

Classical Scalar Fields cont.**1 Fluctuations**

Keeping metric fluctuations in the correspondence between the stress energy tensor and its components leads to a generalization of last weeks problem set

$$\begin{aligned}
 \delta\rho_\phi &= a^{-2}(\dot{\phi}_0\dot{\phi}_1 - \dot{\phi}_0^2 A) + V'\phi_1, \\
 \delta p_\phi &= a^{-2}(\dot{\phi}_0\dot{\phi}_1 - \dot{\phi}_0^2 A) - V'\phi_1, \\
 (\rho_\phi + p_\phi)(v_\phi - B) &= a^{-2}k\dot{\phi}_0\phi_1, \\
 p_\phi\pi_\phi &= 0,
 \end{aligned} \tag{1}$$

where A is the time-time metric perturbation and B is the scalar time-space metric perturbation in covariant perturbation theory.

- (1) Using the gauge transformation properties of the stress energy components derive the gauge transformation properties of ϕ_1 . Hint: look at the velocity component. Argue that your result had to be true given that a scalar field is a scalar field!
- (2) Show that the adiabatic sound speed

$$c_\phi^2 \equiv \dot{p}_\phi/\dot{\rho}_\phi = 1 + \frac{2V'\dot{\phi}}{3(\rho_\phi + p_\phi)} \left(\frac{\dot{a}}{a}\right)^{-1} \tag{2}$$

This looks like a “bad thing” since c_ϕ^2 is not guaranteed to be positive. An imaginary sound speed means accelerated collapse and a scalar field is supposed to be the most gravitationally stable type of matter possible - hence its utility in the dark energy game.

- (3) Show that pressure fluctuation

$$\delta p_\phi = \delta\rho_\phi + 3(\rho_\phi + p_\phi)\frac{v_\phi - B}{k}\frac{\dot{a}}{a}(1 - c_\phi^2). \tag{3}$$

Argue that in the right coordinate system, in this case the comoving gauge the relevant sound speed squared

$$\delta p_\phi/\delta\rho_\phi = 1. \tag{4}$$

The sound speed relevant for gravitational collapse in this gauge is the speed of light. The density fluctuation in the comoving gauge really is what you want to think of as the non-relativistic density perturbation - e.g. it obeys the usual Poisson equation when related to the Newtonian gravitational potential. Thus a (slowly rolling) scalar field is gravitationally stable (smooth) inside the horizon.

- (4) Use the continuity equation to derive the equation of motion for the scalar field perturbation

$$\ddot{\phi}_1 = -2\frac{\dot{a}}{a}\dot{\phi}_1 - (k^2 + a^2V'')\phi_1 + (\dot{A} - 3\dot{H}_L - kB)\dot{\phi}_0 - 2Aa^2V'. \tag{5}$$

Give the expression in the Newtonian and synchronous gauges. What happens to ϕ_1 in the comoving gauge? (hint: re-examine the expression for the energy flux).