

• Consequences : Stress free

$\int' = 0$  if stress gradients negligible and no smooth components (curvature is only component smooth by defn others can be approximately smooth due to stress gradients preventing collapse)

$\Rightarrow \int = \text{const}$  Bardeen curvature conserved

$\Phi + \Psi = S_{\pi} \equiv -8\pi G a^2 \rho \pi / k^2$  shorthand for ans. stress Einstein Eqs

$\Rightarrow \Phi = \underbrace{\left(1 - \frac{\rho}{a} \int \frac{da}{\rho}\right)}_{\text{if } \rho \propto a^{-3(1+w)}} \int + \underbrace{\frac{\rho}{a} \int \frac{da}{\rho} S_{\pi}}_{\text{anisotropic stress}} + \underbrace{C \frac{\rho}{a}}_{\text{decaying mode}}$

$= \left(1 - \frac{2}{5+3w}\right) \int$

$\Phi = \frac{3+3w}{5+3w} \int$   $\Rightarrow$   $\omega = 1/3 \Rightarrow \Phi = \frac{2}{5} \int$   
 $\omega = 0 \Rightarrow \Phi = \frac{3}{5} \int$   
 $\omega \rightarrow -1 \Rightarrow \Phi \rightarrow 0$

$\Phi = 4\pi G a^2 \delta\rho$   
 $\delta\rho \propto a^{-2}$   
 $\frac{\delta\rho}{\rho} \propto a^{-2+3(1+w)}$   
 $\propto a^{1+3w}$

$\Rightarrow \frac{\delta\rho}{\rho}$  grows at a rate to keep potential const (as  $k \rightarrow 0$ )

even for  $-1/3 > w > -1$  "dark energy" if  $w \neq -1$  dark energy cannot be perf. smooth

• Why is  $\Phi$  constant?

• Consider an initial  $\Phi_i = -\Psi_i$

• Generates a potential flow

$$\dot{v} \sim -k\Psi_i$$

$$v \sim -(k\eta)\Psi_i$$

• Generates a density perturbation

$$(\delta\rho)' \sim -(\rho+p)kv$$

$$\delta\rho \sim -(k\eta)^2(\rho+p)\Psi_i$$

• Generates a curvature perturbation

$$k^2\Phi = 4\pi G a^2 \delta\rho$$

$$= -4\pi G a^2 (k\eta)^2(\rho+p)\Psi_i$$

$$\eta = \int \frac{da}{a^2 H} = \frac{1}{aH}$$

$$\eta^2 \approx \frac{1}{a^2 H^2} \approx \frac{3}{8\pi G} \frac{1}{\rho a^2}$$

$$\Phi \sim -\Psi_i \quad \text{remains the same}$$

• If stress gradients oppose gravity

$$v < -(k\eta)\Psi_i$$

and potential/curvature decays

## • Newtonian Density Perturbation

$$\Phi \sim \frac{1}{(k\eta)^2} \left. \frac{\delta\rho}{\rho} \right|_{\text{comoving}}$$

$$\Phi \gg \left. \frac{\delta\rho}{\rho} \right|_{\text{comoving}} \quad \text{if } (k\eta) \ll 1 \quad (\text{superhorizon})$$

$\therefore$  comoving density perturbation and pressure perturbations negligible outside horizon NOT TRUE FOR NEWTONIAN

$$\bar{\Psi} \gg \left. \frac{-\delta\rho}{\rho + \bar{\rho}} \right|_{\text{comoving}} = \delta$$

relate by gauge transformation A

$$\Psi = \delta\eta - \dot{T} - \frac{\dot{a}}{a} T$$

$$T = -v/k$$

$$= \frac{\dot{v}}{k} + \frac{\dot{a}}{a} \frac{v}{k}$$

$$v|_{\text{comov}} = v|_{\text{newt}}$$

$$a\bar{\Psi} = \frac{d}{d\eta} \left[ \frac{av}{k} \right]$$

$$\frac{av}{k} = \int (a\bar{\Psi}) d\eta = \bar{\Psi} | dt$$

$$\frac{v}{k} = \frac{\bar{\Psi} t}{a}$$

$$\therefore \delta\eta = \frac{\delta t}{a} \quad \frac{\delta t}{t} = -\bar{\Psi}$$

gauge transformation follows from  $\bar{\Psi}$  as a time-time perturbation

$$\begin{aligned}
 \delta \rho_i |_{\text{Newt}} &= \delta \rho_i |_{\text{comov}} - \dot{\rho}_i T \\
 &= 3(1+w_i) \rho_i T \frac{\dot{a}}{a} \\
 &= -3(1+w_i) \rho_i \left( \frac{\dot{a}}{a} \frac{1}{a} \Psi t \right) \quad \frac{\dot{a}}{a} \frac{t}{a} = \frac{1}{a} \frac{da}{dt} t \\
 &= -\frac{2(1+w_i)}{(1+w)} \bar{\Psi} \rho_i \quad = Ht \\
 & \quad \quad \quad = \frac{2}{3} \frac{1}{1+w}
 \end{aligned}$$

$$\therefore \frac{\delta \rho_i}{\rho_i} |_{\text{Newt}} \sim \bar{\Psi} \text{ for } k\eta \ll 1$$

eg. radiation  $\Theta_0 \equiv \frac{\delta \rho_\gamma}{4 \rho_\gamma} \quad 1+w_\gamma = 4/3$

$$\Theta_0 = -\frac{2}{3} \frac{1}{(1+w)} \bar{\Psi}$$

$$\begin{aligned}
 \Theta_0 + \bar{\Psi} &= \frac{1}{3} \left[ \frac{-2 + 3 + 3w}{1+w} \right] \bar{\Psi} \\
 &= \frac{1}{3} \left[ \frac{1+3w}{1+w} \right] \bar{\Psi}
 \end{aligned}$$

if  $w=0$  (matter dominated)

$$\boxed{\Theta_0 + \bar{\Psi} = \frac{1}{3} \bar{\Psi}} \quad \text{"Sachs-Wolfe Effect"}$$

## • Smooth Components

defn:  $\delta\rho_s |_{\text{comov}} \ll \delta\rho_m$  even if  $\rho_s > \rho_m$

we obtain this when stress gradients prevent collapse inside (sound) horizon

derived equations with  $\rho_K \equiv \frac{-3}{8\pi G a^2} K$

generalize to arbitrary smooth component

$$\delta = \Phi + 2(\Psi - \Phi') \frac{\rho_m + \rho_s}{\rho_m'}$$

$$\delta' = (\Psi - \Phi') \frac{\rho_s'}{\rho_m'}$$

Eliminate  $\delta$ :

$$\Phi'' + \left(1 - \frac{\rho_m''}{\rho_m'} + \frac{1}{2} \frac{(\rho_m + \rho_s)'}{\rho_m + \rho_s}\right) \Phi' + \left(\frac{1}{2} \frac{2\rho_m' + \rho_s'}{\rho_m + \rho_s} - \frac{\rho_m''}{\rho_m'}\right) \Phi = 0$$

$\Rightarrow$  decay in  $\Phi$

interpretation: potential flow  $\delta\rho_m \sim \rho_m (k\eta)^2 \Phi$   
 Poisson  $\Phi = (k\eta)^{-2} \frac{\delta\rho_m}{(\rho_m + \rho_s)}$   
 gradual decay

# Effectively Smooth Component Domination

inside horizon stress gradients prevent collapse

→ decay in potential

$$\Phi = C_1 a^{-1} + C_2 a^{-1} \int da a \frac{\rho'_{\text{m}}}{\sqrt{\rho_{\text{m}} + \rho_{\text{s}}}}$$

$\rho_{\text{m}}$  stress free matter  
i.e. CDM

$$\sim \int da a^{3(1+w)/2} a^{-2}$$

$a^{-3/2(1-w)}$  ← decay for  $w < 1$

$$\Rightarrow \frac{\delta \rho_{\text{m}}}{\rho_{\text{m}}} \propto a^{-1/2(1-3w)} \quad (w = 1/3 \Rightarrow \ln a)$$

# • Transfer Function

the transfer function accounts for the scale-dependent growth of structure due to stress gradients

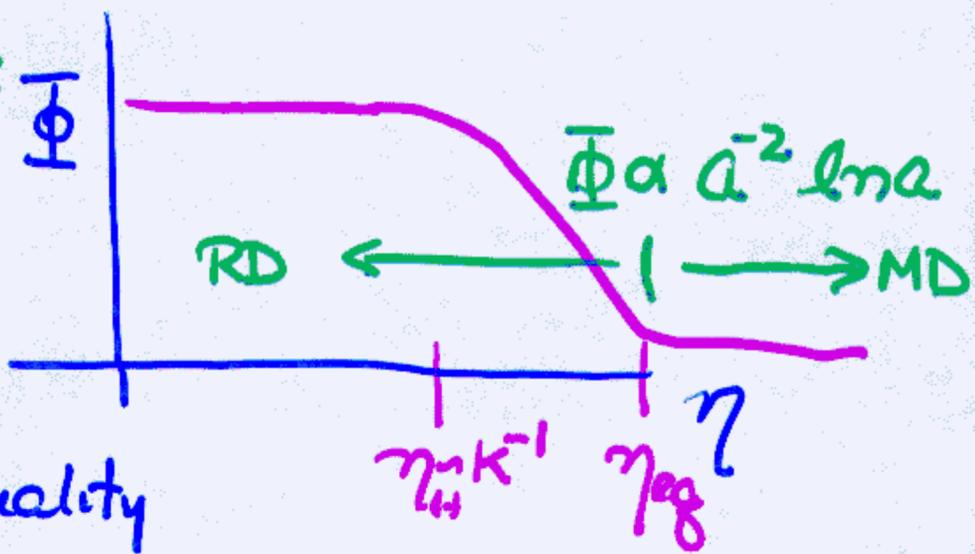
$$T(k) \propto \frac{\Phi(k, a_{\text{now}})}{\Phi(k, a_{\text{initial}})}$$

conventionally normalized to

$$\lim_{k \rightarrow 0} T(k) = 1$$

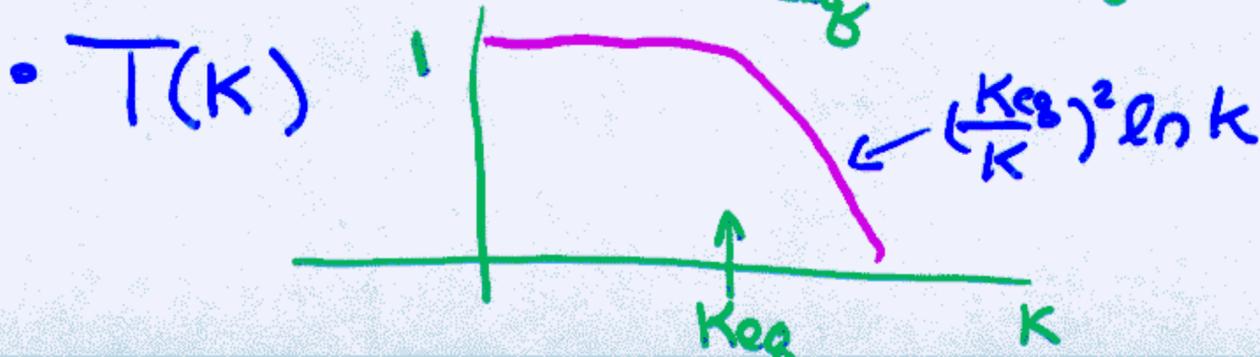
## • Radiation Domination:

- decays as  $a^{-2} \ln a$  between horizon crossing  $\eta_H k^{-1}$  and matter-radiation equality



- RD  $\eta \propto a \therefore \left(\frac{a_H}{a_{\text{eq}}}\right) = \left(\frac{\eta_H}{\eta_{\text{eq}}}\right) = \left(\frac{1}{k \eta_{\text{eq}}}\right)$

- Suppression in  $k$ :  $\left(\frac{a_H}{a_{\text{eq}}}\right)^2 \ln(a_{\text{eq}}/a_H) \propto k^{-2} \ln k$



- Stress Free Vector Modes

$$\left[ \frac{d}{d\eta} + 4\frac{a'}{a} \right] \left[ (\rho + p)(\sqrt{\pm 1}) - B^{(\pm 1)} \right] / \kappa = 0$$

$$\Rightarrow \sqrt{\pm 1} - B^{(\pm 1)} \propto a^{-4} (\rho + p)^{-1} \quad \text{decay unless continuously generated}$$

- Stress Free Tensor Modes

$$\left[ \frac{d^2}{d\eta^2} + 2\frac{a'}{a} \frac{d}{d\eta} + (\kappa^2 + 2\kappa) \right] H_T^{(\pm 2)} = 0$$

$$H_T^{(\pm 2)} = C_1 H_1 + C_2 H_2$$

$$H_1 \propto \chi^{-m} j_m(\chi) \quad \chi = \sqrt{\kappa^2 - 2\kappa} \eta$$

$$H_2 \propto \chi^{-m} n_m(\chi) \quad m = \frac{(1 - 3w)}{(1 + 3w)}$$

$$w > -1/3$$

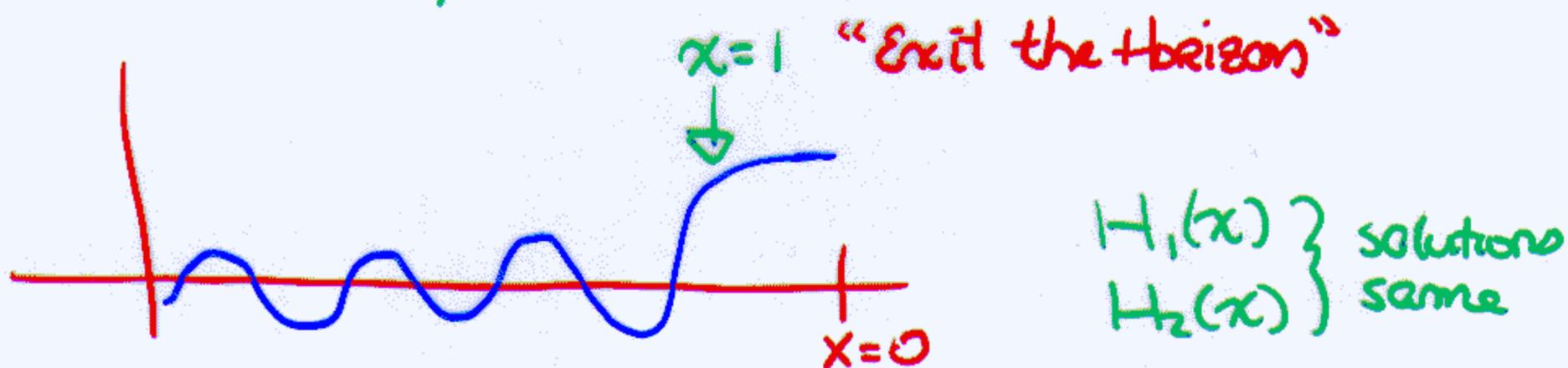
gravity waves constant about horizon  $\chi \ll 1$  and then oscillate & damp

(important for gravity wave Ce's)

$$\omega < -1/3$$

Reverse: gravity waves oscillate and then freeze in at some value

$$\chi = -k \int_{\eta}^{\infty} d\eta = -k(\eta_{\text{end}} - \eta)$$



This is exactly the behavior of scalar field fluctuations: field oscillations freeze in as they "exit the horizon"

Link between scalar field fluctuations & gravitational waves include generation of quantum fluctuations