Astro 321: Problem Set 10

Power Spectrum

Evolve a set of k-modes from the initial epoch to recombination $a_* = 10^{-3}$. Choose your modes to be spaced logarithmically from $k\eta_0$=0.1 to $k\eta_* = 50$, so that you capture the long wavelength behavior. Choose the number of modes so that you capture the oscillatory structure at the highest k while not wasting computation time.

- Plot the three-dimensional log power spectrum $\Delta^2_{\Theta+\Psi}(k)$ of the effective temperature $\Theta + \Psi$ against k; take initial conditions where $\Delta^2_\zeta = 25 \times 10^{-10}$ or renormalize your output appropriately.
- Make a rough translation to angular frequency by taking $\ell = k\eta_0$ and plot the crude angular power spectrum $\Delta^2_{\Theta+\Psi}(\ell)$
- Increase $\Omega_b h^2$ by 20% and explain the change in the peak structure.
- Increase h by 20% and explain the change in the peak structure.

Extra Credit 1

As we learned in class the angular power spectrum does not really have zeros in it. Above we have have neglected projection effects and contributions from the Doppler effect. In reality

$$C_\ell = \frac{2}{\pi} \int d\ln k \, \Delta^2_{\Theta_\ell}(k) \tag{1}$$

where the log-power contributed to ℓ is given through

$$\Theta_\ell \equiv [\Theta + \Psi](\eta_*)(2\ell+1)j_\ell(k\Delta\eta) + v_\gamma(\eta_*)[\ell j_{\ell-1}(k\Delta\eta) - (\ell+1)j_{\ell+1}(k\Delta\eta)] \tag{2}$$

where $\Delta\eta = \eta_0 - \eta_*$. This is the integral method of calculating CMB anisotropies. Spline the table of k-modes above, calculate the integral and plot $\ell(\ell+1)C_\ell/2\pi$. You may find the code for fast j_ℓ generation by Arthur Kosowsky (see web site) helpful.

Extra Credit 2

Plot out the power spectrum $\Delta^2_{\pi_\gamma}(\ell)$ under the crude projection approximation $\ell = k\eta_0$. Discuss the implications for polarization