Astro 448: Problem Set 3

Due October 24

1 Problem 1: Green's Method

- (a) From the continuity and Euler equations of the joint photon-baryon system (see notes) show

$$\ddot{\Theta} + \frac{\dot{R}}{1+R}\dot{\Theta} + k^2 c_s^2 \Theta = F(\eta) \tag{1}$$

$$F(\eta) = -\ddot{\Phi} - \frac{\dot{R}}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi \tag{2}$$

- (b) Take the solutions

$$\theta_a = (1+R)^{-1/4}\cos(ks) \tag{3}$$
$$\theta_b = (1+R)^{-1/4}\sin(ks) \tag{4}$$

  and show that they solve the homogeneous $F = 0$ equation in the adiabatic approximation

- (c) Use the Greens method to construct the particular solution

$$\Theta(\eta) = C_1\theta_a(\eta) + C_2\theta_b(\eta) + \int_0^\eta d\eta' \frac{\theta_a(\eta')\theta_b(\eta) - \theta_a(\eta)\theta_b(\eta')}{\theta_a(\eta')\dot{\theta}_b(\eta') - \dot{\theta}_a(\eta')\theta_b(\eta')} F(\eta') \tag{5}$$

  and give the expression in terms of $\Theta(0)$, $\dot{\Theta}(0)$, R, $\dot{R}(0)$, s. Think of this as taking a set of impulsive forces on the oscillator and propagating their effect into a temperature perturbation at a later time.

- (d) Evalute the general solution for R, Ψ, Φ all constant.

- (e) What you expect to happen *qualitatively* to the acoustic oscillations for initial conditions where there are no gravitational potentials initially and $\Phi = \Psi$ only becomes substantial after horizon crossing $k\eta = 1$ (take $C_1 = C_2 = 0$). Remember that the Greens solution causally propagates an impulsive force. Argue that the appearance of a first peak that is consistent with adiabatic initial conditions is strong argument for inflation.