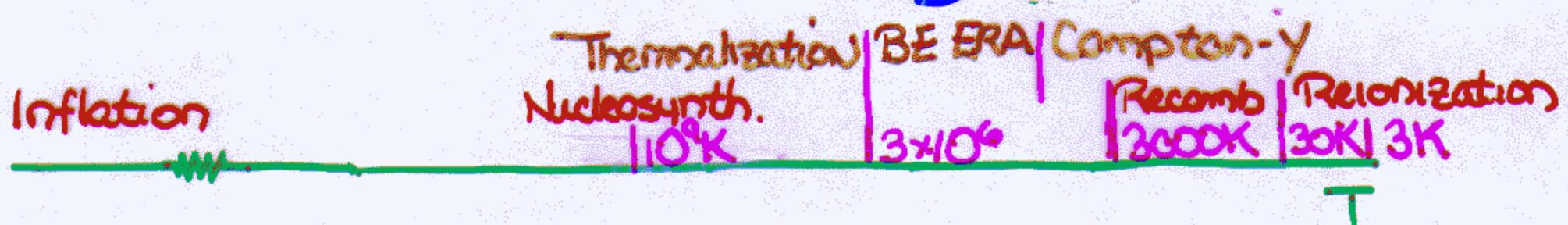


Thermal History



Qualitative Survey

1. Inflation produces a nearly homogeneous CMB solving the horizon problem
2. Big Bang nucleosynthesis
Gamow (1948!) predicted CMB / its temperature from the existence of light elements + baryon density today
3. Photon creating processes can thermalize any distortions to the black body $z \gg \text{few} \times 10^6$
4. Compton scattering can bring any distortions to a Bose-Einstein distribution $z > 10^4$
5. Inverse Compton scattering off hot electrons can produce a Compton- γ distortion $z < 10^4$

6. Recombination $z \approx 10^3$

neutral hydrogen forms anisotropies frozen in

7. Dark Ages $10^3 < z \leq 10$

Compton drag, Compton cooling

8. Reionization $z \sim 10$

Optically thin to Thomson/Compton scattering

Technical Details

- Nucleosynthesis & the Prediction of the CMB temperature

Reverse the $D/H \rightarrow \Omega_b h^2$ argument

BBN gives photon/baryon ratio
+ baryons in stars, etc $\rightarrow T$

Deuterium:



Binding
 $B = 2.2 \text{ MeV}$

at $T \sim 10^9 \text{ K}$ ($\sim 100 \text{ keV}$)

not enough photons to dissociate D

want: probability of n capture

1. Large enough to produce light elements
2. Not too large so as to leave no D ($\rightarrow \text{He}$)

$$\therefore \langle \sigma v \rangle n t \sim 1 \text{ at } T \sim 10^9 \text{ K}$$

\Rightarrow get density at $T \sim 10^9$
use density now

\Rightarrow expansion factor

$\Rightarrow T_{\text{now}} = 10^9 / \text{expansion factor}$

Numbers

$$\langle \sigma v \rangle \approx 4.6 \times 10^{-20} \text{ cm}^3 \text{ s}^{-1}$$

$$a_B = 7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$$

$$t \approx \left(\frac{3c^2}{32\pi G a_B T^4} \right)^{1/2}$$

$$G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$$

$$c = 3 \times 10^{10} \text{ cm s}^{-1}$$

$$\approx \left(\frac{3 \cdot 3^2 \cdot 10^{20}}{3 \cdot 3 \cdot 10 \cdot 10^{-7} \cdot 10^{-14} \cdot 10^{36}} \right)^{1/2} \text{ s}$$

$$\approx (3 \times 10^4)^{1/2} \text{ s} \approx 200 \text{ s}$$

$$n \approx \frac{1}{5 \cdot 2 \cdot 10^{-20} \cdot 10^2} \text{ cm}^{-3} \approx 1 \times 10^{17} \text{ cm}^{-3}$$

$$n_{*}^{\text{now}} \approx 10^{-7} \text{ cm}^{-3} \quad \text{visible baryons in stars}$$

$$n/n_{*}^{\text{now}} = 10^{24} \Rightarrow a_{\text{BBN}} \approx 10^{-8}$$

$$T_{\text{now}} = 10^9 \cdot a_{\text{BBN}} \approx 10 \text{ K}$$

modern perspective $\langle \sigma v \rangle n t \approx 60$ $T = 10^9$
 $n_b/n_{*} \approx 10$

\therefore if light elements formed in big bang
 \rightarrow CMB exists and has temperature
 in the K range

- Compton Scattering: main process coupling CMB to visible matter

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} = 6.65 \times 10^{-25} \text{ cm}^2$$

coupling to protons down by $(m_e/m_p)^2$



$$n_e n_b \sigma_T \equiv \frac{d\tau}{dt} = \text{scattering rate in physical time} \quad (c=1)$$

$$n_e n_b \sigma_T a = a \frac{d\tau}{dt} = \frac{d\tau}{d\eta} = \dot{\tau} = \text{scattering rate in conformal time}$$

$\dot{\tau}$ also scattering probability per unit distance

$$\dot{\tau}^{-1} = 2.17 \times 10^6 (1 - \gamma_0/2) \left[n_e \frac{\Omega_b h^2}{0.02} \right]^{-1} a^2 \text{ Mpc}$$

visible universe today $H_0^{-1} = 3000 h^{-1} \text{ Mpc}$

universe today is optically thin to Thomson scattering

• Optical Depth

$$\tau = \int_{\eta(z)}^{\eta} \dot{\tau} d\eta = 1 \quad \text{defines last scattering } z_*$$

$$= \int_a^1 da \left(\frac{\dot{\tau}(a)}{a^2 H(a)} \right)$$

assume H in matter dominated epoch $H^2 = H_0^2 \Omega_m a^{-3}$

$$\tau \approx 2.4 \times 10^{-3} (1 - Y_p/2) \kappa_e \left(\frac{\Omega_b h^2}{0.02} \right) \left(\frac{\Omega_m h^2}{0.15} \right)^{-1/2} (1+z)^{3/2}$$

$$\tau(z_*) = 1 \quad \text{nb. v. same as comparing } \frac{d\tau}{dz} \text{ to } H \text{ (scattering vs expansion)}$$

$$z_* = 56 (1 - Y_p/2)^{-2/3} \kappa_e^{-2/3} \left(\frac{\Omega_b h^2}{0.02} \right)^{-2/3} \left(\frac{\Omega_m h^2}{0.15} \right)^{1/3}$$

Reionization $z \sim 10$ optically thin

• Compton Drag on Baryons

Momentum Exchange Rate

$$\dot{\tau}_{\text{drag}} = \dot{\tau} \left(\frac{4 \rho_b}{3 \rho_\gamma} \right)$$

$$\tau_{\text{drag}}(z_{\text{drag}}) = 1$$

$$z_{\text{drag}} = 180 (1 - Y_p/2)^{2/5} \kappa_e^{-2/5} \left(\frac{\Omega_m h^2}{0.15} \right)^{1/5}$$

$z > z_{\text{drag}}$ Baryons dynamically coupled to photons

at reionization drag unimportant \therefore baryons track dark matter

Kinetic Coupling

rest frame of electron
change in energy negligible



in the background frame

$$\frac{\delta P}{P} = \frac{1 - \vec{v}_b \cdot \vec{\gamma}'}{1 - \vec{v}_b \cdot \vec{\gamma}} - 1$$

$\approx v_b \cdot (\vec{\gamma} - \vec{\gamma}') + (\vec{v}_b \cdot \vec{\gamma}) v_b \cdot (\vec{\gamma} - \vec{\gamma}') + \dots$

1. Coherent Flow

Average over incoming direction

$$\frac{\delta P}{P} = \frac{\delta T}{T} = \vec{\gamma} \cdot \vec{v}_b \quad \text{a dipole distribution} \equiv v_b$$

once dipole formed scattering does nothing
(isotropic in electron rest frame)

Rate of momentum transfer
drag

relativistic momentum density

$$\dot{\tau} \left(\frac{4}{3} \rho_\gamma \right) (v_b - v_\gamma)$$

$$\dot{\tau} \left(\frac{4}{3} \rho_\gamma / \rho_0 \right) (v_b - v_\gamma)$$

2. Thermal Velocity $\langle v_b^2 \rangle = 3kT_e/mec^2$

Energy transfer at $\Theta(v_b^2)$

\Rightarrow thermal coupling

needed for
perturbation eqns.

• Thermal Coupling

f = distribution function of photons

$$= \frac{1}{e^{x+\mu}-1} \quad \text{Bose Einstein Distribution}$$

$$x = h\nu/kT_e = P/T_e$$

if chemical potential $\mu=0$ then
Planck blackbody

$$\rho_\gamma = \frac{1}{\pi^2} \int f p^3 dp \quad (= a_B T_e^4)$$

$$n_\gamma = \frac{1}{\pi^2} \int f p^2 dp \quad (\propto T_e^3)$$

Kompaneets Equation

$$\frac{\partial f}{\partial t} = \frac{d\tau}{dt} \frac{T_e}{m_e} \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f + f^2 \right) \right]$$

- Number density conserved

$$\frac{\partial n_\gamma}{\partial t} \propto \frac{\partial}{\partial t} \int f x^2 dx = \frac{d\tau}{dt} \frac{T_e}{m_e} \int dx \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f + f^2 \right) \right]$$

$$= 0$$

- Energy exchange

$$a^4 \rho_\gamma \frac{d\tau}{dt} = 4 \frac{d\tau}{dt} \frac{T_e}{m_e} \left(1 - \frac{T_{\text{eff}}}{T_e} \right)$$

where

$$T_{\text{eff}} = \frac{1}{4\rho_0\pi^2} \int_0^{\infty} p^4 f(1+f) dp$$

$$\text{for } f = \frac{1}{e^{p/T_{\text{eff}} + \mu} - 1}$$

$$\frac{\partial f}{\partial(p/T_{\text{eff}})} = \frac{-e^{p/T_{\text{eff}} + \mu}}{(e^{p/T_{\text{eff}} + \mu} - 1)^2} = -(f + f^2)$$

$$\begin{aligned} \therefore T_{\text{eff}} &= \frac{-1}{4\rho_0\pi^2} \int_0^{\infty} p^4 \frac{\partial f}{\partial(p/T_{\text{eff}})} dp \\ &= +\frac{T_{\text{eff}}}{\rho_0} \frac{1}{\pi^2} \int p^3 f dp \\ &= T_{\text{eff}} \quad \checkmark \end{aligned}$$

Rate of Energy Exchange

$$\frac{d\tau_K}{dt} \equiv 4 \frac{T_e}{m_e} \frac{dt}{dt}$$

$$\tau_K(\bar{z}_K) = 1$$

$$\bar{z}_K = 5 \times 10^4 (1 - \gamma_p/2)^{-1/2} \chi_e^{-1/2} \left(\frac{\Omega_b h^2}{0.02} \right)^{1/2}$$

$\bar{z} \gg \bar{z}_K$ energy exchange rapid enough to maintain **KINETIC EQUILIBRIUM** between photons & electrons

Compton Cooling (electron's perspective)

Energy Conservation

$$d(\rho a^3) + p da^3 = 0$$

$$p_\gamma = \frac{1}{3} \rho_\gamma$$

$$\rho_e = n_e m_e + \frac{3}{2} n_e T_e$$

$$p_e = n_e T_e$$

ignore H_e for simplicity

$$a^3 dp_\gamma + \frac{4}{3} \rho_\gamma da^3 + \frac{3}{2} a^3 (n_e + n_H) dT_e + (n_e + n_H) T_e da^3 = 0$$

$$-\frac{3}{2} n_b (1 + x_e) dT_e = dp_\gamma + 4 \rho_\gamma \frac{da}{a} + 3(1 + x_e) n_b T_e \frac{da}{a}$$

$$\therefore \frac{dT_e}{dt} = \underbrace{-2 \frac{da}{dt} \frac{1}{a} T_e}_{\text{adiabatic cooling}} - \underbrace{\frac{2}{3 n_b (1 + x_e)} \frac{1}{a^4} \frac{da^4 p_\gamma}{dt}}_{\text{Compton cooling}}$$

$$T_e \propto a^{-2}$$

(i.e. $T_e \sim mv^2$)
 $v \propto a^{-1}$

$$\frac{dT_e}{dt} = -2 H T_e - \underbrace{\frac{8 \sigma x_e \rho_\gamma}{3 (1 + x_e) m_e}}_{\tau_{\text{cool}}^{-1}} (T_e - T_{\text{eff}})$$

$$H = \tau_{\text{cool}}^{-1}$$

$$\text{at } (1 + Z_{\text{cool}}) \approx 9 (\Omega_m h^2)^{1/5} \left[\frac{(1 + x_e)}{2 x_e} \right]^{2/5}$$

Compton cooling very effective $T_{\text{cool}} \approx T_e$ until recently

Kinetic vs Thermal Equilibrium

for $z \gg z_K$ Compton redistribution of photon energies effective

$z \ll z_K$
Compton- γ dist.
Problem Set 1

$$\left. \frac{\partial f}{\partial t} \right|_{\text{Compton}} \approx 0 = \frac{d\tau_K}{dt} \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f^2 + f \right) \right]$$

recall that for a BE distribution

$$\frac{\partial f}{\partial x} = -f(1+f)$$

\therefore a BE distribution remains BE and is eq. soln

\therefore if a non-vanishing chemical potential exists it will be preserved
none is observed

A Planck function cannot be achieved by Compton scattering alone since it preserves photon number

At higher temperatures, radiative processes become effective



$$z_{\text{therm}} \approx 2 \times 10^6 (1 - \gamma_p/2)^{-2/5} \left(\frac{\Omega_b h^2}{0.02} \right)^{2/5}$$

- Fully ionized universe Compton scattering very efficient and we would not see beyond $z \sim 50$ in the CMB

but

- Recombination $e^- + p \rightarrow H + \gamma$

Saha Equilibrium

$$n_i = g_i \left(\frac{m_i T_i}{2\pi} \right)^{3/2} e^{(\mu_i - m_i)/T_i}$$

$$\mu_e + \mu_p = \mu_H$$

$$\therefore \frac{n_e n_p}{n_H n_b} = \frac{x_e^2}{1 - x_e} = \frac{1}{n_b} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-\frac{(m_e + m_p - m_H)T}{T}} \quad \underline{13.6 \text{ eV}}$$

predicts dramatic drop in ionization at $T \sim 0.3 \text{ eV}$ and log dependence on parameters

In fact recombination freezes out & non equilibrium processes important

recombination directly to ground $\rightarrow \gamma$ that just reionizes another H

recombination proceeds by redshifting out of the line and 2γ decay from $2S$

Multilevel atom: Seager et al (1999)

Rees (1968)