Astro 448: Final Problem Set (8)

Due Dec. 5

1 Beyond Slow Roll

Work through the straightforward but tedious algebra skipped in class. From the equation of motion for ϕ_1 in the spatially unperturbed (or flat) gauge $H_L = H_T = 0$, the Einstein equations, and the homogeneous scalar field equation, show

$$\ddot{u} + (k^2 - \frac{\ddot{z}}{z})u = 0 \tag{1}$$

where $u = a\phi_1$ and $z = a(\dot{a}/a)^{-1}\dot{\phi}_0$. You may find the relations proved in class (in notes) helpful.

2 Projection Kernels and the Integral Approach

A general source to the temperature anisotropy is a function of both position $\mathbf{x}$ and direction $\hat{\mathbf{n}}$ and so can be decomposed into the harmonics

$$G_j^m = (-i)^j \sqrt{\frac{4\pi}{2j+1}} Y_j^m(\hat{\mathbf{n}}) \exp(i\mathbf{k}\cdot\mathbf{x}) \,. \tag{2}$$

- (a) Expand the plane wave into spherical harmonics and spherical bessel functions. You may simplify the algebra by taking $\mathbf{k} \parallel \mathbf{z}$ so the plane wave is azimuthally symmetric.
- (b) Consider the $j = 1$ in the source. Using Clebsch-Gordon coefficients reexpress the mode in the form

$$G_j^m = \sum_\ell (-i)^\ell \sqrt{4\pi(2\ell+1)} \alpha_{j\ell}^{(m)} Y_\ell^m \,. \tag{3}$$

  and give α for $m = 0, \pm 1$ in terms of the spherical bessel function. This is the projection kernel used in the integral approach for the Doppler effect from potential flow and vorticity respectively.