Appendix B

Useful Quantities and Relations

B.1 FRW Parameters

The expansion rate is given by the Hubble parameter

$$
\begin{aligned}
H^2 &\equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \left(\frac{\dot{a}}{a}\frac{a_0}{a}\right)^2 \\
&= \left(\frac{a_0}{a}\right)^4 \frac{a_{eq}+a}{a_{eq}+a_0}\Omega_{\rm m} H_0^2 - \left(\frac{a_0}{a}\right)^2 K + \Omega_\Lambda H_0^2,
\end{aligned} \tag{B.1}
$$

where the curvature is $K = -H_0^2(1-\Omega_{\rm m}-\Omega_\Lambda)$. The value of the Hubble parameter today, for different choices of the fundamental units (see Tab. B.1), is expressed as

$$
\begin{aligned}
H_0 &= 100h\,{\rm kms^{-1}Mpc^{-1}} \\
&= 2.1331\times10^{-42}h\,{\rm GeV} \\
&= (2997.9)^{-1}h\,{\rm Mpc}^{-1} \\
&= (3.0857\times10^{17})^{-1}h\,{\rm s}^{-1} \\
&= (9.7778)^{-1}h\,{\rm Gyr}^{-1}.
\end{aligned} \tag{B.2}
$$

Present day densities in a given particle species X are measured in units of the critical density $\rho_X(a_0) = \Omega_X\rho_{\rm crit}$, where

$$
\begin{aligned}
\rho_{\rm crit} = 3H_0^2/8\pi G &= 1.8788\times10^{-29}h^2\,{\rm g\,cm^{-3}} \\
&= 8.0980\times10^{-47}h^2\,{\rm GeV}^4 \\
&= 1.0539\times10^{4}h^2\,{\rm eV\,cm^{-3}} \\
&= 1.1233\times10^{-5}h^2{\rm protons\,cm^{-3}} \\
&= 2.7754\times10^{11}h^2{\rm M_\odot\,Mpc^{-3}}.
\end{aligned} \tag{B.3}
$$

For the CMB,

$$
n_{\gamma 0} = 399.3\Theta_{2.7}^3\,{\rm cm}^{-3},
$$

1 s	$= 9.7157\times10^{-15}$ Mpc
1 yr	$= 3.1558\times10^{7}$ s
1 Mpc	$= 3.0856\times10^{24}$ cm
1 AU	$= 1.4960\times10^{13}$ cm
1 K	$= 8.6170\times10^{-5}$ eV
1 $M_\odot$	$= 1.989\times10^{33}$ g
1 GeV	$= 1.6022\times10^{-3}$ erg
	$= 1.7827\times10^{-24}$ g
	$= (1.9733\times10^{-14}\,{\rm cm})^{-1}$
	$= (6.5821\times10^{-25}\,{\rm s})^{-1}$

Planck's constant	$\hbar = 1.0546\times10^{-27}\,{\rm cm^2\,g\,s^{-1}}$
Speed of light	$c = 2.9979\times10^{10}\,{\rm cm\,s^{-1}}$
Boltzmann's constant	$k_B = 1.3807\times10^{-16}\,{\rm erg\,K^{-1}}$
Fine structure constant	$\alpha = 1/137.036$
Gravitational constant	$G = 6.6720\times10^{-8}\,{\rm cm^3\,g^{-1}\,s^{-2}}$
Stefan-Boltzmann constant	$\sigma = ac/4 = \pi^2k_B^4/60\hbar^3c^2$
	$a = 7.5646\times10^{-15}\,{\rm erg\,cm^{-3}K^{-4}}$
Thomson cross section	$\sigma_T = 8\pi\alpha^2/3m_e^2 = 6.6524\times10^{-25}\,{\rm cm^2}$
Electron mass	$m_e = 0.5110$ MeV
Neutron mass	$m_n = 939.566$ MeV
Proton mass	$m_p = 938.272$ MeV

Table B.1: Physical Constants and Conversion factors

$$
\begin{aligned}
\rho_{\gamma 0} &= 4.4738\times10^{-34}\Theta_{2.7}^4\,{\rm g\,cm^{-3}}, \\
\Omega_\gamma &= 2.3812\times10^{-5}h^{-2}\Theta_{2.7}^4,
\end{aligned} \tag{B.4}
$$

and for the neutrinos

$$
\begin{aligned}
\rho_{\nu 0} &= [(1-f_\nu)^{-1}-1]\rho_{\gamma 0}, \\
\Omega_\nu &= [(1-f_\nu)^{-1}-1]\Omega_\gamma,
\end{aligned} \tag{B.5}
$$

with $(1-f_\nu)^{-1} = 1.68$ for the standard model, or for the total radiation

$$
\begin{aligned}
\rho_{r0} &= (1-f_\nu)^{-1}\rho_{\gamma 0}, \\
\Omega_r &= (1-f_\nu)^{-1}\Omega_\gamma.
\end{aligned} \tag{B.6}
$$

B.2 Time Variables

Throughout the text we use four time variables interchangeably, they are a the scale factor, z the redshift, η the conformal time, and t the coordinate time. In addition, three dimensionless time parameterizations are useful to consider: χ the development angle in an open universe, D the relative amplitude of pressureless matter fluctuations, and τ the optical depth to Compton scattering.

B.2.1 Scale Factor and Redshift

The scale factor $a(t)$ describes the state of expansion and is the fundamental measure of time in the Hubble equation (B.1) since it controls the energy density of the universe. In this Appendix, we leave the normalization of a free to preserve generality. However, the normalization applied in §4, §5, §6, and Appendix A is $a_{eq} = 1$. The conversion factor between the more commonly employed normalization $a_0 = 1$ is

$$\begin{aligned} \frac{a_{eq}}{a_0} &= \frac{\Omega_r}{\Omega_\mathrm{m} - \Omega_r} \\ &= 2.38 \times 10^{-5} (\Omega_\mathrm{m} h^2)^{-1} (1 - f_\nu)^{-1} \Theta_{2.7}^4 . \end{aligned} \tag{B.7}$$

The redshift z is defined by $(1+z) = a_0/a$ and serves the same role as the scale factor normalized to the present. We give the scale factor normalized to 3/4 at baryon-photon equality a special symbol R given the frequency of its appearance in equations related to Compton scattering. More explicitly,

$$\begin{aligned} R &= \frac{3}{4}\frac{\rho_b}{\rho_\gamma} = (1-f_\nu)^{-1} \frac{3}{4}\frac{\Omega_b}{\Omega_\mathrm{m}}\frac{a}{a_{eq}} \\ &= 31.5 \Omega_b h^2 \Theta_{2.7}^{-4} (z/10^3)^{-1} . \end{aligned} \tag{B.8}$$

Epochs of interest for the CMB are listed in Tab. B.2 by their redshifts.

B.2.2 Conformal Time

By definition, the conformal time $\eta = \int dt/a$ is related to the scale factor as

$$\eta = \int \frac{da}{a} \frac{1}{H} \frac{a_0}{a} . \tag{B.9}$$

Note that in these $c = 1$ units, the conformal time doubles as the comoving size of the horizon. In an open universe, it is also related to the development angle by

$$\chi = \sqrt{-K}\eta . \tag{B.10}$$

Asymptotic relations are often useful for converting values. Before curvature or Λ domination, the conformal time

$$\begin{aligned} \eta &= \frac{2\sqrt{2}}{k_{eq}} \left(\sqrt{1 + a/a_{eq}} - 1 \right) \\ &= 2(\Omega_\mathrm{m} H_0^2)^{-1/2} (a_{eq}/a_0)^{1/2} \left(\sqrt{1 + a/a_{eq}} - 1 \right) , \end{aligned} \tag{B.11}$$

Epoch	Definition
$z_* = 10^3 \Omega_b^{-0.027/(1+0.11\ln\Omega_b)}$ $\Omega_0 = 1$	Last scattering (recomb.)
$= 10^2 (\Omega_\mathrm{m} h^2/0.25)^{1/3} (x_e \Omega_b h^2/0.0125)^{-2/3}$	Last scattering (reion.)
$z_d = 160 (\Omega_\mathrm{m} h^2)^{1/5} x_e^{-2/5}$	Drag epoch
$z_{eq} = 4.20 \times 10^4 \Omega_\mathrm{m} h^2 (1 - f_\nu) \Theta_{2.7}^{-4}$	Matter-radiation equality
$z_{b\gamma} = 3.17 \times 10^4 \Omega_b h^2 \Theta_{2.7}^{-4}$	Baryon-photon equality
$z_H = (1 + z_{eq})\{4(k/k_{eq})^2/[1 + (1 + 8(k/k_{eq})^2)^{1/2}]\} - 1$	Hubble length crossing
$z = (1 - \Omega_\mathrm{m} - \Omega_\Lambda)/\Omega_0 - 1$	Matter-curvature equality
$z = (\Omega_\Lambda/\Omega_\mathrm{m})^{1/3} - 1$	Matter-Λ equality
$z = [\Omega_\Lambda/(1 - \Omega_\mathrm{m} - \Omega_\Lambda)]^{1/2} - 1$	Curvature-Λ equality
$z_{cool} = 9.08 \Theta_{2.7}^{-16/5} f_{cool}^{2/5} (\Omega_\mathrm{m} h^2)^{1/5} - 1$	Compton cooling era
$z > 4\sqrt{2} z_K$	Bose-Einstein era
$z < z_K/8$	Compton-y era
$z_K = 7.09 \times 10^3 (1 - Y_p/2)^{-1/2} (x_e \Omega_b h^2)^{-1/2} \Theta_{2.7}^{1/2}$	Comptonization epoch
$z_{\mu,dc} = 4.09 \times 10^5 (1 - Y_p/2)^{-2/5} (x_e \Omega_b h^2)^{-2/5} \Theta_{2.7}^{1/5}$	Dbl. Compton therm. epoch
$z_{\mu,br} = 5.60 \times 10^4 (1 - Y_p/2)^{-4/5} (x_e \Omega_b h^2)^{-6/5} \Theta_{2.7}^{13/5}$	Bremss. therm. epoch
$\Theta_{2.7} = T_0/2.7\mathrm{K} \simeq 1.01$	Temperature Scaling
$Y_p = 4 n_{He}/n_b \simeq 0.23$	Helium mass fraction
$(1 - f_\nu)^{-1} = 1 + \rho_\nu/\rho_\gamma \to 1.68132$	Neutrino density correction
$k_{eq} = (2 \Omega_\mathrm{m} H_0^2 a_0/a_{eq})^{1/2}$	Equality Hubble wavenumber
$= 9.67 \times 10^{-2} \Omega_\mathrm{m} h^2 (1 - f_\nu)^{1/2} \Theta_{2.7}^{-2}$ Mpc^{-1}	
$f_{cool} = x_e^{-1}[(1 + x_e)/2 - (3 + 2x_e) Y_p/8](1 - Y_p/2)^{-1}$	Cooling correction factor

Table B.2: Critical Redshifts

Critical epochs are also denoted as the corresponding value in the coordinate time t, scale factor a, and conformal time η. The neutrino fraction f_ν is given for three families of massless neutrinos and the standard thermal history. The Hubble crossing redshift z_H is given for the matter and radiation dominated epochs.

and reduces to

$$\eta = \begin{cases} (\Omega_r H_0^2)^{-1/2} a/a_0 & \text{RD} \\ 2(\Omega_{\rm m} H_0^2)^{-1/2}(a/a_0)^{1/2}, & \text{MD} \end{cases} \quad \text{(B.12)}$$

where $\Omega_r/\Omega_{\rm m} \simeq a_{eq}/a_0$. In a $\Lambda = 0$ universe, it also has an asymptotic solution for $a \gg a_{eq}$

$$\begin{aligned} \eta &= \frac{1}{\sqrt{-K}} \cosh^{-1}\left[1 + \frac{2(1-\Omega_{\rm m})}{\Omega_{\rm m}}\frac{a}{a_0}\right] \qquad \text{MD/CD} \\ \lim_{\Omega_0 \to 0} \eta_0 &\to (-K)^{-1/2}\ln(4/\Omega_{\rm m}), \end{aligned} \quad \text{(B.13)}$$

and thus the horizon scale is larger than the curvature scale $(-K)^{-1/2}$ for low Ω_0 universes. In a flat universe,

$$\eta_0 \simeq 2(\Omega_{\rm m} H_0^2)^{-1/2}(1 + \ln \Omega_{\rm m}^{0.085}), \qquad \Omega_{\rm m} + \Omega_\Lambda = 1 \quad \text{(B.14)}$$

and the horizon goes to a constant $\eta = 2.8\ H_0^{-1}\Omega_{\rm m}^{-1/3}(1-\Omega_{\rm m})^{-1/6}$ as $a/a_0 \to \infty$.

B.2.3 Coordinate Time

The coordinate time is defined in terms of the scale factor as,

$$t = \int \frac{da}{a}\frac{1}{H}. \quad \text{(B.15)}$$

It also takes on simple asymptotic forms, *e.g.*

$$t = \frac{2}{3}(\Omega_{\rm m} H_0^2)^{-1/2} a_0^{-3/2}[(a+a_{eq})^{1/2}(a - 2a_{eq}) + 2a_{eq}^{3/2}]. \qquad \text{RD/MD} \quad \text{(B.16)}$$

Explicitly, this becomes

$$\begin{aligned} t &= \frac{1}{2}(\Omega_{\rm m} H_0^2)^{-1/2}(a_0/a_{eq})^{1/2}(a/a_0)^2 \qquad \text{RD} \\ &= 2.4358 \times 10^{19}\Theta_{2.7}^{-2}(1+z)^{-2}\text{s}. \end{aligned} \quad \text{(B.17)}$$

and

$$\begin{aligned} t &= \frac{2}{3}(\Omega_{\rm m} H_0^2)^{-1/2}(a/a_0)^{3/2} \ \text{MD} \\ &= 2.0571 \times 10^{17}(\Omega_{\rm m} h^2)^{-1/2}(1+z)^{-3/2}\text{s}. \end{aligned} \quad \text{(B.18)}$$

The expansion time, defined as H^{-1} scales similarly

$$\begin{aligned} t_{exp} &= (\Omega_{\rm m} H_0^2)^{-1/2}(a/a_0)^2 a_0^{1/2}(a+a_{eq})^{-1/2} \\ &= 4.88 \times 10^{19}(z + z_{eq} + 2)^{-1/2}\Theta_{2.7}^{-2}(1+z)^{-3/2}\text{s}. \end{aligned} \quad \text{(B.19)}$$

For $\Lambda = 0$ universes, the coordinate time at late epochs when radiation can be neglected is given by

$$t = H_0^{-1}\left[\frac{(1+\Omega_{\rm m} z)^{1/2}}{(1-\Omega_{\rm m})(1+z)} - \frac{\Omega_{\rm m}}{2(1-\Omega_{\rm m})^{3/2}}\cosh^{-1}\left(\frac{2(1-\Omega_{\rm m})}{\Omega_{\rm m}(1+z)} + 1\right)\right]. \qquad \text{MD/CD} \quad \text{(B.20)}$$

In particular, the age of the universe today is

$$t_0 = H_0^{-1}(1-\Omega_0)^{-1}\left[1 - \frac{\Omega_0}{2}(1-\Omega_0)^{-1/2}\cosh(2/\Omega_0 - 1)\right], \qquad \Omega_\Lambda = 0 \quad \text{(B.21)}$$

where the factor in square brackets goes to unity as $\Omega_0 \to 0$. This should be compared with the flat $\Omega_0 + \Omega_\Lambda = 1$ result

$$t_0 = \frac{2}{3}H_0^{-1}(1-\Omega_0)^{-1/2}\ln\left[\frac{1+\sqrt{1-\Omega_0}}{\sqrt{\Omega_0}}\right], \qquad \Omega_0 + \Omega_\Lambda = 1, \quad \text{(B.22)}$$

which diverges logarithmically as $\Omega_0 \to 0$. Finally a microphysical time scale of interest for the CMB,

$$\begin{aligned} t_C &= (d\tau/dt)^{-1} = (x_e n_e \sigma_T)^{-1} \\ &= 4.4674 \times 10^{18}(1 - Y_p/2)^{-1}(x_e \Omega_b h^2)^{-1}(1+z)^{-3}\text{s}, \end{aligned} \quad \text{(B.23)}$$

is the Compton mean free time between scatterings.

B.2.4 Growth Function

The amplitude of matter fluctuations undergoing pressureless growth is another useful parameterization of time. It is given by equation (5.9) as

$$D = \frac{5}{2}\Omega_{\rm m}\frac{a_0}{a_{eq}}g(a)\int \frac{da}{a}\frac{1}{g^3(a)}\left(\frac{a_0}{a}\right)^2, \quad \text{(B.24)}$$

where the dimensionless, "pressureless" Hubble parameter is

$$g^2(a) = \left(\frac{a_0}{a}\right)^3 \Omega_{\rm m} + \left(\frac{a_0}{a}\right)^2 (1 - \Omega_{\rm m} - \Omega_\Lambda) + \Omega_\Lambda. \quad \text{(B.25)}$$

In the matter or radiation-dominated epoch, $D = a/a_{eq}$ by construction. In a $\Lambda = 0$ universe, D becomes

$$D = \frac{5}{2x_{eq}}\left[1 + \frac{3}{x} + \frac{3(1+x)^{1/2}}{x^{3/2}}\ln[(1+x)^{1/2} - x^{1/2}]\right], \quad \text{(B.26)}$$

where $x = (\Omega_0^{-1} - 1)a/a_0$. Fitting formulae for the growth factor, valid for the general case, are occasionally useful [26]:

$$\frac{D_0}{a_0} \simeq \frac{5}{2}\Omega_{\rm m}\left[\Omega_{\rm m}^{4/7} - \Omega_\Lambda + \left(1 + \frac{1}{2}\Omega_{\rm m}\right)\left(1 - \frac{1}{70}\Omega_\Lambda\right)\right]^{-1}, \quad \text{(B.27)}$$

$$\frac{d\ln D}{d\ln a} \simeq \left[\frac{\Omega_{\rm m}(1+z)^3}{\Omega_{\rm m}(1+z)^3 - (\Omega_{\rm m} + \Omega_\Lambda - 1)(1+z)^2 + \Omega_\Lambda}\right]^{4/7}. \quad \text{(B.28)}$$

The latter relation is often employed to relate the velocity to the density field.

Experiment	ℓ_0	ℓ_1	ℓ_2	$Q_{\rm flat}(\mu$K)	Ref.
COBE	–	–	18	19.9 ± 1.6	[62]
FIRS	–	–	30	19 ± 5	[57]
Ten.	20	13	30	26 ± 6	[70]
SP94	67	32	110	26 ± 6	[68]
SK	69	42	100	29 ± 6	[119]
Pyth.	73	50	107	37 ± 12	[49]
ARGO	107	53	180	25 ± 6	[43]
IAB	125	60	205	61 ± 27	[131]
MAX-2 (γUMi)	158	78	263	74 ± 31	[2]
MAX-3 (γUMi)	158	78	263	50 ± 11	[67]
MAX-4 (γUMi)	158	78	263	48 ± 11	[44]
MAX-3 (μPeg)	158	78	263	19 ± 8	[117]
MAX-4 (σHer)	158	78	263	39 ± 8	[32]
MAX-4 (ιDra)	158	78	263	39 ± 11	[32]
MSAM2	143	69	234	40 ± 14	[30]
MSAM3	249	152	362	39 ± 12	[30]

Table B.3: Anisotropy Data Points

A compilation of anisotropy measurements from [146]. The experimental window function peaks at ℓ_0 and falls to half power at ℓ_1 and ℓ_2. Points are plotted in Fig. 1.3.

$\tilde P(\tilde k)$ is the form most often quoted in the literature [175, 82, 134].

The power spectrum may also be expressed in terms of the bulk velocity field. At late times, pressure can be neglected and the total continuity equation (5.6) reduces to

$$\begin{aligned} kV_T &= -\dot\Delta_T \\ &= -\frac{\dot a}{a}\frac{d\ln D}{d\ln a}\Delta_T, \end{aligned} \tag{B.54}$$

and in particular

$$kV_T(\eta_0,k) = -H_0 \frac{d\ln D}{d\ln a}\bigg|_{\eta_0} \Delta_T(\eta_0,k), \tag{B.55}$$

or

$$P_V(k) \equiv |V_T(\eta_0,k)|^2 = H_0^2 \left(\frac{d\ln D}{d\ln a}\right)^2\bigg|_{\eta_0} P(k), \tag{B.56}$$

for the velocity power spectrum. Recall from equation (B.27) that $d\ln D/d\ln a \simeq \Omega_0^{0.6}$ in an open universe.

B.4.2 Anisotropies

The anisotropy power spectrum C_ℓ is given by

$$\frac{2\ell+1}{4\pi}C_\ell = \int \frac{dk}{k} T_\ell^2(k) \times \begin{cases} k^3|\Phi(0,k)|^2 & \text{adiabatic} \\ k^3|S(0,k)|^2, & \text{isocurvature} \end{cases} \tag{B.57}$$

where $T_\ell(k)$ is the radiation transfer function from the solution to the Boltzmann equation. Examples are given in §6. The power measured by a given experiment with a window function W_ℓ has an ensemble average value of

$$\left(\frac{\Delta T}{T}\right)^2_{rms} = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell W_\ell. \tag{B.58}$$

Only if the whole sky is measured at high signal to noise does the variance follow the "cosmic variance" prediction of a χ^2 with $2\ell+1$ degrees of freedom. Real experiments make noisy measurements of a fraction of the sky and therefore require a more detailed statistical treatment. To employ likelihood techniques, we must assume some underlying power spectrum. In order to divorce the measurement from theoretical prejudice, experimental results are usually quoted with a model independent choice. The two most common conventions are the gaussian autocorrelation function $C_{\rm gacf}(\theta) = C_0 \exp(-\theta^2/2\theta_c^2)$ and the "flat" power spectrum motivated by the Sachs-Wolfe tail of adiabatic models (see *e.g.* [174]),

$$\begin{aligned} C_{\ell\rm gacf} &= 2\pi C_0\theta_c^2 \exp[-\ell(\ell+1)\theta_c^2/2], \\ C_{\ell\rm flat} &= \frac{24\pi}{5}\left(\frac{Q_{\rm flat}}{T_0}\right)^2 [\ell(\ell+1)]^{-1}. \end{aligned} \tag{B.59}$$

The two power estimates are thus related by

$$Q_{\rm flat}^2 \frac{6}{5}\sum_\ell \frac{2\ell+1}{\ell(\ell+1)}W_\ell = C_0\theta_c^2 \frac{1}{2}\sum_\ell (2\ell+1)\exp[-\ell(\ell+1)\theta_c^2/2]W_\ell. \tag{B.60}$$

The current status of measurements is summarized in Tab. B.3 [146].

B.4.3 Large Scale Structure

Large scale structure measurements probe a smaller scale and have yet another set of normalization conventions based on the two point correlation function of astrophysical objects

$$\xi_{ab}(\mathbf{x}) = \langle \delta\rho_a(\mathbf{x}'+\mathbf{x})\delta\rho_b(\mathbf{x}')/\bar\rho_a\bar\rho_b\rangle\,. \tag{B.61}$$

If all objects are clustered similarly, then all $\xi_{aa} = \xi$ and the two-point correlation function is related to the underlying power spectrum by

$$\begin{aligned} \xi(r) &= \frac{V}{2\pi^2}\int \frac{dk}{k} k^3 P(k) X_\nu^0(\sqrt{-K}r) \\ &\simeq \frac{V}{2\pi^2}\int \frac{dk}{k} k^3 P(k) \frac{\sin(kr)}{kr}, \end{aligned} \tag{B.62}$$

where the approximation assumes that scales of interest are well below the curvature scale. The normalization of the power spectrum is often quoted by the Nth moment of the correlation function $J_N(r) = \int_0^r \xi(x) x^{(N-1)} dx$ which implies

$$J_3(r) = \frac{V}{2\pi^2}\int \frac{dk}{k} P(k)(kr)^2 j_1(kr). \tag{B.63}$$

For reference, $j_1(x) = x^{-2}\sin x - x^{-1}\cos x$. Another normalization convention involves the rms density fluctuation in spheres of constant radii

$$\sigma^2(r) = \frac{V}{2\pi^2} \int \frac{dk}{k} k^3 P(k) \left(\frac{3j_1(kr)}{kr}\right)^2 . \tag{B.64}$$

The observed galaxy distribution implies that

$$J_3(10h^{-1}\mathrm{Mpc}) \simeq 270h^{-3}\mathrm{Mpc}^3 \tag{B.65}$$

$$\sigma_8 \equiv \sigma(8h^{-1}\mathrm{Mpc}) = \begin{cases} 1.1 \pm 0.15 & \text{optical [109]} \\ 0.69 \pm 0.04. & \text{IRAS [55]} \end{cases} \tag{B.66}$$

The discrepancy between estimates of the normalization obtained by different populations of objects implies that they may all be biased tracers of the underlying mass. The simplest model for bias assumes $\xi_{aa} = b_a^2 \xi$ with constant b. Peacock & Dodds [122] find that the best fit to the Abell cluster (A), radio galaxy (R), optical galaxy (O), and IRAS galaxy (I) data sets yields $b_A : b_R : b_O : b_I = 4.5 : 1.9 : 1.3 : 1$.