The CMB Power Spectrum

A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND DAMPING TAIL FROM THE 2500-SQUARE-DEGREE SPT-SZ SURVEY

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Photo Credit: Daniel and Dana; Sunset 2011
a.k.a.
A MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND DAMPING TAIL FROM THE 2500-SQUARE-DEGREE SPT-SZ SURVEY

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ABSTRACT

We present a measurement of the cosmic microwave background (CMB) temperature power spectrum using data from the recently completed South Pole Telescope Sunyaev-Zel’dovich (SPT-SZ) survey. This measurement is made from observations of 2540 deg^2 of sky with arcminute resolution at 150 GHz, and improves upon previous measurements using the SPT by tripling the sky area. We report CMB temperature anisotropy power over the multipole range 650 < ℓ < 3000. We fit the SPT bandpowers, combined with the results from the seven-year Wilkinson Microwave Anisotropy Probe (WMAP7) data release, with a six-parameter ΛCDM cosmological model and find that the two datasets are consistent and well fit by the model. Adding SPT measurements significantly improves ΛCDM parameter constraints, and in particular tightens the constraint on the angular sound horizon θ_s by a factor of 2.7. The impact of gravitational lensing on the CMB power spectrum is detected with 8.1σ, the most significant detection to date. The inferred amplitude of the lensing spectrum is consistent with the ΛCDM prediction. This sensitivity of the SPT+WMAP7 data to lensing by large-scale structure at low redshifts allows us to constrain the mean curvature of the observable universe with CMB data alone to be Ω_k = −0.003±0.014. Using the SPT+WMAP7 data, we measure the spectral index of scalar fluctuations to be n_s = 0.9623 ± 0.0097 in the ΛCDM model, a 3.9σ preference for a scale-dependent spectrum with n_s < 1. The SPT measurement of the CMB damping tail helps break the degeneracy that exists between the tensor-to-scalar ratio r and n_s in large-scale CMB measurements, leading to an upper limit of r < 0.18 (95% C.L.) in the ΛCDM+r model. Adding low-redshift measurements of the Hubble constant (H_0) and the baryon acoustic oscillation (BAO) feature to the SPT+WMAP7 data leads to further improvements. The combination of SPT+WMAP7+H_0+BAO constrains n_s = 0.9538 ± 0.0081 in the ΛCDM model, a 5.7σ detection of n_s < 1, and places an upper limit of r < 0.11 (95% C.L.) in the ΛCDM+r model. These new constraints on n_s and r have significant implications for our understanding of inflation, which we discuss in the context of selected single-field inflation models.

Subject headings: cosmology – cosmology:cosmic microwave background – cosmology: observations – large-scale structure of universe
What I hope to answer:

1) How does one actually measure the CMB power spectrum?

2) How does the SPT measurement fit into the rest of the field?

3) How does one constrain cosmology with the CMB power spectrum?
Part 1 (of 3):

How to measure a CMB Power Spectrum
Step 1: Build a 10-meter telescope at the South Pole
Why the South Pole?

- Atmospheric transparency and stability:
  - Extremely high (~10,500 feet), dry, and cold.
  - Sun below horizon for 6 months.
- Unique geographical location:
  - Observe the clearest views through the Galaxy, 24/365
  - Clean horizon.
- Excellent support from existing research station.
The South Pole Telescope

- Millimeter-wavelength telescope
- Located at the South Pole (dry)
- 10 meter primary mirror
- high-angular resolution (~1 arcminute)

photo by Dana Hrubes
Step 2: Observe 2500 deg$^2$ of sky
SPT-SZ 2500 deg$^2$ survey

Status: finished in Nov. 2011.
Observation Strategy:

- 19 observation fields
- 3 frequencies (90, 150, and 220 GHz)
- Avoid the Galaxy
Observation Strategy:
Sweep in azimuth, then step in elevation
Observe 1 field to desired noise level, then move to next field
The result is maps of 19 fields,
The result is maps of 19 fields, per field X ~200.

ra22h30dec-55
Single Observation

Monday, November 5, 2012
The result is maps of 19 fields, mostly noise (atmosphere).
How do we calculate a power spectrum from the maps?

- **Cross-correlate** and average all pairs of observations (noise doesn’t correlate)

- Correct for **filtering of time-ordered data**, PSF, mode-coupling from finite sky.

- Estimate **errors** from simulations and data.
How do we calculate a power spectrum from the maps?

- Cross-correlate and average all pairs of observations (noise doesn’t correlate)
- Correct for filtering of time-ordered data, PSF, mode-coupling from finite sky.
- Estimate errors from simulations and data.
SPT “Pseudo-Cl” Analysis

Key concept:

The CMB signal is spatially correlated, while the noise is *uncorrelated*.

By cross-correlating observations, the CMB signal can be recovered.
SPT “Pseudo-Cl” Analysis

Fourier Transform

Basic Idea:

1) 2D Fourier Transform* each map

\[
\hat{D}^{AB}_b \equiv \left\langle \frac{\ell(\ell + 1)}{2\pi} H_\ell \text{Re}[\tilde{m}^A_\ell \tilde{m}^B*_\ell] \right\rangle_{\ell \in b}
\]

*Flat Sky approximation
Basic Idea:

1) 2D Fourier Transform each map

2) Cross-correlate all pairs of maps
SPT “Pseudo-Cl” Analysis

Noise weight

Basic Idea:
1) 2D Fourier Transform each map
2) Cross-correlate all pairs of maps

Account for anisotropic noise

\[ \hat{D}_{AB}^b \equiv \left\langle \frac{\ell(\ell+1)}{2\pi} H_\ell H_\ell \text{Re}[\hat{m}_\ell^A \hat{m}_\ell^B^*] \right\rangle_{\ell \in b} \]
Basic Idea:
1) 2D Fourier Transform each map

2) Cross-correlate all pairs of maps

3) Average over ell (assume isotropy)
Basic Idea:

1) 2D Fourier Transform each map

2) Cross-correlate all pairs of maps

3) Average over ell (assume isotropy)

4) Average over pairs A&B

\[ \hat{D}^{AB}_b \equiv \left\langle \frac{\ell(\ell + 1)}{2\pi} H_\ell \text{Re}[\tilde{m}^A_\ell \tilde{m}^{B*}_\ell] \right\rangle_{\ell \in b} \]
SPT “Pseudo-Cl” Analysis

Average over Fields

Basic Idea:

1) 2D Fourier Transform each map

2) Cross-correlate all pairs of maps

3) Average over ell (assume isotropy)

4) Average over pairs A&B

5) Average over the 19 fields

\[
\hat{D}_{AB}^{\ell} \equiv \left\langle \frac{\ell(\ell + 1)}{2\pi} H_{\ell} \text{Re}[\tilde{m}^A_{\ell} \tilde{m}^B_{\ell}^*] \right\rangle_{\ell \epsilon b}
\]
How do we calculate a power spectrum from the maps?

- **Cross-correlate** and average all pairs of observations (noise doesn’t correlate)

- Correct for **filtering of time-ordered data, PSF, mode-coupling** from finite sky.

- Estimate **errors** from simulations and data.
SPT “Pseudo-Cl” Analysis

In Equations:

\[ \tilde{a}_{\ell m} = \int d\hat{n} \left[ \Delta T^i(\hat{n})W(\hat{n}) \right] Y_{\ell m}(\hat{n}) \]

\[ \tilde{C}^{ii}_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m}|^2 \]

Need to explicitly account for:

\[ \left< \tilde{C}^{ii}_{\ell} \right> = B^2_{\ell} \left< C_{\ell'} \right> \]
In Equations:

\[
\tilde{a}_{\ell m} = \int d\hat{n} \left[ \Delta T^i(\hat{n}) W(\hat{n}) \right] Y_{\ell m}(\hat{n})
\]

\[
\tilde{C}_{\ell}^{ii} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m}|^2
\]

Need to explicitly account for:

• Experimental beam shape
In Equations:

$$\tilde{a}^i_{\ell m} = \int d\hat{n} \left[ \Delta T^i(\hat{n}) W(\hat{n}) \right] Y_{\ell m}(\hat{n})$$

$$\tilde{C}^{ii}_{\ell} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |\tilde{a}^i_{\ell m}|^2$$

Need to explicitly account for:
- Experimental beam shape
- Filtering of timestream data

$$\langle \tilde{C}^{ii}_{\ell} \rangle = F_{\ell} B^2_{\ell} \langle C_{\ell'} \rangle$$
Account for timestream filtering: “Transfer Function”

- Simulate 100 CMB skies (healpix)
- Project onto flat-sky maps for 19 fields
- Calculate the power spectrum from simulated maps
- Transfer Function
  \[ F_l = \frac{C_l^{\text{input}}}{C_l^{\text{measured}}} \]
SPT "Pseudo-Cl" Analysis

In Equations:

\[ \tilde{a}_{\ell m}^i = \int d\hat{n} \left[ \Delta T^i(\hat{n})W(\hat{n}) \right] Y_{\ell m}(\hat{n}) \]

\[ \tilde{C}_{\ell}^{ii} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m}^i|^2 \]

Need to explicitly account for:

- Experimental beam shape
- Filtering of timestream data
- Masking for unwanted sources
Account for masking: “Mode coupling”

- Apodize edges of each field
- Mask point-sources
  (Unmasked point-sources are accounted for in the cosmological fits)

Analytically calculate the mode coupling:
SPT “Pseudo-Cl” Analysis

In Equations:
\[
\tilde{a}_{\ell m}^i = \int d\hat{n} \left[ \Delta T^i(\hat{n}) W(\hat{n}) \right] Y_{\ell m}(\hat{n})
\]
\[
\tilde{C}_{\ell}^{ii} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m}^i|^2
\]

Need to explicitly account for:
• Experimental beam shape
• Filtering of timestream data
• Masking for unwanted sources
• Avoid noise bias with cross-spectra (no noise model required)
How do we calculate a power spectrum from the maps?

- **Cross-correlate** and average all pairs of observations (noise doesn’t correlate)

- Correct for **filtering of time-ordered data, PSF, mode-coupling** from finite sky.

- Estimate **errors** from simulations and data.
Bandpower uncertainties contain contributions from: **noise, sample variance, and other uncertainties**

\[ \text{Cov} = \text{Cov\_noise} + \text{Cov\_sv} + \text{other uncertainties} \]
Bandpower uncertainties contain contributions from: **noise, sample variance, and other uncertainties**

\[
\text{Cov} = \text{Cov\_noise} + \text{Cov\_sv} + \text{other uncertainties}
\]

1) **Cov\_noise is the variance between the cross-spectra:**

\[
\text{cov\_ij} = \langle (dl\_i - \mu) \times (dl\_j - \mu) \rangle
\]

Where \( \mu = \langle dl \rangle \)
Bandpower uncertainties contain contributions from: \textbf{noise, sample variance, and other uncertainties}

\[ \text{Cov} = \text{Cov}_{\text{noise}} + \text{Cov}_{\text{sv}} + \text{other uncertainties} \]

1) \text{Cov}_{\text{noise}} is the variance between the cross-spectra:

\[ \text{cov}_{ij} = \langle (d\ell_i - \mu) \times (d\ell_j - \mu) \rangle \]

Where \( \mu = \langle d\ell \rangle \)

2) \text{Cov}_{\text{sv}} is calculated directly from the 100 independent (2500 deg\(^2\)) simulated spectra.
Bandpower uncertainties contain contributions from: noise, sample variance, and other uncertainties

\[ \text{Cov} = \text{Cov}\_\text{noise} + \text{Cov}\_\text{sv} + \text{other uncertainties} \]

1) Cov\_noise is the variance between the cross-spectra:

\[ \text{cov}\_ij = <(dl\_i - \mu) \times (dl\_j - \mu)> \]

   Where \( \mu = <dl> \)

2) Cov\_sv is calculated directly from the 100 independent (2500 deg\(^2\)) simulated spectra.

3) Other uncertainties:
   Beam uncertainties: ell-dependent, up to 10% of error bars.
   Calibration uncertainties: 2.6% in power

Monday, November 5, 2012
Put this all together and you get ...
One Field

PS, ra22h30dec-55
Five Fields...

PS, 2008–09 fields

$D_\ell$ ($\mu K^2$)

$\ell$

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Nineteen Fields!  Full 2500 deg^2, run_07
CMB Power Spectrum

Fourier transform this

and you get....

SPT, Full Survey
Story, et al., 2012

Angular Frequency (multipole)

Power (µK²)
Part 2 (of 3): Context for the SPT power spectrum
WMAP7 Figure from Larson et al.
WMAP

$C_l$ vs Multipole moment $l$

1 DEGREE

SMALLER SCALES....

SPT ADDS INFORMATION AT SMALL ANGULAR SCALES HERE AND BEYOND

WMAP7 Figure from Larson et al.
Recent measurement from SPT

“damping tail”
Strong experimental progress

(10 years after first detection by COBE)

$\ell(\ell+1)C_\ell/2\pi (\mu K^2)$

ACBAR
CBI
VSA
DASI
Boomerang
MAXIMA
ARCHEOPS

$\ell$
Strong experimental progress
Strong experimental progress

First result from the full SPT survey!  

Story, et al., 2012

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Strong experimental progress

First result from the full SPT survey!

Story, et al., 2012

SV limited

WMAP7

SPT, Full Survey

2012

First result from the full SPT survey!

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Vs. other small-scale experiments
State of the art

Story, et al., 2012
arXiv:1210.7231

Reichardt et al., 2012
arXiv:1111:0932

![Graph showing power spectra for various experiments: WMAP 7-year, SPT 220 GHz, SPT 150 GHz, and SPT 95 GHz. The x-axis represents angular frequency (multipole) in degrees, and the y-axis represents power in $\mu K^2$. Different experiments are distinguished by different markers and colors.](image-url)
Part 3 (of 3):

How the SPT Power Spectrum constrains cosmology
The SPT spectrum adds 3 important pieces:

(1) Peak position (angular scale) and size
(2) Damping scale (slope of damping tail)
(3) Height of 3rd peak
What’s in a power spectrum?

Peak positions:
Angular size of sound horizon at recombination

Power spectrum plot with data from WMAP7 and SPT, Full Survey.
What’s in a power spectrum?

Damping scale:
Angular size of Silk damping length at recombination
(Photons diffuse out of over-dense regions, and carry baryons with them.)

Damping scale:
Angular size of Silk damping length at recombination
(Photons diffuse out of over-dense regions, and carry baryons with them.)
What’s in a power spectrum?

Angular Frequency (multipole)

Relative height of peaks:
e.g. ratio of baryon to dark matter density

Power (μK^2)

SPT, Full Survey

Relative height of peaks:
e.g. ratio of baryon to dark matter density
Fitting a model

Markov Chain Monte Carlo (MCMC)

- **CosmoMC** (Lewis & Bridle 2002)
- **CAMB** (Lewis et al. 2000)
- **PICO** (Fendt & Wandelt 2007)
The baseline model: $\Lambda$CDM

Cosmological Parameters

- **CMB** for a flat $\Lambda$CDM model with six parameters:
  \[
  \{\Omega_b h^2, \Omega_c h^2, \theta_s, \tau, n_s, \Delta^2_R\}
  \]
The baseline model: $\Lambda$CDM

**Cosmological Parameters**

- **CMB** for a flat $\Lambda$CDM model with six parameters: $\{\Omega_b h^2, \Omega_c h^2, \theta_s, \tau, n_s, \Delta^2_R\}$

**Foreground/Nuisance Parameters**

- **Sunyaev-Zel’dovich effect** [Thomson/Compton scattering off electrons in large-scale structure]
- **Poisson** [random point sources (galaxies)]
- **Clustered** point sources.

9 parameters (6 cosmo., 3 “nuisance”)
Fitting a model

Models

- Baseline: LCDM
- Extensions: LCDM+Alens
- LCDM+$\Omega_K$
- LCDM+r

$N_{eff}, \sum m_v, \omega, dn_s/dlnk$, etc $\rightarrow H12$
Fitting a model

Datasets

- **Baryon Acoustic Oscillations (BAO)**
  - WiggleZ survey (Blake et al. 2011)
  - SDSS-II (DR7) (Padmanabhan et al. 2012)
  - BOSS (Anderson et al. 2012)

- **Local Expansion Rate \( (H_0) \)**
  - Riess et al. 2012
Consistency between datasets

LCDM, 3 different datasets

BAO observable $r_s/D_v$

$H_0$ observable
Consistency between datasets

LCDM, 3 different datasets

- CMB and $H_0$ differ by 0.3 $\sigma$.
- CMB and BAO differ by 1.5 $\sigma$.
- (CMB+BAO) and $H_0$ differ by 1.8 $\sigma$.
- (CMB+$H_0$) and BAO differ by 2.1 $\sigma$.
Consistency between datasets

LCDM, 3 different datasets

Possible Explanations:
- statistical fluctuations
- systematic errors / bias
- evidence for a departure from ΛCDM
Part 3a:
Standard model of Cosmology, $(\Lambda \text{CDM})$
Quality of fit

WMAP & SPT are consistent with each other, and ΛCDM provides good fit to joint dataset.

\[ X^2 = 45.9 / 39, \text{ pte} = 0.21 \]
Basic $\Lambda$CDM results: WMAP7

WMAP7 (Baryons, Dark matter, Sound horizon)

WMAP7 (Optical depth, Slope & Amplitude of primordial fluctuations)

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Basic $\Lambda$CDM results: WMAP7+SPT

Rel. improve. from SPT:

- $\Omega_b h^2$: 1/1.5
- $\Omega_c h^2$: 1/1.4
- $100 \theta_s$: 1/2.7

- $\tau$: 1/1.07
- $n_s$: 1/1.4
- $10^9 \Delta_R^2$: 1/1.2
Part 3b : Gravitational Lensing ($\Lambda$CDM+$A_{lens}$)
Reionization, first stars

Recombination, CMB

Inflation?

Galaxies, many more stars

Large-Scale Structure, accelerated expansion

Cosmic Timeline

(300 kyr) (0.5 Gyr) (1.6 Gyr) (6.0 Gyr) (13.8 Gyr)

z~1000 z~10 z~4 z~1

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Gravitational Lensing of the CMB

Paths of CMB photons are bent by gravity of $z \sim 2$ matter.
Recombination, CMB

Inflation?

Recombination, CMB

Paths of CMB photons are bent by gravity of z~2 matter.

Distance to CMB and statistical properties of CMB known very accurately, so effects of lensing can be isolated.

Novel method for studying large-scale structure at z~2.
Unlensed vs lensed CMB

Duncan Hanson
via Kendrick Smith

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Unlensed vs lensed CMB

Duncan Hanson
via Kendrick Smith

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Unlensed vs lensed CMB

Small effect!

How do you detect it?
Lensing Smooths the Acoustic Peaks

"Lensing knob" turned from 0X to 9X.
Lensing Smooths the Acoustic Peaks

\[ C_\ell^\psi \rightarrow A_{\text{lens}} C_\ell^\psi \]

\[ A_{\text{lens}} = 0.86^{+0.15}_{-0.13} \]

Consistent with expected level (\( A_{\text{lens}} = 1.0 \))

SPT+WMAP give 8.1\( \sigma \) detection of CMB lensing.

SPT

WMAP7

CMB+foregrounds

CMB
Part 3c: Curvature and Dark Energy
(\(\Lambda CDM + \Omega_K\))
Curvature and Dark Energy:
$\Omega_K - \Omega_\Lambda$ degeneracy in large-scale data

Source: WMAP5: arXiv 0803.0586

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Curvature and Dark Energy:
\( \Omega_K - \Omega_\Lambda \) degeneracy in large-scale data

The data is consistent with:
- **Young Universe**
  - \((\text{big } \Omega_\Lambda, \Omega_K>0, \text{ large } H_0)\)
- **Old Universe**
  - \((\Omega_\Lambda =0, \Omega_K<0, \text{ small } H_0)\)
- **Flat Universe**

Source: WMAP5: arXiv 0803.0586
Curvature and Dark Energy:

$\Omega_K - \Omega_\Lambda$ degeneracy in large-scale data

The data is consistent with:

- **Young Universe**
  - (big $\Omega_\Lambda$, $\Omega_K > 0$, large $H_0$)

- **Old Universe**
  - ($\Omega_\Lambda = 0$, $\Omega_K < 0$, small $H_0$)

- **Flat Universe**

Break the degeneracy:

- low-redshift distance measures (BAO, $H_0$)
- Gravitational lensing of the CMB

Source: WMAP5: arXiv 0803.0586
Curvature and Dark Energy:

$\Omega_K - \Omega_\Lambda$ degeneracy in large-scale data

- **SPT+WMAP7:** $\Omega_K = -0.003 \pm 0.014 - 0.018$
- **CMB+BAO+H0:** $\Omega_K = -0.0059 \pm 0.0040$

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Curvature and Dark Energy:

$\Omega_K - \Omega_\Lambda$ degeneracy in large-scale data

$\Omega_K = -0.003 \pm 0.014 - 0.018$

$\Omega_\Lambda > 0$ at 5.4 $\sigma$

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Part 3d: Inflation ($\Lambda$CDM+r)
Initial perturbations

Scalar perturbations (already included)

- Perturbations in the energy density.
- The *only* perturbations which form structure due to gravitational instability (therefore only ones required in a minimal model)

\[ \Delta_R^2(k) = \Delta_R^2(k_0) \left( \frac{k}{k_0} \right)^{n_s-1} \]
Initial perturbations

Scalar perturbations (already included)

• Perturbations in the energy density.
• The only perturbations which form structure due to gravitational instability (therefore only ones required in a minimal model)

Vector perturbations

• “Eddies” in matter with $\nabla \times \mathbf{v} \neq 0$ and $\nabla \cdot \mathbf{v} = 0$
• Exponentially damped by expansion of Universe (negligible with inflation, but can be significant for defect/string models)
Initial perturbations

Scalar perturbations (already included)

- Perturbations in the energy density.
- The only perturbations which form structure due to gravitational instability (therefore only ones required in a minimal model)

Vector perturbations

- Gravity waves - transverse-traceless metric perturbations
- Generally predicted by inflation models; amplitude related to energy scale at which inflation occurs.
- Convention: amplitude parameter is $r$, the ratio of tensor to scalar power.

\[ \Delta^2_h(k) = \Delta^2_h(k_0) \left( \frac{k}{k_0} \right)^{n_t} \]
SPT Power Spectrum has 2 experimental handles:

\[ \Delta_R^2(k) = \Delta_R^2(k_0) \left( \frac{k}{k_0} \right)^{n_s-1} \]

\[ \Delta_h^2(k) = \Delta_h^2(k_0) \left( \frac{k}{k_0} \right)^{n_t} \]

\[ r = \frac{\Delta_h^2(k)}{\Delta_R^2(k)} \bigg|_{k=0.002 \text{ Mpc}^{-1}} \]

2 Parameters: \( n_s \), and \( r \)
SPT Power Spectrum has 2 experimental handles:

\[ \Delta_R^2(k) = \Delta_R^2(k_0) \left( \frac{k}{k_0} \right)^{n_s - 1} \]

\[ \Delta_h^2(k) = \Delta_h^2(k_0) \left( \frac{k}{k_0} \right)^{n_t} \]

\[ r = \left. \frac{\Delta_h^2(k)}{\Delta_R^2(k)} \right|_{k=0.002 \text{ Mpc}^{-1}} \]

2 Parameters: \( n_s \), and \( r \)
Inflation makes predictions for $n_S$ and $r$

1) The potential describes the model

*Figure from Baumann et al. 2009 (arXiv:0811.3919v2)
Inflation makes predictions for $n_s$ and $r$

1) The potential describes the model
2) $\Delta \phi$ separates “large-field” and “small-field” models.

The dividing line is ($r = 0.01$)
Inflation makes predictions for $n_s$ and $r$

"Slow-roll" parameters:

\[
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2}{2} \frac{\dot{\phi}^2}{H^2} \approx \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2
\]

\[
|\eta| \approx M_{pl}^2 \left| \frac{V''}{V} \right|
\]
Inflation makes predictions for $n_S$ and $r$

“Slow-roll” parameters:

$$
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2}{2} \frac{\dot{\phi}^2}{H^2} \approx \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2
$$

$$
|\eta| \approx M_{pl}^2 \left| \frac{V''}{V} \right|
$$

$$
n_S = 1 - 4\epsilon_{CMB} + 2\eta_{CMB}
$$

$$
r = 13.7\epsilon_{CMB}
$$
Inflation makes predictions for $n_s$ and $r$

```
V(\phi)
```

"Slow-roll" parameters:

\[
\epsilon \equiv -\frac{\ddot{H}}{H^2} = \frac{M_{pl}^2}{2} \frac{\dot{\phi}^2}{H^2} \approx \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2
\]

\[
|\eta| \approx M_{pl}^2 \left| \frac{V''}{V} \right|
\]

\[
n_s = 1 - 4\epsilon_{CMB} + 2\eta_{CMB}
\]

Inflation must end: $n_s \neq 1$
Inflation makes predictions for $n_s$ and $r$

"Slow-roll" parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2}{2} \frac{\dot{\phi}^2}{H^2} \approx \frac{M_{pl}^2}{2} \left(\frac{V'}{V}\right)^2$$

$$|\eta| \approx M_{pl}^2 \left|\frac{V''}{V}\right|.$$ 

$r = 13.7\epsilon_{CMB}.$

$$V^{1/4} = 1.06 \times 10^{16} \text{ GeV} \left(\frac{r_*}{0.01}\right)^{1/4}$$

"Energy Scale of Inflation"
In Summary:

Measure $n_s, r \Rightarrow$ constrain inflation

\[ n_s = 1 - 4\epsilon_{CMB} + 2\eta_{CMB} \]

\[ r = 13.7\epsilon_{CMB}. \]
Tensor perturbations:
Damping tail measurements break the $n_s - r$ degeneracy
SPT constraints
\( \Lambda \)CDM Results

Most significant detection of a departure from scale-invariance!

\( n_s: \)
- WMAP7 = 0.969 ± 0.014
- WMAP7+SPT (CMB) = 0.962 ± 0.010
- CMB+BAO+H_0 = 0.954 ± 0.008

\[ P(n_s > 1): \]
- WMAP7 = 1.4e-2 (2.2 \( \sigma \))
- +SPT = 3.9e-4 (3.9 \( \sigma \))
- +BAO+H_0=1.4e-9 (5.9 \( \sigma \))
Best limits on tensors!

No evidence for tensors yet; 95% upper limits are:

- **WMAP**: $r < 0.36$
- **WMAP+SPT**: $r < 0.18$
- **CMB+BAO+H0**: $r < 0.11$
Data Constraints

Weakened degeneracy between $n_s$ and $r$
Inflation Models

Models of Inflation

Exponential:

\[ V(\Phi) \sim \exp(\sqrt{\Phi^2/p}) \]
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Chaotic:
\[ V(\Phi) \sim \Phi^p \]
Inflation Models

Models of Inflation

Exponential:
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Chaotic:
\[ V(\Phi) \sim \Phi^p \]

Hilltop:
\[ V(\Phi) \sim 1-\Phi^2 \]
Exponential: 
\[ V(\Phi) \sim \exp\left(\sqrt{\Phi^2/p}\right) \]

Chaotic: 
\[ V(\Phi) \sim \Phi^p \]

Hilltop: 
\[ V(\Phi) \sim 1 - \Phi^2 \]

Small-field: 
\[ r < 0.01 \]
Summary of results from Story et al. 2012:

- Most precise measurement of the CMB damping tail from $650 < \ell < 3000$
- Detect gravitational lensing at $8.1\,\sigma$, consistent with $\Lambda$CDM (highest significance to date from CMB alone)
- Measure curvature to 1.5% accuracy with CMB alone
- Measure $n_s < 1$ at $5.9\,\sigma$ (1st >5\,\sigma detection)
- Constrain $r < 0.11$ (CMB+BAO+$H_0$)
- Place constraints on simple models of inflation

*Further extensions explored in H12*
Thanks!
**2D Kweights:** We apply a weight-mask to each map in fourier-space to down-weight noisy regions.

The 2d kweight is defined as:

\[
\text{weight}_{2d} = \frac{1}{(GS\{\text{cl} \_ \text{th}\} + GS\{\text{noise/TF}\})^2}
\]
In Pictures, the 2d kweight looks like:

\[
\frac{1}{(GS\{\text{image 1}\} + GS\{\text{image 2}\})^2}
\]
Uncertainty in Inflation predictions