Astro 321

Lecture Notes Set 8

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Halo Bias

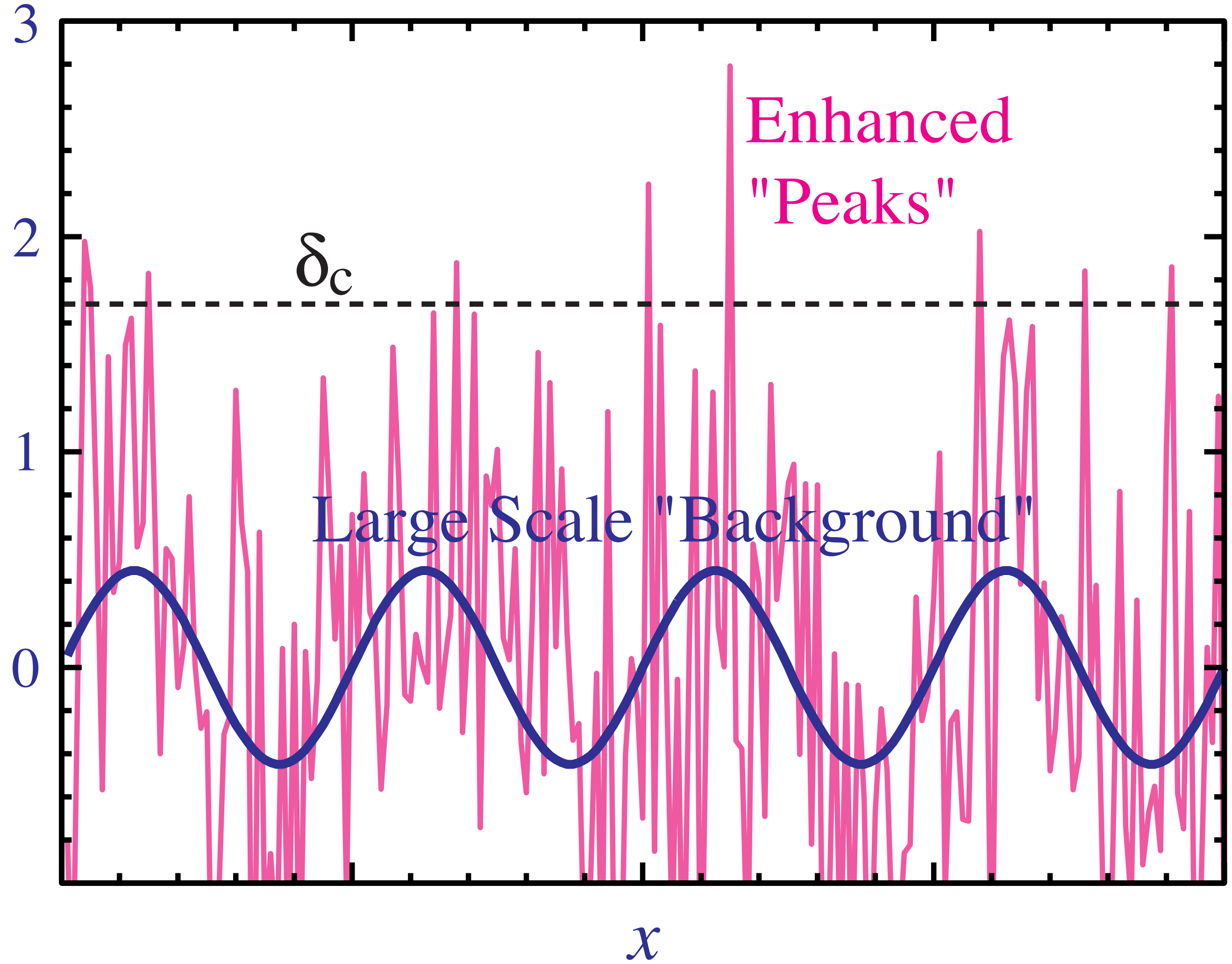
- If halos are formed without regard to the underlying density fluctuation and move under the gravitational field then their number density is an unbiased tracer of the dark matter density fluctuation

$$\left(\frac{\delta n}{n}\right)_{\rm halo} = \left(\frac{\delta \rho}{\rho}\right)$$

- However spherical collapse says the probability of forming a halo depends on the initial density field
- Large scale density field acts as “background” enhancement of probability of forming a halo or “peak”
- Peak-Background Split (Efstathiou 1998; Cole & Kaiser 1989; Mo & White 1997)

Peak-Background Split

- Schematic Picture:



Perturbed Mass Function

- Density fluctuation split

$$\delta = \delta_b + \delta_p$$

- Lowers the threshold for collapse

$$\delta_{cp} = \delta_c - \delta_b$$

so that $\nu = \delta_{cp}/\sigma$

- Taylor expand number density $n_M \equiv dn/d\ln M$

$$n_M + \frac{dn_M}{d\nu}\frac{d\nu}{d\delta_b}\delta_b \ldots = n_M\left[1 + \frac{(\nu^2-1)}{\sigma\nu}\right]$$

if mass function is given by Press-Schechter

$$n_M \propto \nu\exp(-\nu^2/2)$$

Halo Bias

- Halos are biased tracers of the "background" dark matter field with a bias $b(M)$ that is given by spherical collapse and the form of the mass function

$$\frac{\delta n_M}{n_M} = [1 + b(M)]\,\delta$$

- For Press-Schechter

$$b(M) = 1 + \frac{\nu^2 - 1}{\delta_c}$$

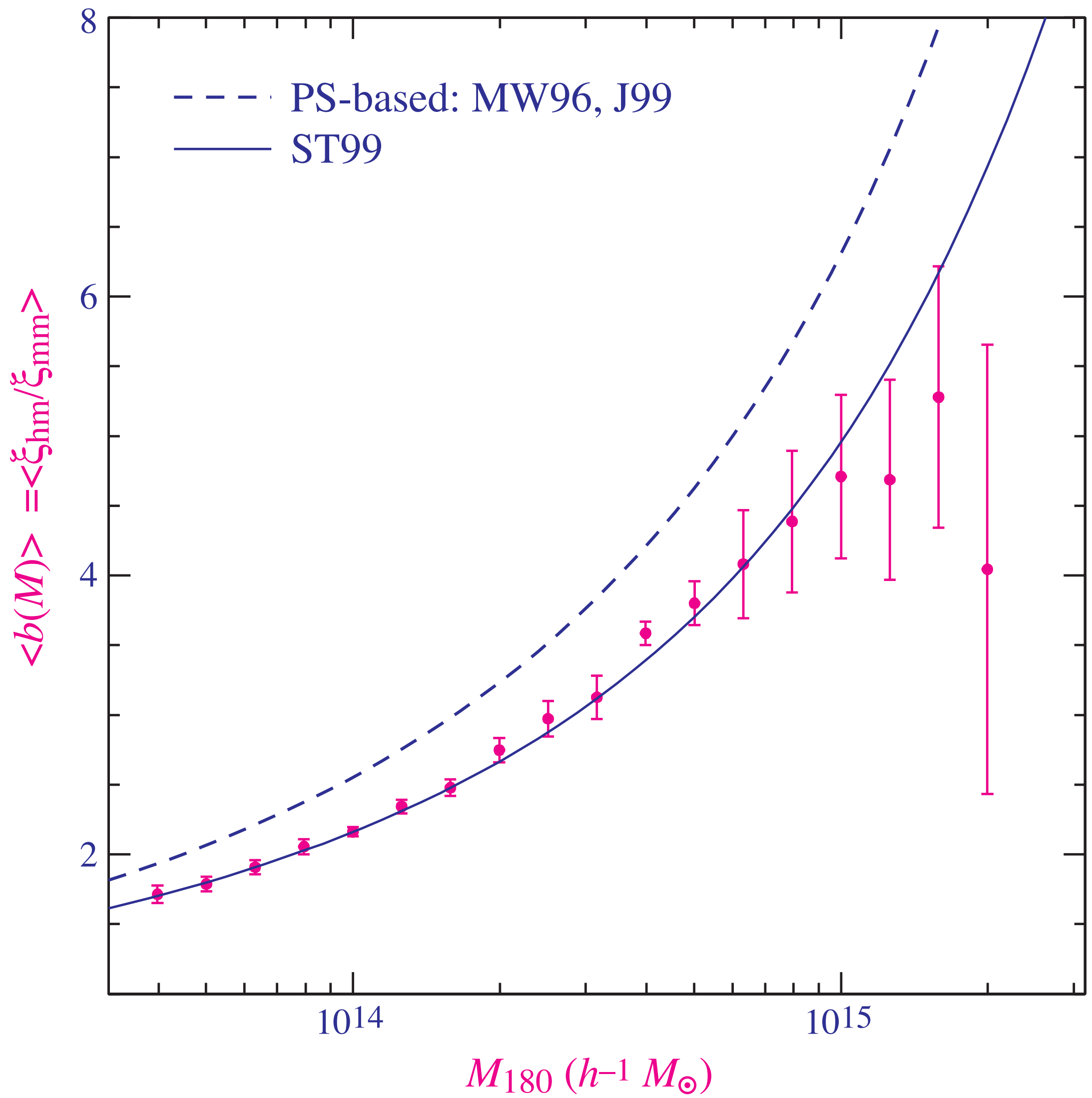
- Improved by the Sheth-Torman mass function

$$b(M) = 1 + \frac{a\nu^2 - 1}{\delta_c} + \frac{2p}{\delta_c[1 + (a\nu^2)^p]}$$

with $a = 0.75$ and $p = 0.3$ to match simulations.

Numerical Bias

- Example of halo bias from a simulation (from Hu & Kravstov 2002)



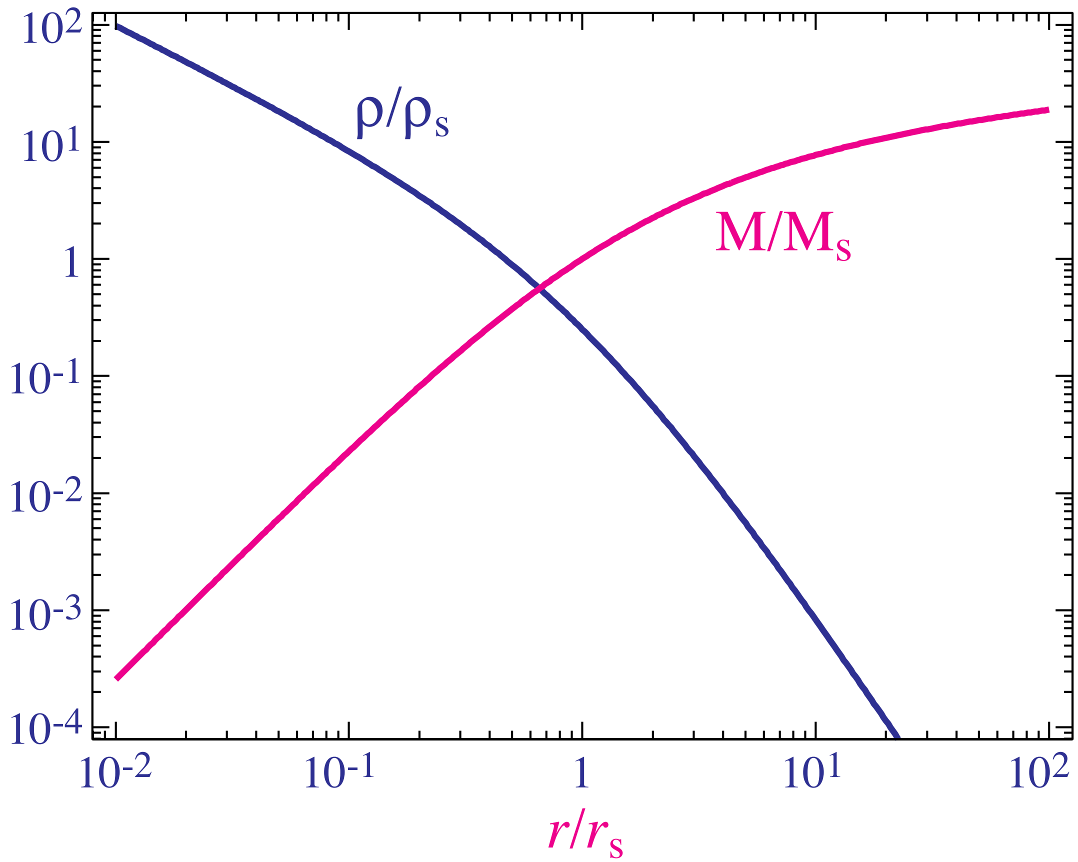
What is a Halo?

- Mass function and halo bias depend on the definition of mass of a halo
- Agreement with simulations depend on how halos are identified
- Other observables (associated galaxies, X-ray, SZ) depend on the details of the density profile
- Fortunately, simulations have shown that halos take on a near universal form in their density profile at least on large scales.

NFW Halo

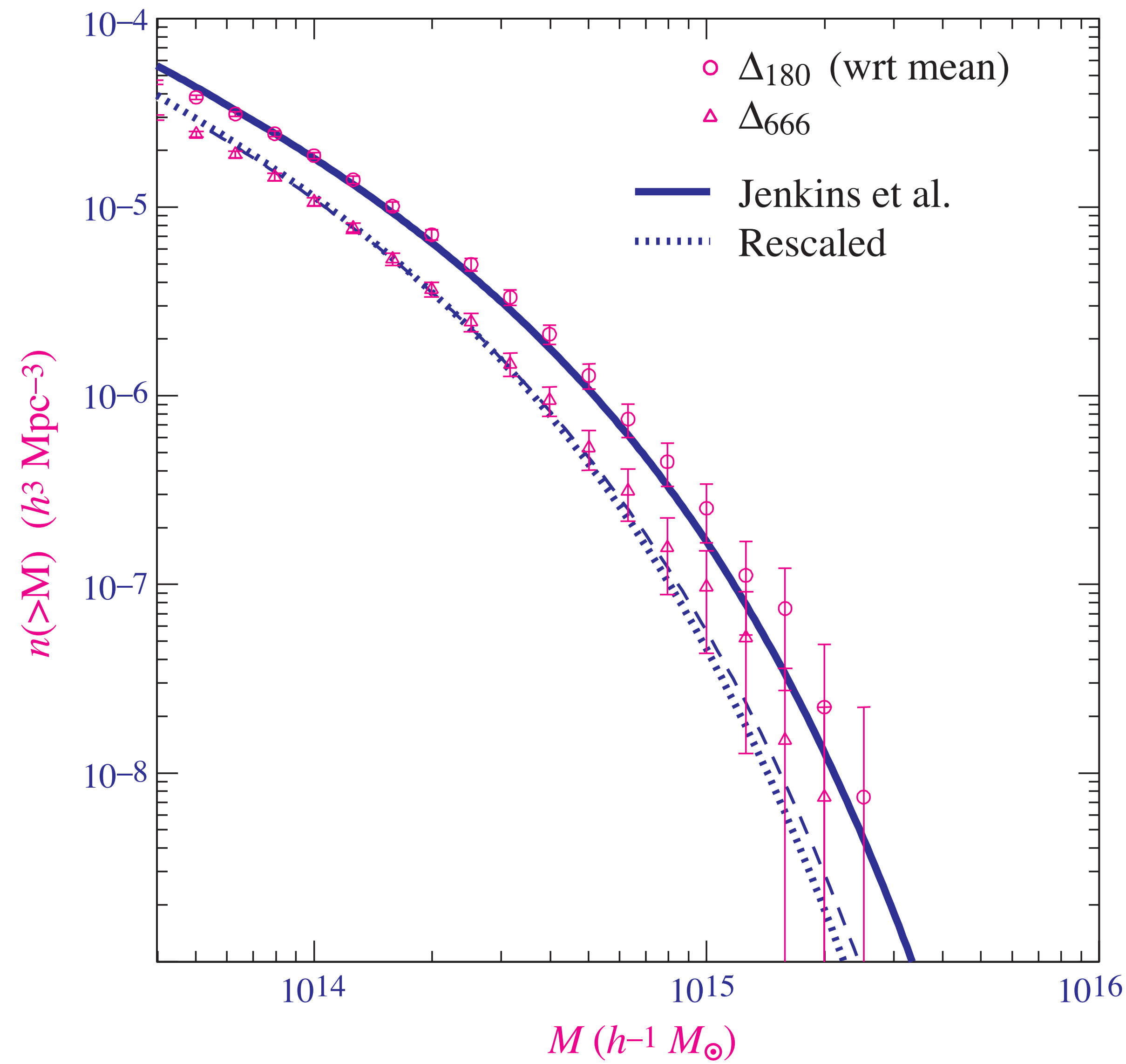
- Density profile well-described by (Navarro, Frenk & White 1997)

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$



Transforming the Masses

- NFW profile gives a way of transforming different mass definitions



Lack of Concentration?

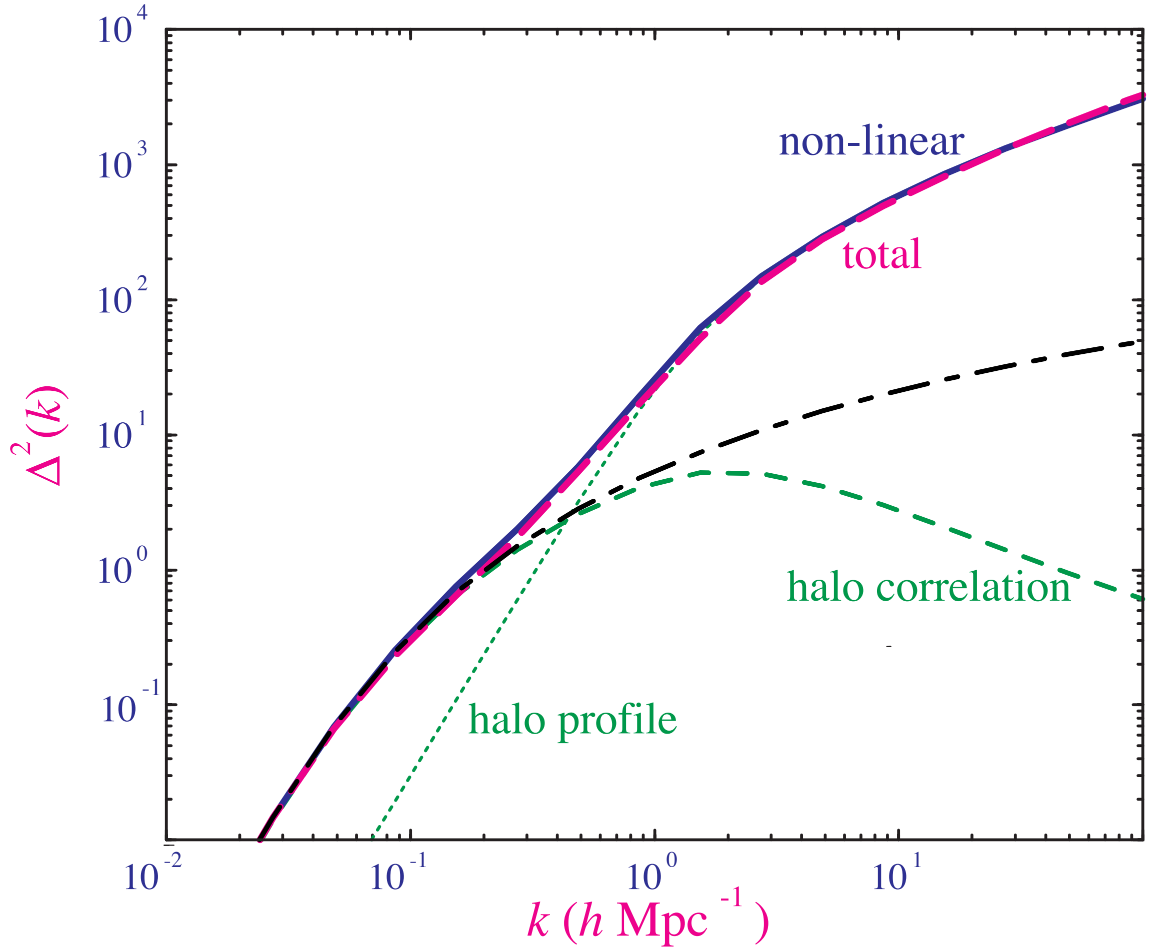
- NFW parameters may be recast into M_v, the mass of a halo out to the virial radius r_v where the overdensity wrt mean reaches $\Delta_v = 180$.
- Concentration parameter

$$c \equiv \frac{r_v}{r_s}$$

- CDM predicts $c \sim 10$ for M_* halos. Too centrally concentrated for galactic rotation curves?
- Possible discrepancy has lead to the exploration of dark matter alternatives: warm (m ~keV) dark matter, self-interacting dark-matter, annihillating dark matter, ultra-light “fuzzy” dark matter, $\ldots$

The Halo Model

- NFW halos, of abundance n_M given by mass function, clustered according to the halo bias $b(M)$ and the linear theory $P(k)$
- Power spectrum example:



Non-Linear Power Spectrum

- Non-linear power spectrum is composed of dark matter halos that are clustered according to the halo bias + the clustering due to the halo density profile

$$P_{\rm nl}(k,z) = I_2^2(k,z)P(k,z) + I_1(k,z)$$

where

$$I_2(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right) \frac{dn}{d\ln M} b(M) y(k,M)$$

$$I_1(k,z) = \int d\ln M \left(\frac{M}{\rho_m(z=0)}\right)^2 \frac{dn}{d\ln M} y^2(k,M)$$

and y is the Fourier transform of the halo profile with $y(0,M)=1$

$$y(k,M) = \frac{1}{M}\int_0^{r_h} dr 4\pi r^2 \rho(r,M) \frac{\sin(kr)}{kr}$$

Galaxy Power Spectrum

- For galaxies, one defines a halo occupation distribution which determines the number of galaxies (satisfying a certain observational criteria) that can occupy a halo of mass M
- Take a simple example of a mass selection on the galaxies, then $N(M) = 0$ for $M < M_{\rm th}$ and above threshold $N(M) = C + S(M)$ where $C = 1$ accounts for the central galaxy and satellite galaxies follow a poisson distribution with mean $S(M) \approx M/30M_{\rm th}$

Galaxy Power Spectrum

- Then assuming that satellites are distributed according to the mass profile

$$P_{\rm gal}(k,z) = I_2^2(k,z)P(k,z) + I_1(k,z)$$

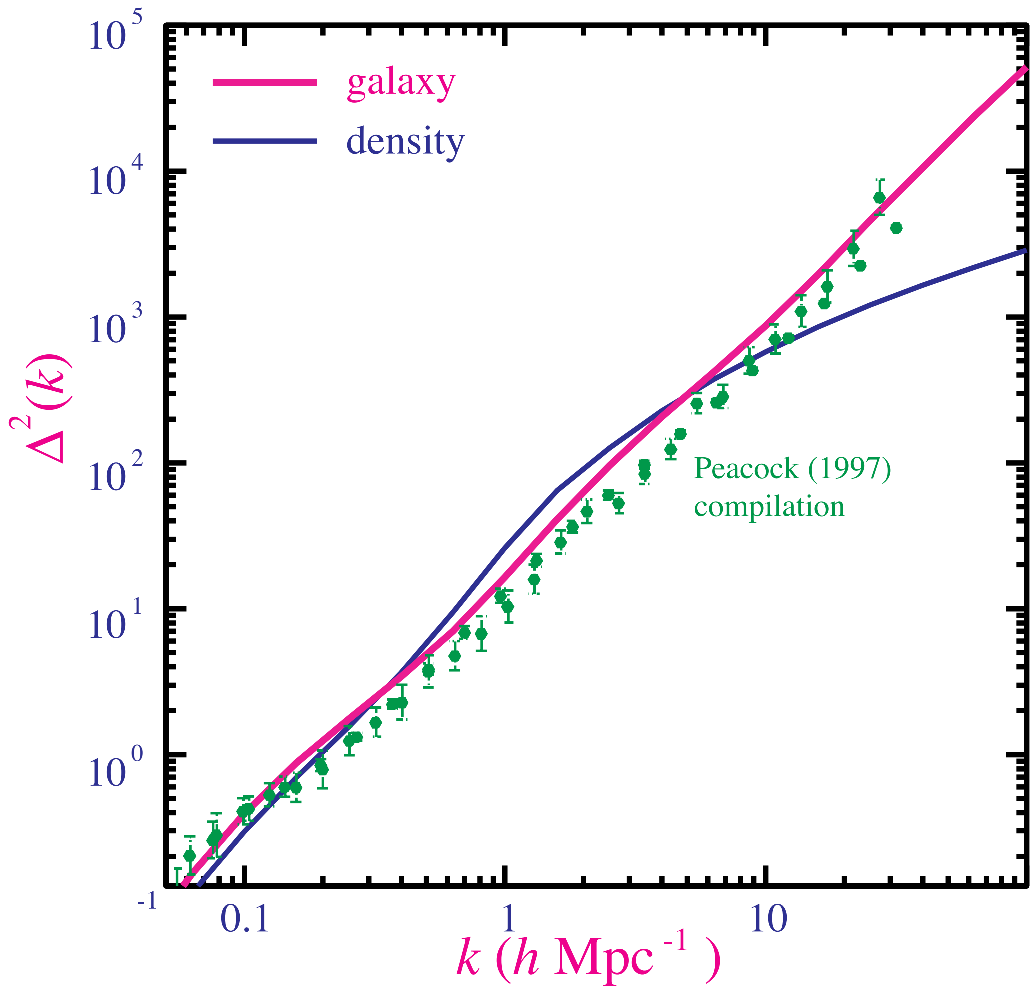
where

$$I_2(k,z) = \frac{1}{n_{\rm gal}}\int d\ln M \frac{dn}{d\ln M} b(M)[C + y(k,M)S(M)]$$
$$I_1(k,z) = \frac{1}{n_{\rm gal}^2}\int d\ln M \frac{dn}{d\ln M}[S^2(M)y^2(k,M) + 2CS(M)y(k,M)]$$

- Break between the one and two halo regime first seen by SDSS

Galaxy Power Spectrum

- Example (Seljak 2001)



- An explanation of the nearly power law galaxy spectrum

Incredible, Extensible Halo Model

- An industry developed to build semi-analytic models for wide variety of cosmological observables based on the halo model
- Idea: associate an observable (galaxies, gas, ...) with dark matter halos
- Let the halo model describe the statistics of the observable
- The overextended halo model?

Halo Temperature

- Motivate with isothermal distribution, correct from simulations

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

- Express in terms of virial mass M_v enclosed at virial radius r_v

$$M_v = \frac{4\pi}{3} r_v \rho_m \Delta_v = \frac{2}{G} r_v \sigma^2$$

- Eliminate r_v, temperature $T \propto \sigma^2$ velocity dispersion^2
- Then $T \propto M_v^{2/3}(\rho_m \Delta_v)^{1/3}$ or

$$\left(\frac{M_v}{10^{15} h^{-1} M_\odot}\right) = \left[\frac{f}{(1+z)(\Omega_m \Delta_v)^{1/3}} \frac{T}{1\text{keV}}\right]^{3/2}$$

- Theory (X-ray weighted): $f \sim 0.75$; observations $f \sim 0.54$. Difference is crucial in determining cosmology from cluster counts!