

Astro 321

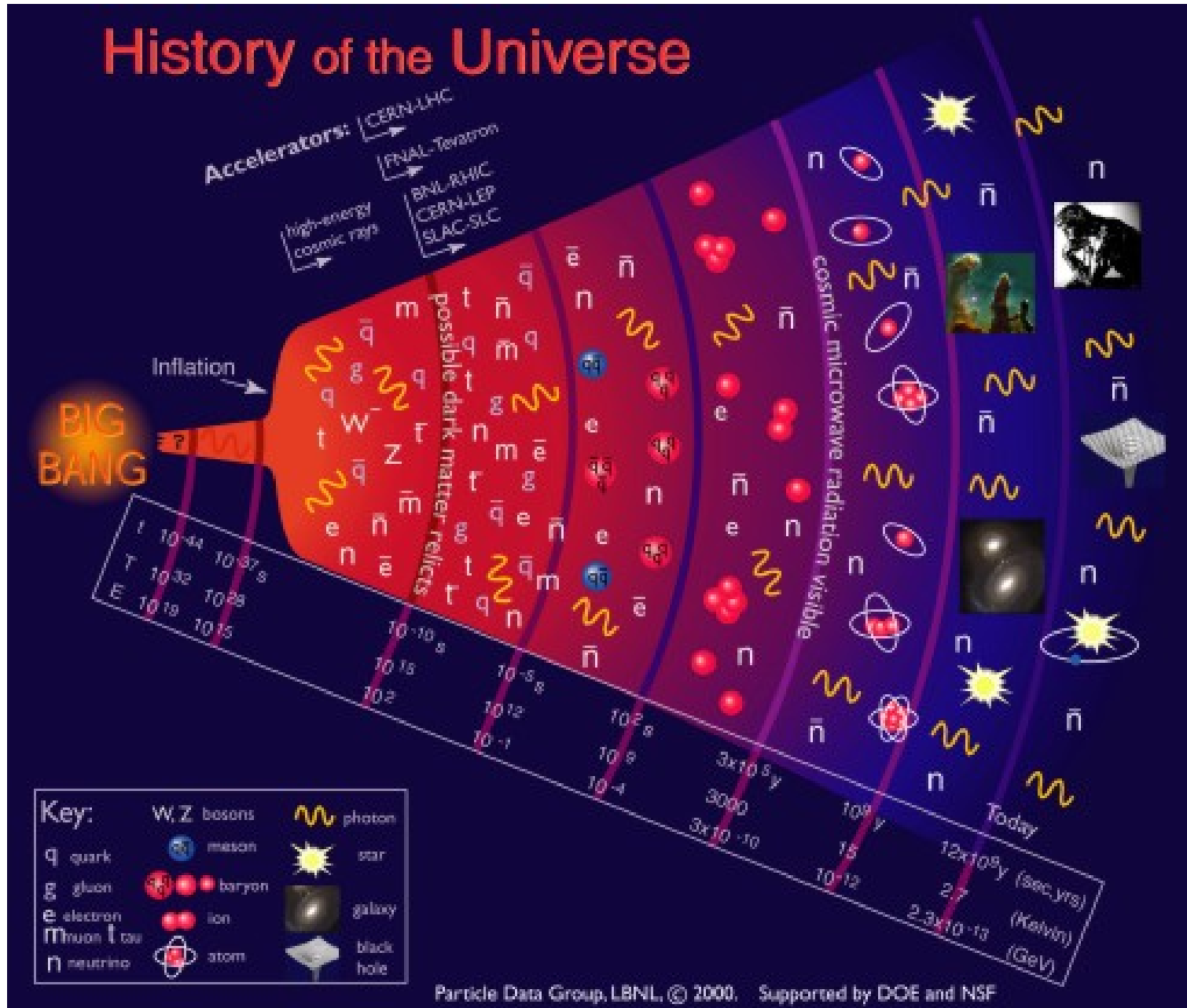
Set 2: Thermal History

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Macro vs Micro Description

- In the first set of notes, we used a **macroscopic** description.
- **Gravity** only cares about **bulk properties**: energy density, momentum density, pressure, anisotropic stress – **stress tensor**
- Matter and radiation is composed of particles whose properties can be described by their **phase space distribution** or occupation function
- Macroscopic properties are integrals or **moments** of the **phase space distribution**
- Particle **interactions** involve the evolution of the phase space distribution
- Rapid interactions drive distribution to **thermal equilibrium** but must compete with the expansion rate of universe
- **Freeze out**, the origin of species

Brief Thermal History

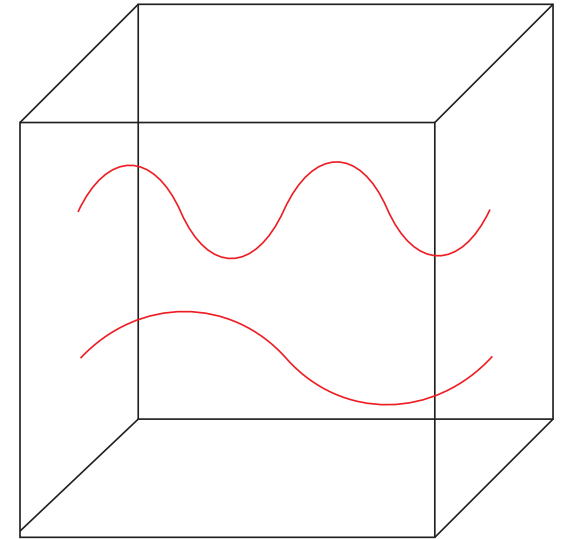


Fitting in a Box

- Counting momentum states with momentum q and de Broglie wavelength

$$\lambda = \frac{h}{q} = \frac{2\pi\hbar}{q}$$

- In a discrete volume L^3 there is a discrete set of states that satisfy periodic boundary conditions
- We will hereafter set $\hbar = c = 1$
- As in Fourier analysis



$$e^{2\pi i x/\lambda} = e^{iqx} = e^{iq(x+L)} \rightarrow e^{iqL} = 1$$

Fitting in a Box

- Periodicity yields a discrete set of allowed states

$$Lq = 2\pi m_i, \quad m_i = 1, 2, 3\dots$$

$$q_i = \frac{2\pi}{L} m_i$$

- In each of 3 directions

$$\sum_{m_{xi} m_{yj} m_{zk}} \rightarrow \int d^3 m$$

- The differential number of allowed momenta in the volume

$$d^3 m = \left(\frac{L}{2\pi} \right)^3 d^3 q$$

Density of States

- The total number of states allows for a number of internal degrees of freedom, e.g. spin, quantified by the degeneracy factor g
- Total density of states:

$$\frac{dN_s}{V} = \frac{g}{V} d^3 m = \frac{g}{(2\pi)^3} d^3 q$$

- If all states were occupied by a single particle, then particle density

$$n_s = \frac{N_s}{V} = \frac{1}{V} \int dN_s = \int \frac{g}{(2\pi)^3} d^3 q$$

Distribution Function

- The distribution function f quantifies the occupation of the allowed momentum states

$$n = \frac{N}{V} = \frac{1}{V} \int f dN_s = \int \frac{g}{(2\pi)^3} f d^3q$$

- f , aka phase space occupation number, also quantifies the density of particles per unit phase space $dN/(\Delta x)^3(\Delta q)^3$
- For photons, the spin degeneracy $g = 2$ accounting for the 2 polarization states
- Energy $E(q) = (q^2 + m^2)^{1/2}$
- Momentum \rightarrow frequency $q = 2\pi/\lambda = 2\pi\nu = \omega = E$ (where $m = 0$ and $\lambda\nu = c = 1$)

Bulk Properties

- Integrals over the distribution function define the bulk properties of the collection of particles
- Number density

$$n(\mathbf{x}, t) \equiv \frac{N}{V} = g \int \frac{d^3q}{(2\pi)^3} f$$

- Energy density

$$\rho(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} E(q) f$$

where $E^2 = q^2 + m^2$

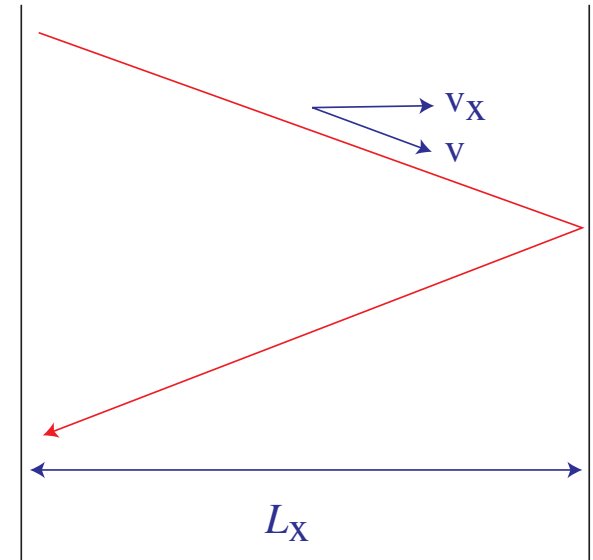
- Momentum density

$$(\rho + p)\mathbf{v}(\mathbf{x}, t) = g \int \frac{d^3q}{(2\pi)^3} \mathbf{q} f$$

Bulk Properties

- Pressure: particles bouncing off a surface of area A in a volume spanned by L_x : per momentum state

$$\begin{aligned} p_q &= \frac{F}{A} = \frac{N_{\text{part}}}{A} \frac{\Delta q}{\Delta t} \\ &\quad (\Delta q = 2|q_x|, \quad \Delta t = 2L_x/v_x,) \\ &= \frac{N_{\text{part}}}{V} |q_x| |v_x| = \frac{N_{\text{part}}}{V} \frac{|q||v|}{3} \\ &\quad (v = \gamma m v / \gamma m = q/E) \\ &= \frac{N_{\text{part}}}{V} \frac{q^2}{3E} \end{aligned}$$



Bulk Properties

- So that summed over occupied momenta states

$$p(\mathbf{x}, t) = g \int \frac{d^3 q}{(2\pi)^3} \frac{|q|^2}{3E(q)} f$$

- Pressure is just one of the quadratic in q moments, in particular the isotropic one
- The remaining 5 components are the anisotropic stress (vanishes in the background)

$$\pi^i_j(\mathbf{x}, t) = g \int \frac{d^3 q}{(2\pi)^3} \frac{3q^i q_j - q^2 \delta^i_j}{3E(q)} f$$

- We shall see that these are related to the 5 quadrupole moments of the angular distribution

Bulk Properties

- These are more generally the components of the stress-energy tensor

$$T^{\mu}_{\nu} = g \int \frac{d^3q}{(2\pi)^3} \frac{q^{\mu} q_{\nu}}{E(q)} f$$

- 0-0: energy density
- 0- i : momentum density
- $i - i$: pressure
- $i \neq j$: anisotropic stress
- In the FRW background cosmology, isotropy requires that there be only a net energy density and pressure

Observable Properties

- Only get to measure luminous properties of the universe. For photons mass $m = 0$, $g = 2$ (units: $J m^{-3}$)

$$\rho(\mathbf{x}, t) = 2 \int \frac{d^3q}{(2\pi)^3} q f = 2 \int dq d\Omega \left(\frac{q}{2\pi} \right)^3 f$$

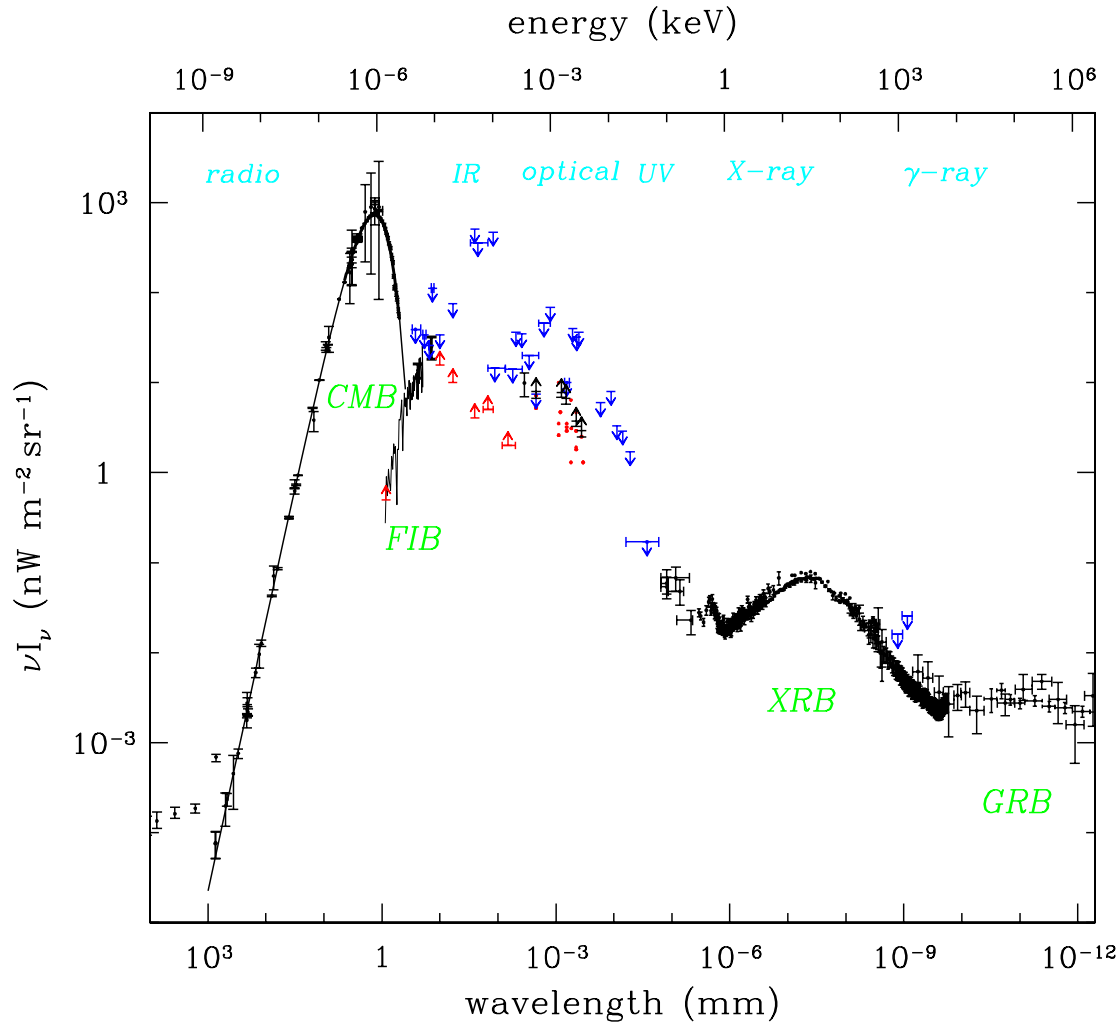
- Spectral energy density (per unit frequency $q = h\nu = \hbar 2\pi\nu = 2\pi\nu$, solid angle)

$$u_\nu = \frac{d\rho}{d\nu d\Omega} = 2(2\pi)\nu^3 f$$

- Photons travelling at speed of light so that $u_\nu = I_\nu = 4\pi\nu^3 f$ the specific intensity or brightness, energy flux across a surface, units of $W m^{-2} Hz^{-1} sr^{-1}$ (SI); $ergs s^{-1} cm^{-2} Hz^{-1} sr^{-1}$ (cgs)

Diffuse Extragalactic Light

- νI_ν peaks in the microwave mm-cm region: CMB black body
 $T = 2.725 \pm 0.002K$ or $n_\gamma = 410 \text{ cm}^{-3}$, $\Omega_\gamma = 2.47 \times 10^{-5} h^{-2}$.



Observable Properties

- Integrate over frequencies for total intensity

$$I = \int d\nu I_\nu = \int d \ln \nu I_\nu$$

νI_ν often plotted since it shows peak under a log plot; I and νI_ν have units of $\text{W m}^{-2} \text{sr}^{-1}$ and is independent of choice of frequency unit

- Flux density (specific flux): integrate over the solid angle of a radiation source, units of $\text{W m}^{-2} \text{Hz}^{-1}$ or Jansky = $10^{-26} \text{W m}^{-2} \text{Hz}^{-1}$

$$F_\nu = \int_{\text{source}} I_\nu d\Omega$$

a.k.a. spectral energy distribution

Observable Properties

- Flux integrate over frequency, units of W m^{-2}

$$F = \int d \ln \nu \nu F_\nu$$

- Flux in a frequency band S_b measured in terms of magnitudes (optical), set to some standard zero point per band

$$m_b - m_{\text{norm}} = 2.5 \log_{10}(F_{\text{norm}}/F_b) \approx \ln(F_{\text{norm}}/F_b)$$

- Luminosity: integrate over area assuming isotropic emission or beaming factor, units of W

$$L = 4\pi d_L^2 F$$

Liouville Equation

- Liouville theorem states that the phase space distribution function is conserved along a trajectory in the absence of particle interactions

$$\frac{Df}{Dt} = \left[\frac{\partial}{\partial t} + \frac{d\mathbf{q}}{dt} \frac{\partial}{\partial \mathbf{q}} + \frac{d\mathbf{x}}{dt} \frac{\partial}{\partial \mathbf{x}} \right] f = 0$$

subtlety in expanding universe is that the de Broglie wavelength of particles changes with the expansion so that

$$q \propto a^{-1}$$

- Homogeneous and isotropic limit

$$\frac{\partial f}{\partial t} + \frac{dq}{dt} \frac{\partial f}{\partial q} = \frac{\partial f}{\partial t} - H(a) \frac{\partial f}{\partial \ln q} = 0$$

Energy Density Evolution

- Integrate Liouville equation over $g \int d^3q / (2\pi)^3 E$ to form

$$\begin{aligned}
 \frac{\partial \rho}{\partial t} &= H(a)g \int \frac{d^3q}{(2\pi)^3} E q \frac{\partial}{\partial q} f \\
 &= H(a)g \int \frac{d\Omega}{(2\pi)^3} \int dq q^3 E \frac{\partial}{\partial q} f \\
 &= -H(a)g \int \frac{d\Omega}{(2\pi)^3} \int dq \frac{d(q^3 E)}{dq} f \\
 &= -H(a)g \int \frac{d\Omega}{(2\pi)^3} \int dq (3q^2 E + q^3 \frac{dE}{dq}) f \\
 d(E^2 = q^2 + m^2) &\rightarrow E dE = q dq \\
 &= -3H(a)g \int \frac{d^3q}{(2\pi)^3} (E + \frac{q^2}{3E}) f = -3H(a)(\rho + p)
 \end{aligned}$$

as derived previously from energy conservation

Boltzmann Equation

- Boltzmann equation says that Liouville theorem must be modified to account for collisions

$$\frac{Df}{Dt} = C[f]$$

- Heuristically

$$C[f] = \text{particle sources} - \text{sinks}$$

- Collision term: integrate over phase space of incoming particles, connect to outgoing state with some interaction strength

Boltzmann Equation

- Form:

$$C[f] = \int d(\text{phase space}) [\text{energy-momentum conservation}] \\ \times |M|^2 [\text{emission} - \text{absorption}]$$

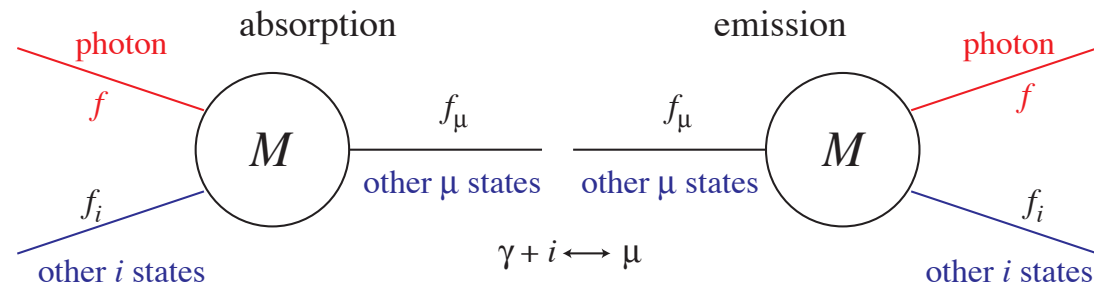
- Matrix element M , assumed T [or CP] invariant
- (Lorentz invariant) phase space element

$$\int d(\text{phase space}) = \prod_i \frac{g_i}{(2\pi)^3} \int \frac{d^3 q_i}{2E_i}$$

- Energy conservation: $(2\pi)^4 \delta^{(4)}(q_1 + q_2 + \dots)$

Boltzmann Equation

- Emission - absorption term involves the particle occupation of the various states
- For concreteness: take f to be the photon distribution function
- Interaction ($\gamma + \sum i \leftrightarrow \sum \mu$); sums are over all incoming and outgoing other particles



- [emission-absorption] $+$ = boson; $-$ = fermion

$$\prod_i \prod_\mu f_\mu (1 \pm f_i)(1 \pm f) - \prod_i \prod_\mu (1 \pm f_\mu) f_i f$$

Boltzmann Equation

- Photon Emission: $f_\mu(1 \pm f_i)(1 + f)$

f_μ : proportional to number of emitters

$(1 \pm f_i)$: if final state is occupied and a fermion, process blocked;
if boson the process enhanced

$(1 + f)$: final state factor for photons: “1”: spontaneous emission
(remains if $f = 0$); “+ f ”: stimulated and proportional to the
occupation of final photon

- Photon Absorption: $-(1 \pm f_\mu)f_i f$

$(1 \pm f_\mu)$: if final state is occupied and fermion, process blocked; if
boson the process enhanced

f_i : proportional to number of absorbers

f : proportional to incoming photons

Boltzmann Equation

- If interactions are rapid they will establish an equilibrium distribution where the distribution functions no longer change

$$C[f_{\text{eq}}] = 0$$

- Solve by inspection

$$\prod_i \prod_\mu f_\mu (1 \pm f_i)(1 \pm f) - \prod_i \prod_\mu (1 \pm f_\mu) f_i f = 0$$

- Try $f_a = (e^{-E_a/T} \mp 1)^{-1}$ so that $(1 \pm f_a) = e^{-E_a/T} (e^{-E_a/T} \mp 1)^{-1}$

$$e^{-\sum(E_i+E)/T} - e^{-\sum E_\mu/T} = 0$$

and energy conservation says $E + \sum E_i = \sum E_\mu$, so identity is satisfied if the constant T is the same for all species

Boltzmann Equation

- If the interaction does not create or destroy particles of type f (or types $i, \mu \dots$) then the distribution

$$f_{\text{eq}} = (e^{-(E-\mu)/T} \mp 1)^{-1}$$

also solves the equilibrium equation: e.g. a scattering type reaction

$$\gamma_E + i \rightarrow \gamma_{E'} + j$$

$$\sum E_i + (E - \mu) = \sum E_j + (E' - \mu) = 0$$

since the chemical potential μ does not depend on the photon energy, likewise if f is a fermion

- Not surprisingly, this is the Fermi-Dirac distribution for fermions and the Bose-Einstein distribution for bosons

Boltzmann Equation

- Even more generally, for a single reaction, the other species can carry chemical potentials too so long as

$$\sum \mu_i + \mu = \sum \mu_\nu$$

the law of mass action is satisfied

- This general rule applies to interactions that freely create or destroy the particles - e.g. $\gamma + e^- \rightarrow 2\gamma + e^-$

$$\mu_e + \mu = \mu_e + 2\mu \rightarrow \mu = 0$$

so that the chemical potential is driven to zero if particle number is not conserved in interaction

Maxwell Boltzmann Distribution

- For the nonrelativistic limit $E = m + \frac{1}{2}q^2/m$, $E/T \gg 1$ so both distributions go to the Maxwell-Boltzmann distribution

$$f_{\text{eq}} = \exp[-(m - \mu)/T] \exp(-q^2/2mT)$$

- Here it is even clearer that the chemical potential μ is the normalization parameter for the number density of particles whose number is conserved.
- μ and n can be used interchangeably

Poor Man's Boltzmann Equation

- Non expanding medium

$$\frac{\partial f}{\partial t} = \Gamma (f - f_{\text{eq}})$$

where Γ is some rate for collisions

- Add in expansion in a homogeneous medium

$$\frac{\partial f}{\partial t} + \frac{dq}{dt} \frac{\partial f}{\partial q} = \Gamma (f - f_{\text{eq}})$$

$$(q \propto a^{-1} \rightarrow \frac{1}{q} \frac{dq}{dt} = -\frac{1}{a} \frac{da}{dt} = H)$$

$$\frac{\partial f}{\partial t} - H \frac{\partial f}{\partial \ln q} = \Gamma (f - f_{\text{eq}})$$

- So equilibrium will be maintained if collision rate exceeds expansion rate $\Gamma > H$

Non-Relativistic Bulk Properties

- Number density

$$\begin{aligned}n &= g e^{-(m-\mu)/T} \frac{4\pi}{(2\pi)^3} \int_0^\infty q^2 dq \exp(-q^2/2mT) \\&= g e^{-(m-\mu)/T} \frac{2^{3/2}}{2\pi^2} (mT)^{3/2} \int_0^\infty x^2 dx \exp(-x^2) \\&= g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}\end{aligned}$$

- Energy density $E = m \rightarrow \rho = mn$
- Pressure $q^2/3E = q^2/3m \rightarrow p = nT$, ideal gas law

Ultra-Relativistic Bulk Properties

- Chemical potential $\mu = 0$, $\zeta(3) \approx 1.202$
- Number density

$$n_{\text{boson}} = gT^3 \frac{\zeta(3)}{\pi^2} \quad \zeta(n+1) \equiv \frac{1}{n!} \int_0^\infty \frac{x^n}{e^x - 1}$$

$$n_{\text{fermion}} = \frac{3}{4} gT^3 \frac{\zeta(3)}{\pi^2}$$

- Energy density

$$\rho_{\text{boson}} = gT^4 \frac{3}{\pi^2} \zeta(4) = gT^4 \frac{\pi^2}{30}$$

$$\rho_{\text{fermion}} = \frac{7}{8} gT^4 \frac{3}{\pi^2} \zeta(4) = \frac{7}{8} gT^4 \frac{\pi^2}{30}$$

- Pressure $q^2/3E = E/3 \rightarrow p = \rho/3$, $w_r = 1/3$

Entropy Density

- First law of thermodynamics

$$dS = \frac{1}{T}(d\rho(T)V + p(T)dV)$$

so that

$$\left. \frac{\partial S}{\partial V} \right|_T = \frac{1}{T}[\rho(T) + p(T)]$$
$$\left. \frac{\partial S}{\partial T} \right|_V = \frac{V}{T} \frac{d\rho}{dT}$$

- Since $S(V, T) \propto V$ is extensive

$$S = \frac{V}{T}[\rho(T) + p(T)] \quad \sigma = S/V = \frac{1}{T}[\rho(T) + p(T)]$$

Entropy Density

- Integrability condition $dS/dV dT = dS/dT dV$ relates the evolution of entropy density

$$\begin{aligned}\frac{d\sigma}{dT} &= \frac{1}{T} \frac{d\rho}{dT} \\ \frac{d\sigma}{dt} &= \frac{1}{T} \frac{d\rho}{dt} = \frac{1}{T} [-3(\rho + p)] \frac{d \ln a}{dt} \\ \frac{d \ln \sigma}{dt} &= -3 \frac{d \ln a}{dt} \quad \sigma \propto a^{-3}\end{aligned}$$

comoving entropy density is conserved in thermal equilibrium

- For ultra relativistic bosons $s_{\text{boson}} = 3.602 n_{\text{boson}}$; for fermions factor of $7/8$ from energy density.

$$g_* = \sum_{\text{bosons}} g_b + \frac{7}{8} \sum g_f$$

Neutrino Freezeout

- Neutrino equilibrium maintained by weak interactions, e.g.
 $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$
- Weak interaction cross section $T_{10} = T/10^{10} K \sim T/1\text{MeV}$

$$\sigma_w \sim G_F^2 E_\nu^2 \approx 4 \times 10^{-44} T_{10}^2 \text{cm}^2$$

- Rate $\Gamma = n_\nu \sigma_w = H$ at $T_{10} \sim 3$ or $t \sim 0.2\text{s}$
- After neutrino freezeout, electrons and positrons annihilate dumping their entropy into the photons
- Before g_* : $\gamma, e^+, e^- = 2 + \frac{7}{8}(2 + 2) = \frac{11}{2}$
- After g_* : $\gamma = 2$; so conservation of entropy gives

$$g_* T^3 \Big|_{\text{initial}} = g_* T^3 \Big|_{\text{final}} \quad T_\nu = \left(\frac{4}{11} \right)^{1/3} T_\gamma$$

Relic Neutrinos

- Relic number density (zero chemical potential; now required by oscillations & BBN)

$$n_\nu = n_\gamma \frac{3}{4} \frac{4}{11} = 112 \text{cm}^{-3}$$

- Relic energy density assuming one species with finite m_ν :

$$\rho_\nu = m_\nu n_\nu$$

$$\rho_\nu = 112 \frac{m_\nu}{\text{eV}} \text{eV cm}^{-3} \quad \rho_c = 1.05 \times 10^4 h^2 \text{eV cm}^{-3}$$

$$\Omega_\nu h^2 = \frac{m_\nu}{93.7 \text{eV}}$$

- Candidate for dark matter? an eV mass neutrino goes non relativistic around $z \sim 1000$ and retains a substantial velocity dispersion σ_ν .

Hot Dark Matter

- Momenta for a nonrelativistic species redshifts like temperature for a relativistic one, so average momentum is still given by

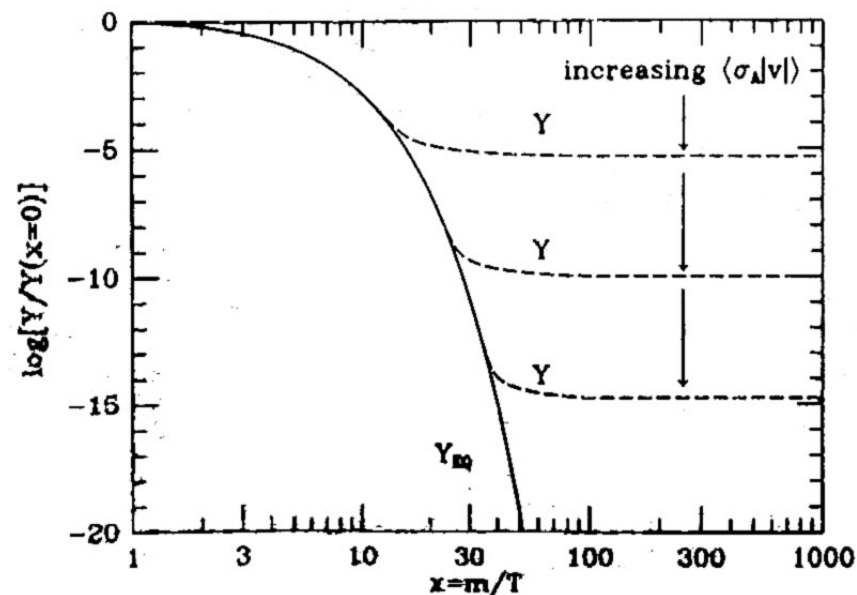
$$\langle q \rangle = 3T_\nu = m\sigma_\nu$$

$$\begin{aligned}\sigma_\nu &= 3 \left(\frac{m_\nu}{1\text{eV}} \right)^{-1} \left(\frac{T_\nu}{1\text{eV}} \right) = 3 \left(\frac{m_\nu}{1\text{eV}} \right)^{-1} \left(\frac{T_\nu}{10^4\text{K}} \right) \\ &= 6 \times 10^{-4} \left(\frac{m_\nu}{1\text{eV}} \right)^{-1} = 200\text{km/s} \left(\frac{m_\nu}{1\text{eV}} \right)^{-1}\end{aligned}$$

- on order the rotation velocity of galactic halos and higher at higher redshift - small objects can't form: top down structure formation – not observed – must not constitute the bulk of the dark matter

Cold Dark Matter

- Problem with neutrinos is they decouple while relativistic and hence have a comparable number density to photons - for a reasonable energy density, the mass must be small



- The equilibrium distribution for a non-relativistic species declines exponentially beyond the mass threshold

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

- Exponential will eventually win soon after $T < m$, suppressing annihilation rates

WIMP Miracle

- Freezeout when annihilation rate equal expansion rate $\Gamma \propto \sigma_A$, increasing annihilation cross section decreases abundance

$$\Gamma = n \langle \sigma_A v \rangle = H$$

$$H \propto T^2 \sim m^2$$

$$\rho_{\text{freeze}} = mn \propto \frac{m^3}{\langle \sigma_A v \rangle}$$

$$\rho_c = \rho_{\text{freeze}} (T/T_0)^{-3} \propto \frac{1}{\langle \sigma_A v \rangle}$$

independently of the mass of the CDM particle

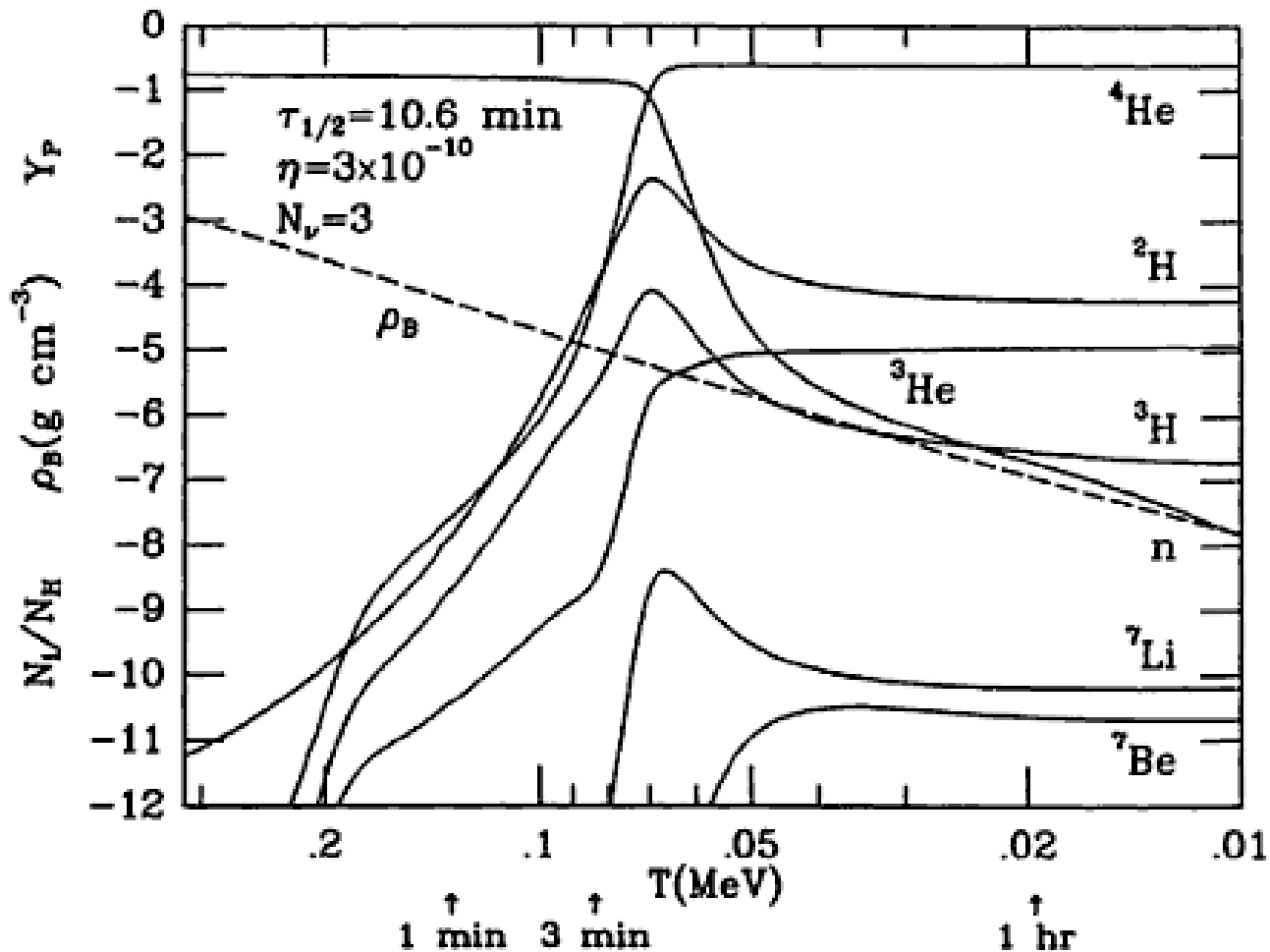
- Plug in some typical numbers for supersymmetric candidates or WIMPs (weakly interacting massive particles) of $\langle \sigma_A v \rangle \approx 10^{-36} \text{ cm}^2$ and restore the proportionality constant $\Omega_c h^2$ is of the right order of magnitude (~ 0.1)!

Axions

- Alternate solution: keep light particle but not created in thermal equilibrium
- Example: axion dark matter - particle that solves the strong CP problem
- Inflation sets initial conditions, fluctuation from potential minimum
- Once Hubble scale smaller than the mass scale, field unfreezes
- Coherent oscillations of the axion field - condensate state. Can be very light $m \ll 1\text{eV}$ and yet remain cold.
- Same reason a quintessence dark energy candidate must be lighter than the Hubble scale today

Big Bang Nucleosynthesis

- Integrating the Boltzmann equation for nuclear processes during first few minutes leads to synthesis and freezeout of light elements



Big Bang Nucleosynthesis

- Most of light element synthesis can be understood through nuclear statistical equilibrium and reaction rates
- Equilibrium abundance of species with mass number A and charge Z (Z protons and $A - Z$ neutrons)

$$n_A = g_A \left(\frac{m_A T}{2\pi} \right)^{3/2} e^{(\mu_A - m_A)/T}$$

- In chemical equilibrium with protons and neutrons

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

$$n_A = g_A \left(\frac{m_A T}{2\pi} \right)^{3/2} e^{-m_A/T} e^{(Z\mu_p + (A - Z)\mu_n)/T}$$

Big Bang Nucleosynthesis

- Eliminate chemical potentials with n_p, n_n

$$e^{\mu_p/T} = \frac{n_p}{g_p} \left(\frac{2\pi}{m_p T} \right)^{3/2} e^{m_p/T}$$

$$e^{\mu_n/T} = \frac{n_n}{g_n} \left(\frac{2\pi}{m_n T} \right)^{3/2} e^{m_n/T}$$

$$n_A = g_A g_p^{-Z} g_n^{Z-A} \left(\frac{m_A T}{2\pi} \right)^{3/2} \left(\frac{2\pi}{m_p T} \right)^{3Z/2} \left(\frac{2\pi}{m_n T} \right)^{3(A-Z)/2} \\ \times e^{-m_A/T} e^{(Z\mu_p + (A-Z)\mu_n)/T} n_p^Z n_n^{A-Z}$$

$$(g_p = g_n = 2; m_p \approx m_n = m_b = m_A/A)$$

$$(B_A = Zm_p + (A-Z)m_n - m_A)$$

$$= g_A 2^{-A} \left(\frac{2\pi}{m_b T} \right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} e^{B_A/T}$$

Big Bang Nucleosynthesis

- Convenient to define abundance fraction

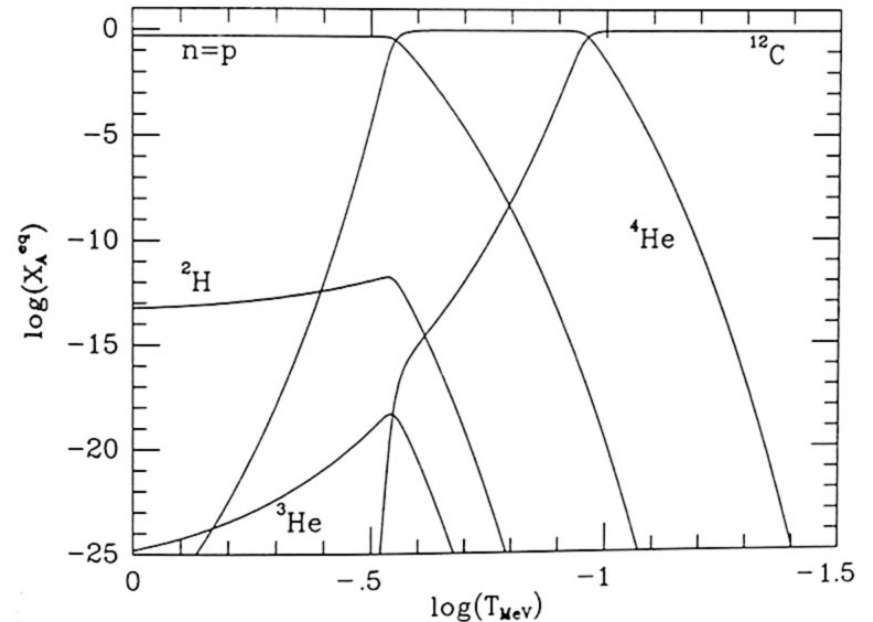
$$\begin{aligned}
 X_A &\equiv A \frac{n_A}{n_b} = A g_A 2^{-A} \left(\frac{2\pi}{m_b T} \right)^{3(A-1)/2} A^{3/2} n_p^Z n_n^{A-Z} n_b^{-1} e^{B_A/T} \\
 &= A g_A 2^{-A} \left(\frac{2\pi n_b^{2/3}}{m_b T} \right)^{3(A-1)/2} A^{3/2} e^{B_A/T} X_p^Z X_n^{A-Z} \\
 &\quad \left(n_\gamma = \frac{2}{\pi^2} T^3 \zeta(3) \quad \eta_{b\gamma} \equiv n_b / n_\gamma \right) \\
 &= A^{5/2} g_A 2^{-A} \left[\left(\frac{2\pi T}{m_b} \right)^{3/2} \frac{2\zeta(3)\eta_{b\gamma}}{\pi^2} \right]^{A-1} e^{B_A/T} X_p^Z X_n^{A-Z}
 \end{aligned}$$

Deuterium

- Deuterium $A = 2$, $Z = 1$, $g_2 = 3$, $B_2 = 2.225$ MeV

$$X_2 = \frac{3}{\pi^2} \left(\frac{4\pi T}{m_b} \right)^{3/2} \eta_{b\gamma} \zeta(3) e^{B_2/T} X_p X_n$$

- Deuterium
“bottleneck” is mainly
due to the low baryon-photon
number of the universe
 $\eta_{b\gamma} \sim 10^{-9}$, secondarily due
to the low binding energy B_2



Deuterium

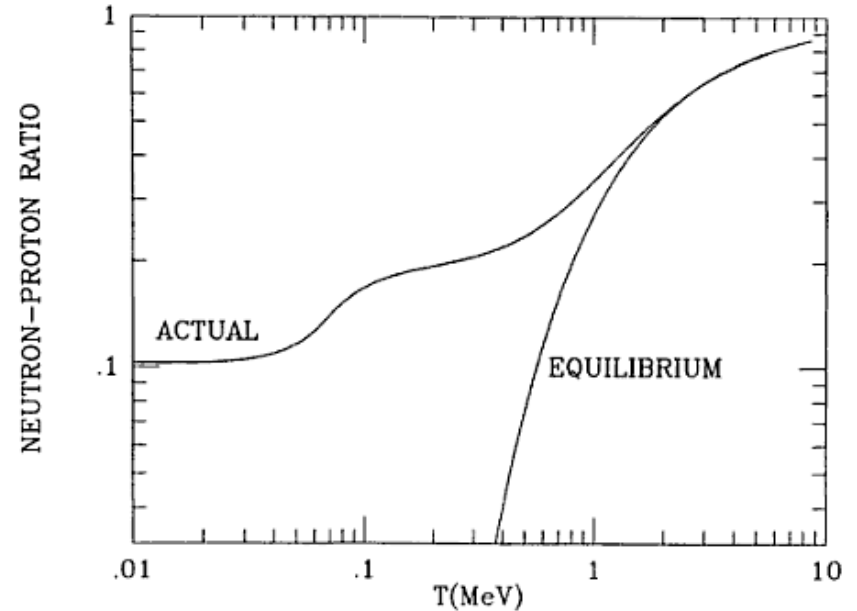
- $X_2/X_p X_n \approx \mathcal{O}(1)$ at $T \approx 100\text{keV}$ or 10^9 K, much lower than the binding energy B_2
- Most of the deuterium formed then goes through to helium via
 $D + D \rightarrow {}^3\text{He} + p$, ${}^3\text{He} + D \rightarrow {}^4\text{He} + n$
- Deuterium freezes out as number abundance becomes too small to maintain reactions $n_D = \text{const.}$ independent of n_b
- The deuterium freezeout fraction $n_D/n_b \propto \eta_{b\gamma}^{-1} \propto (\Omega_b h^2)^{-1}$ and so is fairly sensitive to the baryon density.
- Observations of the ratio in quasar absorption systems give $\Omega_b h^2 \approx 0.02$

Helium

- Essentially all neutrons around during nucleosynthesis end up in Helium
- In equilibrium, the neutron-to-proton ratio is determined by the mass difference

$$Q = m_n - m_p = 1.293 \text{ MeV}$$

$$\frac{n_n}{n_p} = \exp[-Q/T]$$



Helium

- Equilibrium is maintained through weak interactions, e.g.
 $n \leftrightarrow p + e^- + \bar{\nu}, \nu + n \leftrightarrow p + e^-, e^+ + n \leftrightarrow p + \bar{\nu}$ with rate

$$\frac{\Gamma}{H} \approx \frac{T}{0.8\text{MeV}}$$

- Freezeout fraction

$$\frac{n_n}{n_p} = \exp[-1.293/0.8] \approx 0.2$$

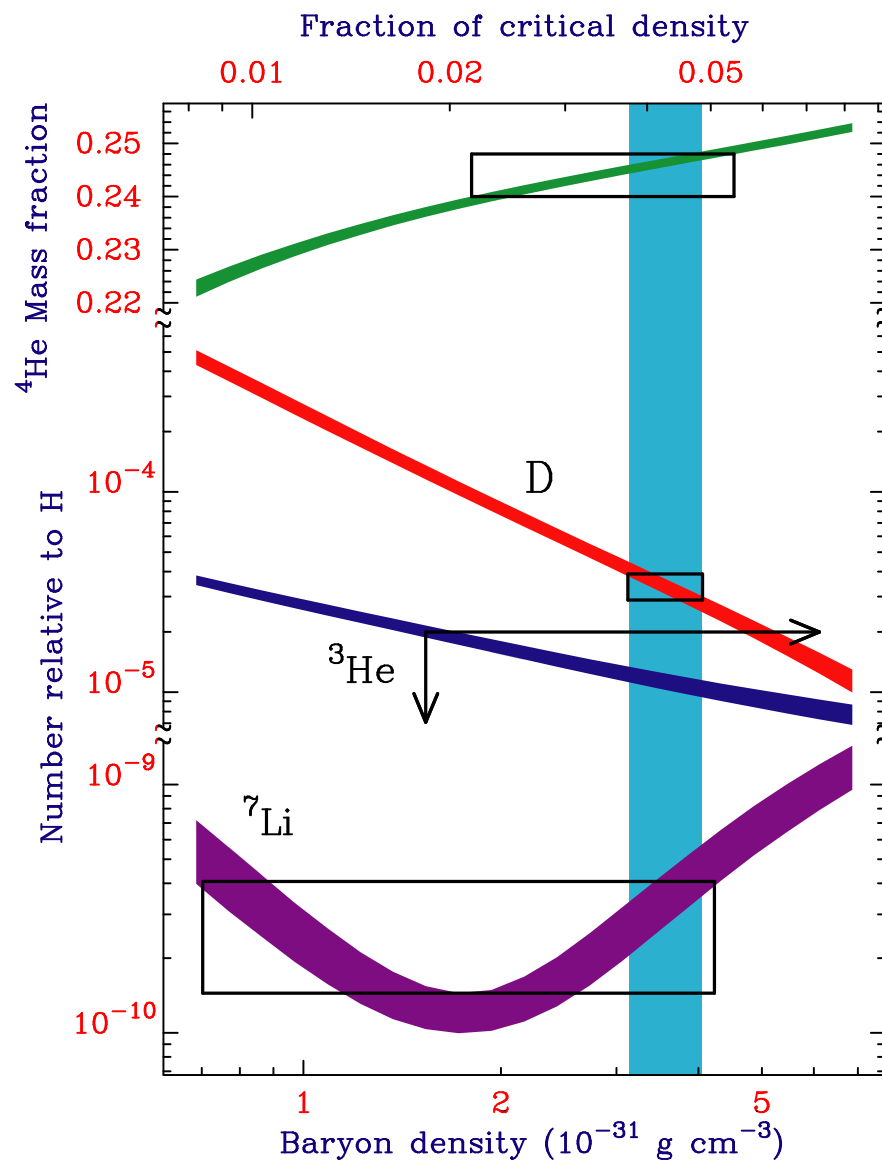
- Finite lifetime of neutrons brings this to $\sim 1/7$ by 10^9K
- Helium mass fraction

$$\begin{aligned} Y_{\text{He}} &= \frac{4n_{\text{He}}}{n_b} = \frac{4(n_n/2)}{n_n + n_p} \\ &= \frac{2n_n/n_p}{1 + n_n/n_p} \approx \frac{2/7}{8/7} \approx \frac{1}{4} \end{aligned}$$

Helium

- Depends mainly on the expansion rate during BBN - measure number of relativistic species
- Traces of ${}^7\text{Li}$ as well. Measured abundances in reasonable agreement with deuterium measure $\Omega_b h^2 = 0.02$ but the detailed interpretation is still up for debate

Light Elements



Burles, Nollett, Turner (1999)

Baryogenesis

- What explains the small, but non-zero, baryon-to-photon ratio?

$$\eta_{b\gamma} = n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2 \approx 6 \times 10^{-10}$$

- Must be a slight excess of baryons b to anti-baryons \bar{b} that remains after annihilation
- Sakharov conditions
 - Baryon number violation: some process must change the net baryon number
 - CP violation: process which produces b and \bar{b} must differ in rate
 - Out of equilibrium: else equilibrium distribution with vanishing chemical potential (processes exist which change baryon number) gives equal numbers for b and \bar{b}
- Expanding universe provides 3; physics must provide 1,2

Baryogenesis

- Example: out of equilibrium decay of some heavy boson X , \bar{X}
- Suppose X decays through 2 channels with baryon number b_1 and b_2 with branching ratio r and $1 - r$ leading to a change in the baryon number per decay of

$$rb_1 + (1 - r)b_2$$

- And \bar{X} to $-b_1$ and $-b_2$ with ratio \bar{r} and $1 - \bar{r}$

$$-\bar{r}b_1 - (1 - \bar{r})b_2$$

- Net production

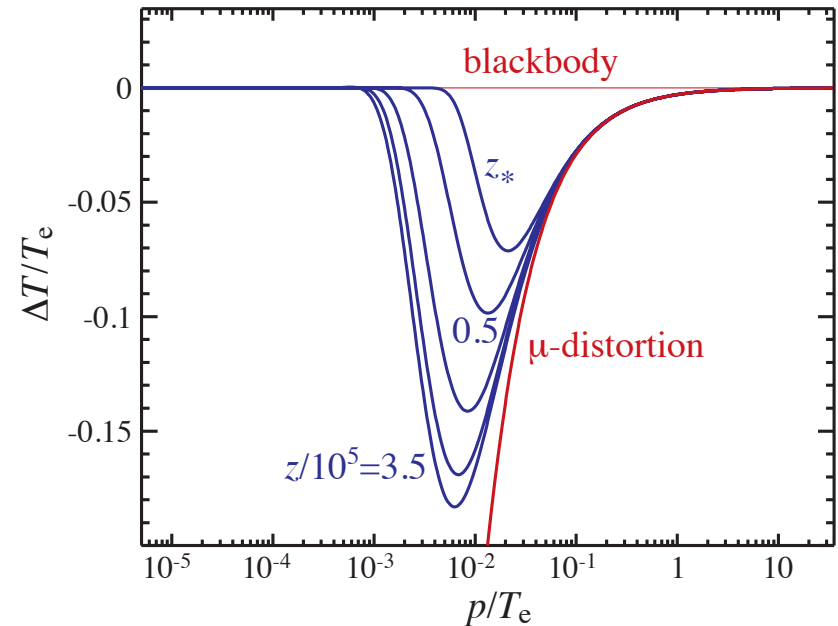
$$\Delta b = (r - \bar{r})(b_1 - b_2)$$

Baryogenesis

- Condition 1: $b_1 \neq 0, b_2 \neq 0$
- Condition 2: $\bar{r} \neq r$
- Condition 3: out of equilibrium decay
- GUT and electroweak (instanton) baryogenesis mechanisms exist
- Active subject of research

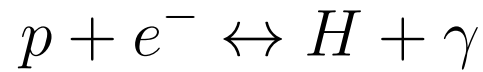
Black Body Formation

- After $z \sim 10^6$, photon creating processes $\gamma + e^- \leftrightarrow 2\gamma + e^-$ and bremsstrahlung $e^- + p \leftrightarrow e^- + p + \gamma$ drop out of equilibrium for photon energies $E \sim T$.
- Compton scattering remains effective in redistributing energy via exchange with electrons
- Out of equilibrium processes like decays leave residual photon chemical potential imprint
- Observed black body spectrum places tight constraints on any that might dump energy into the CMB



Recombination

- Maxwell-Boltzmann distribution determines the equilibrium distribution for reactions, e.g. big-bang nucleosynthesis, recombination:



$$\frac{n_p n_e}{n_H} \approx e^{-B/T} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{(\mu_p + \mu_e - \mu_H)/T}$$

where $B = m_p + m_e - m_H = 13.6\text{eV}$ is the binding energy, $g_p = g_e = \frac{1}{2}g_H = 2$, and $\mu_p + \mu_e = \mu_H$ in equilibrium

- Define ionization fraction

$$n_p = n_e = x_e n_b$$

$$n_H = n_b - n_p = (1 - x_e) n_b$$

Recombination

- Saha Equation

$$\begin{aligned}\frac{n_e n_p}{n_H n_b} &= \frac{x_e^2}{1 - x_e} \\ &= \frac{1}{n_b} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-B/T}\end{aligned}$$

- Naive guess of $T_* = B$ wrong due to the low baryon-photon ratio
– $T_* \approx 0.3\text{eV}$ so recombination at $z_* \approx 1000$
- But the photon-baryon ratio is very low

$$\eta_{b\gamma} \equiv n_b/n_\gamma \approx 3 \times 10^{-8} \Omega_b h^2$$

Recombination

- Eliminate in favor of $\eta_{b\gamma}$ and B/T through

$$n_\gamma = 0.244T^3, \quad \frac{m_e}{B} = 3.76 \times 10^4$$

- Big coefficient

$$\frac{x_e^2}{1 - x_e} = 3.16 \times 10^{15} \left(\frac{B}{T} \right)^{3/2} e^{-B/T}$$

$$T = 1/3\text{eV} \rightarrow x_e = 0.7, \quad T = 0.3\text{eV} \rightarrow x_e = 0.2$$

- Further delayed by inability to maintain equilibrium since net is through 2γ process and redshifting out of line

Recombination

