationship in defining the scalar tilt.

2. Chaotic Inflation and Axion Dark Matter

Consider polynomial chaotic inflation where $V = m^2\phi^2/2$.

- Write down ϵ and δ. Inflation will occur if the initial field $\phi_0(0) = \phi_i$ meets what conditions?
- Write down the slow roll equation in coordinate time ($d^2\phi_0/dt^2 = 0$; $\delta \ll 1$) with $H(\phi)$ ($\epsilon \ll 1$) evaluated with the Friedmann equation.
- Solve for $\phi_0(t)$.
- Solve for $a(t)$ using the $H(\phi)$ relation and assume $a(t=0) = a_i$.
- Write down the curvature power spectrum Δ^2_ζ and Δ^2_h for this model.
- Consider the case where the slow-roll conditions are violated. Specifically, solve for $\phi_0(t)$ in the case where the expansion drag term can be neglected. Write down $\rho_\phi(t)$ and $p_\phi(t)$. What is the time averaged equation of state in this case? An axion behaves in this manner.

If in addition to the inflaton field, an axion field is present during inflation, it will also carry quantum fluctuations. Since the axion carries a negligible energy density during inflation these do not imprint a curvature fluctuation and are hence known as isocurvature perturbations. More specifically, assuming that the axions are the dark matter, these isocurvature fluctuations represent a perturbation in the local number density of dark matter particles to radiation or an "entropy fluctuation". Note that the entropy perturbation is also scale-invariant for the same reason the curvature fluctuations are.