Astro 448: Problem Set 2

Due October 17

1 Problem 1: Thomson scattering

Refer to the FRW reference sheet or Kolb & Turner for numbers and conversion factors.

- (a) Scattering rate. Given the Thomson cross section σ_T and a density of free electrons $n_e = n_b$ (fully ionized hydrogen case), calculate the time in seconds it takes a typical photon to scatter at an epoch a. Compare this time with the Hubble or expansion time $H^{-1}(a)$. [You may assume $a \ll 1$ so that dark energy or curvature is negligible]. Express the ratio in terms of $\Omega_m h^2$ and $\Omega_b h^2$. What is the redshift at which this ratio is unity? Why is this number different from the last scattering epoch claimed in class $z = 1000$.

- (b) Mean free path. Convert your answer in (a) to physical length units. This is the mean free path that a photon travels before scattering off an electron. What is the mean free path λ_c in comoving coordinates as a function of a and $\Omega_b h^2$?

- (c) Diffusion. Photons random walk in the baryons due to the mean free path and so they can only travel a distance $\lambda_D = \sqrt{\lambda_c \eta}$ where η is the conformal time. Calculate λ_D at $z = 1000$ for $\Omega_b h^2 = 0.02$, $h = 0.7$, $\Omega_m = 1/3$ and compare it to η at the same epoch. Since the horizon scale corresponds to $\ell \sim 200$, where in multipole do you expect diffusion effects to become important? Above this scale, photons are trapped in the baryons and the tight coupling or fluid approximation of the system is valid.