Astro 448

Lecture Notes Set 6

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Secondary Anisotropy

- CMB photons traverse the large-scale structure of the universe from $z = 1000$ to the present.
- With the nearly scale-invariant adiabatic fluctuations observed in the CMB, structures form from the bottom up, i.e. small scales first, a.k.a. hierarchical structure formation.
- First objects reionize the universe between $z \sim 7 - 30$
- Main sources of secondary anisotropy
- Gravitational: Integrated Sachs-Wolfe effect (gravitational redshift) and gravitational lensing
- Scattering: peak suppression, large-angle polarization, Doppler effect(s), inverse Compton scattering

Transfer Function

- Transfer function transfers the initial Newtonian curvature to its value today (linear response theory)

$$T(k) = \frac{\Phi(k, a=1)}{\Phi(k, a_{\rm init})} \frac{\Phi(k_{\rm norm}, a_{\rm init})}{\Phi(k_{\rm norm}, a=1})$$

- Conservation of Bardeen curvature: Newtonian curvature is a constant when stress perturbations are negligible: above the horizon during radiation and dark energy domination, on all scales during matter domination
- When stress fluctuations dominate, perturbations are stabilized by the Jeans mechanism
- Hybrid Poisson equation: Newtonian curvature, comoving density perturbation $\Delta \equiv (\delta\rho/\rho)_{\rm com}$ implies Φ decays

$$(k^2 - 3K)\Phi = 4\pi G\rho\Delta \sim \eta^{-2}\Delta$$

Transfer Function

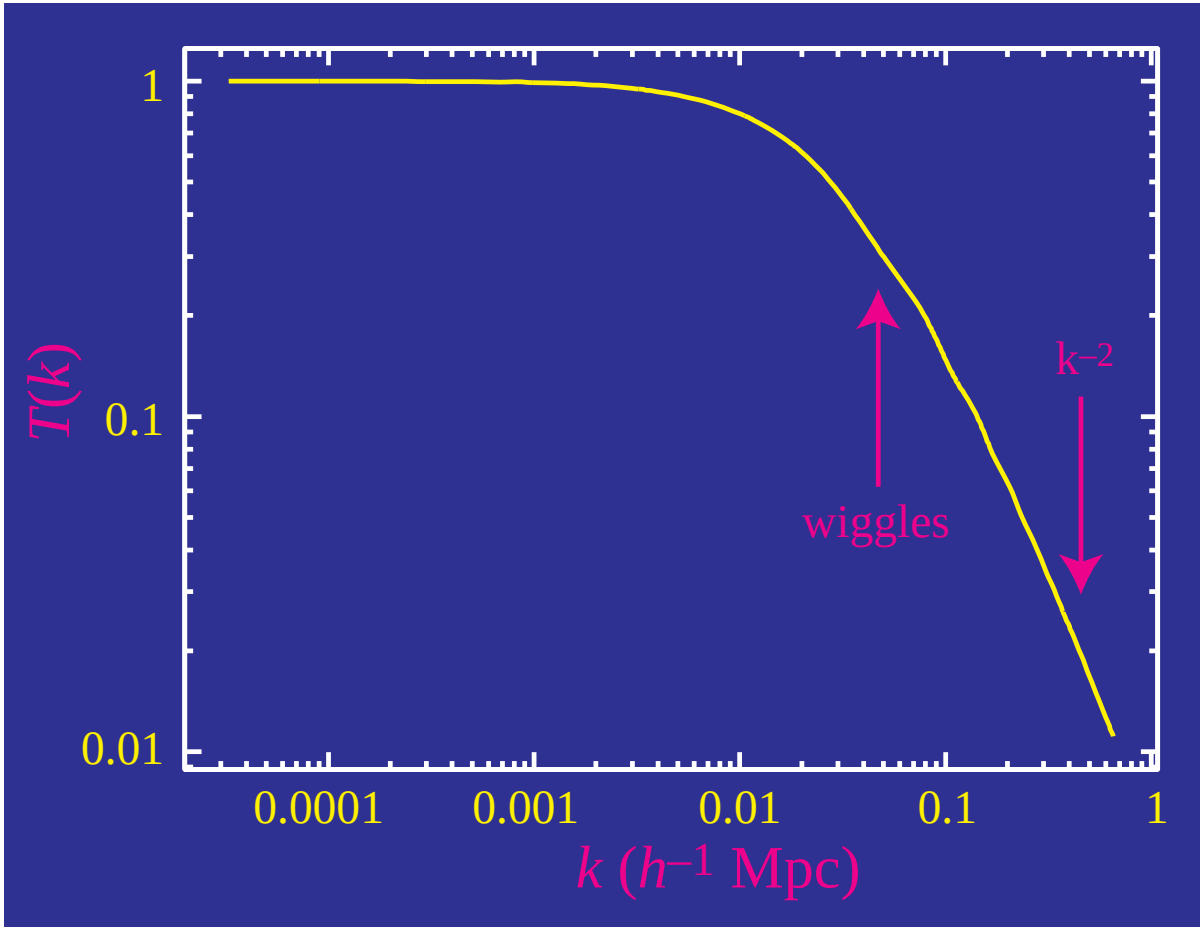
- Matter-radiation example: Jeans scale is horizon scale and Δ freezes into its value at horizon crossing $\Delta_H \approx \Phi_{\rm init}$
- Freezing of Δ stops at $\eta_{\rm eq}$

$$\Phi \sim (k\eta_{\rm eq})^{-2}\Delta_H \sim (k\eta_{\rm eq})^{-2}\Phi_{\rm init}$$

- Conventionally $k_{\rm norm}$ is chosen as a scale between the horizon at matter radiation equality and dark energy domination.
- Small correction since growth with a smooth radiation component is logarithmic not frozen
- Run CMBfast to get transfer function or use fits

Transfer Function

- Transfer function has a k^{-2} fall-off beyond $k_{\rm eq} \sim \eta_{\rm eq}^{-1}$



- Additional baryon wiggles are due to acoustic oscillations at recombination – an interesting means of measuring distances

Growth Function

- Same physics applies to the dark energy dominated universe
- Under the dark energy sound horizon, dark energy density frozen. Potential decays at the same rate for all scales

$$g(a) = \frac{\Phi(k_{\rm norm}, a)}{\Phi(k_{\rm norm}, a_{\rm init})}$$

- Pressure growth suppression: $\delta \equiv \delta\rho_m/\rho_m \propto a\phi$

$$\frac{d^2 g}{d\ln a^2} + \left[\frac{5}{2} - \frac{3}{2}w(z)\Omega_{DE}(z)\right]\frac{dg}{d\ln a} + \frac{3}{2}[1 - w(z)]\Omega_{DE}(z)g = 0\,,$$

where $w \equiv p_{DE}/\rho_{DE}$ and $\Omega_{DE} \equiv \rho_{DE}/(\rho_m + \rho_{DE})$ with initial conditions $g = 1$, $dg/d\ln a = 0$

- As $\Omega_{DE} \to 0$ $g =$const. is a solution. The other solution is the decaying mode, elimated by initial conditions

ISW effect

- Potential decay leads to gravitational redshifts through the integrated Sachs-Wolfe effect
- Intrinsically a large effect since $2\Delta\Phi = 6\Psi_{\rm init}/3$
- But net redshift is integral along along line of sight

$$\frac{\Theta_\ell(k,\eta_0)}{2\ell+1} = \int_0^{\eta_0} d\eta e^{-\tau}[2\dot{\Phi}(k,\eta)]j_\ell(k(\eta_0-\eta))$$
$$= 2\Phi(k,\eta_{MD})\int_0^{\eta_0} d\eta e^{-\tau}\dot{g}(D)j_\ell(kD)$$

- On small scales where $k \gg \dot{g}/g$, can pull source out of the integral

$$\int_0^{\eta_0} d\eta \dot{g}(D)j_\ell(kD) \approx \dot{g}(D=\ell/k)\frac{1}{k}\sqrt{\frac{\pi}{2\ell}}$$

evaluated at peak, where we have used $\int dx j_\ell(x) = \sqrt{\pi/2\ell}$

ISW effect

- Power spectrum

$$C_\ell = \frac{2}{\pi}\int \frac{dk}{k}\frac{k^3\langle\Theta_\ell^*(k,\eta_0)\Theta_\ell(k,\eta_0)\rangle}{(2\ell+1)^2}$$
$$= \frac{2\pi^2}{l^3}\int d\eta D\dot{g}^2(\eta)\Delta_\Phi^2(\ell/D,\eta_{MD})$$

- Or $l^2C_l/2\pi \propto 1/\ell$ for scale invariant potential. This is the Limber equation in spherical coordinates. Projection of $3D$ power retains only the transverse piece.
- Cancellation of redshifts and blueshifts as the photon traverses many crests and troughs of a small scale fluctuation during decay
- Enhancement of the $\ell < 10$ multipoles. Difficult to extract from cosmic variance and galaxy. Current ideas: cross correlation with other tracers of structure

Gravitational Lensing

- Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int_{\eta_*}^{\eta_0} d\eta \, \frac{(D_* - D)}{D \, D_*} \Phi(D\hat{\mathbf{n}}, \eta) \, .$$

  such that the fields are remapped as

$$x(\hat{\mathbf{n}}) \longrightarrow x(\hat{\mathbf{n}} + \nabla\phi) \, ,$$

  where $x \in \{\Theta, Q, U\}$ temperature and polarization.

- Taylor expansion leads to product of fields and Fourier mode-coupling

Flat-sky Treatment

- Talyor expand

$$\Theta(\hat{\mathbf{n}}) = \tilde{\Theta}(\hat{\mathbf{n}} + \nabla\phi)$$

$$= \tilde{\Theta}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{\Theta}(\hat{\mathbf{n}}) + \frac{1}{2}\nabla_i\phi(\hat{\mathbf{n}})\nabla_j\phi(\hat{\mathbf{n}})\nabla^i\nabla^j\tilde{\Theta}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition

$$\phi(\hat{\mathbf{n}}) = \int \frac{d^2l}{(2\pi)^2}\phi(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

$$\tilde{\Theta}(\hat{\mathbf{n}}) = \int \frac{d^2l}{(2\pi)^2}\tilde{\Theta}(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

Flat-sky Treatment

- Mode coupling of harmonics

$$\Theta(\mathbf{l}) = \int d\hat{\mathbf{n}}\, \Theta(\hat{\mathbf{n}}) e^{-i\mathbf{l}\cdot\hat{\mathbf{n}}}$$
$$= \tilde{\Theta}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} \tilde{\Theta}(\mathbf{l}_1) L(\mathbf{l}, \mathbf{l}_1)\,,$$

where

$$L(\mathbf{l}, \mathbf{l}_1) = \phi(\mathbf{l} - \mathbf{l}_1)\,(\mathbf{l} - \mathbf{l}_1)\cdot\mathbf{l}_1$$
$$+ \frac{1}{2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2} \phi(\mathbf{l}_2)\phi^*(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\,(\mathbf{l}_2\cdot\mathbf{l}_1)(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\cdot\mathbf{l}_1\,.$$

- Represents a coupling of harmonics separated by $L \approx 60$ peak of deflection power

Power Spectrum

- Power spectra

$$\langle \Theta^*(\mathbf{l})\Theta(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\, C_l^{\Theta\Theta}\,,$$
$$\langle \phi^*(\mathbf{l})\phi(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\, C_l^{\phi\phi}\,,$$

becomes

$$C_l^{\Theta\Theta} = \left(1 - l^2 R\right)\tilde{C}_l^{\Theta\Theta} + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\tilde{C}^{\Theta\Theta}_{|\mathbf{l}-\mathbf{l}_1|}C^{\phi\phi}_{l_1}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2\,,$$

where

$$R = \frac{1}{4\pi}\int \frac{dl}{l}\, l^4 C_l^{\phi\phi}\,. \tag{1}$$

Smoothing Power Spectrum

- If $\tilde{C}_l^{\Theta\Theta}$ slowly varying then two term cancel

$$\tilde{C}_l^{\Theta\Theta} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} C_l^{\phi\phi} (\mathbf{l}\cdot\mathbf{l}_1)^2 \approx l^2 R \tilde{C}_l^{\Theta\Theta} \,.$$

- So lensing acts to smooth features in the power spectrum. Smoothing kernel is $L \sim 60$ the peak of deflection power spectrum
- Because acoustic feature appear on a scale $l_A \sim 300$, smoothing is a subtle effect in the power spectrum.
- Lensing generates power below the damping scale which directly reflect power in deflections on the same scale

Polarization Lensing

- Polarization field harmonics lensed similarly

$$[Q \pm iU](\hat{\mathbf{n}}) = -\int \frac{d^2 l}{(2\pi)^2} [E \pm iB](\mathbf{l}) e^{\pm 2i\phi_{\mathbf{l}}} e^{\mathbf{l}\cdot\hat{\mathbf{n}}}$$

so that

$$\begin{aligned}
[Q \pm iU](\hat{\mathbf{n}}) &= [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}} + \nabla\phi) \\
&\approx [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) + \nabla_i \phi(\hat{\mathbf{n}}) \nabla^i [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) \\
&\quad + \frac{1}{2} \nabla_i \phi(\hat{\mathbf{n}}) \nabla_j \phi(\hat{\mathbf{n}}) \nabla^i \nabla^j [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}})
\end{aligned}$$

Polarization Power Spectra

- Carrying through the algebra

$$C_l^{EE} = \left(1 - l^2 R\right) \tilde{C}_l^{EE} + \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \left[(\tilde{C}_{l_1}^{EE} + \tilde{C}_{l_1}^{BB}) + \cos(4\varphi_{l_1})(\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_1}^{BB})\right],$$
$$C_l^{BB} = \left(1 - l^2 R\right) \tilde{C}_l^{BB} + \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \left[(\tilde{C}_{l_1}^{EE} + \tilde{C}_{l_1}^{BB}) - \cos(4\varphi_{l_1})(\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_1}^{BB})\right],$$
$$C_l^{\Theta E} = \left(1 - l^2 R\right) \tilde{C}_l^{\Theta E} + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \tilde{C}_{l_1}^{\Theta E} \cos(2\varphi_{l_1}),$$

- Lensing generates B-modes out of the acoustic polaraization E-modes contaminates gravitational wave signature if $E_i < 10^{16}$GeV.

Reconstruction from the CMB

- Correlation between Fourier moments reflect lensing potential

$$\langle x(\mathbf{l})x'(\mathbf{l}')\rangle_{\rm CMB} = f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{l}+\mathbf{l}')\,,$$

  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass

Scattering Secondaries

- Optical depth during reionization

$$\tau \approx 0.066 \left(\frac{\Omega_b h^2}{0.02}\right) \left(\frac{\Omega_m h^2}{0.15}\right)^{-1/2} \left(\frac{1+z}{10}\right)^{3/2}$$

- Anisotropy suppressed as $e^{-\tau}$. Integral solution

$$\frac{\Theta_\ell(k,\eta_0)}{2\ell+1} = \int_0^{\eta_0} d\eta e^{-\tau} S_0^{(0)} j_\ell(k(\eta_0-\eta)) + \ldots$$

- Isotropic (lare scale) fluctuations not supressed since suppression represents isotropization by scattering
- Quadrupole from the Sachs-Wolfe effect scatters into a large angle polarization bump

Doppler Effects

- Velocity fields of 10^{-3} and optical depths of 10^{-2} would imply large Doppler effect due to reionization
- Limber approximation says only fluctuations transverse to line of sight survive
- In linear theory, transverse fluctuations have no line of sight velocity and so Doppler effect is highly suppressed.
- Beyond linear theory: modulate the optical depth in the transverse direction using density fluctuations or ionization fraction fluctuations. Generate a modulated Doppler effect
- Linear fluctuations: Vishniac effect; Clusters: kinetic SZ effect; ionization patches: inhomogeneous reionization effect

Thermal SZ Effect

- Thermal velocities also lead to Doppler effect but first order contribution cancels because of random directions
- Residual effect is of order $v^2\tau \approx T_e/m_e\,\tau$ and can reach a sizeable level for clusters with $T_e \approx 10$keV.
- Raleigh-Jeans decrement and Wien enhancement described by second order collision term in Boltzmann equation: Kompaneets equation
- Clusters are rare objects so contribution to power spectrum suppressed, but may have been detected by CBI/BIMA: extremely sensitive to power spectrum normalization σ_8
- White noise on large-scales $(l < 2000)$, turnover as cluster profile is resolved