Ast 448

Set 2: Polarization and Secondaries

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Stokes Parameters

- Specific intensity is related to quadratic combinations of the electric field.
- Define the intensity matrix (time averaged over oscillations) $\langle \mathbf{E}\,\mathbf{E}^\dagger \rangle$
- Hermitian matrix can be decomposed into Pauli matrices

$$\mathbf{P} = \langle \mathbf{E}\,\mathbf{E}^\dagger \rangle = \frac{1}{2}\left(I\boldsymbol{\sigma}_0 + Q\,\boldsymbol{\sigma}_3 + U\,\boldsymbol{\sigma}_1 - V\,\boldsymbol{\sigma}_2\right)\,,$$

where

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \boldsymbol{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \boldsymbol{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Stokes parameters recovered as $\mathrm{Tr}(\sigma_i \mathbf{P})$
- Choose units of temperature for Stokes parameters $I \to \Theta$

Stokes Parameters

- Consider a general plane wave solution

$$\mathbf{E}(t,z) = E_1(t,z)\hat{\mathbf{e}}_1 + E_2(t,z)\hat{\mathbf{e}}_2$$
$$E_1(t,z) = A_1 e^{i\phi_1} e^{i(kz-\omega t)}$$
$$E_2(t,z) = A_2 e^{i\phi_2} e^{i(kz-\omega t)}$$

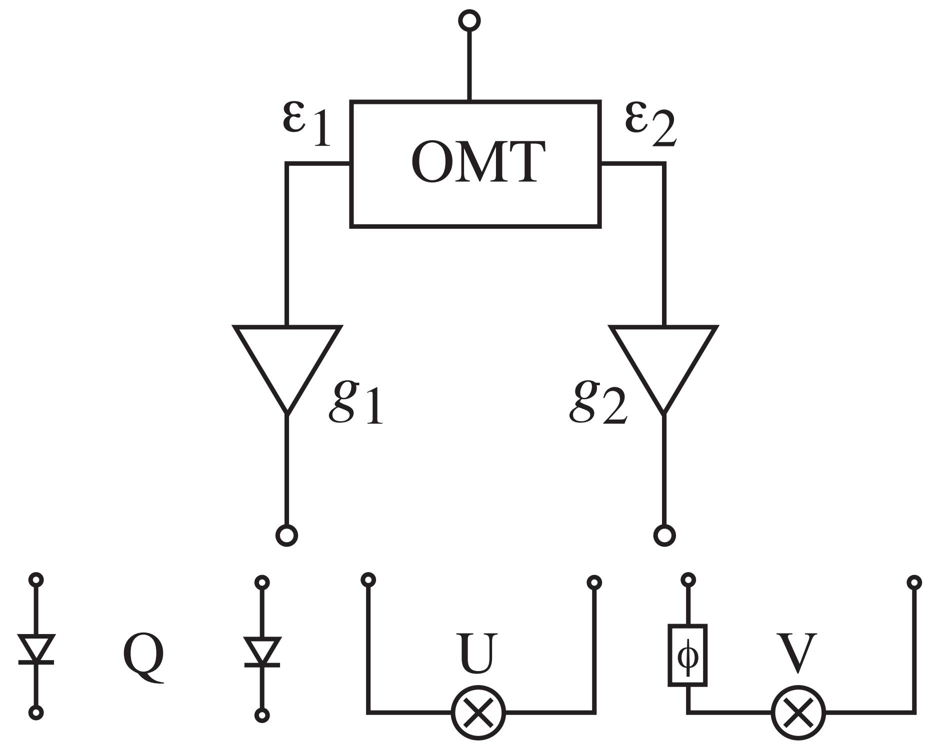
- Explicitly:

$$I = \langle E_1 E_1^* + E_2 E_2^* \rangle = A_1^2 + A_2^2$$
$$Q = \langle E_1 E_1^* - E_2 E_2^* \rangle = A_1^2 - A_2^2$$
$$U = \langle E_1 E_2^* + E_2 E_1^* \rangle = 2A_1 A_2 \cos(\phi_2 - \phi_1)$$
$$V = -i\,\langle E_1 E_2^* - E_2 E_1^* \rangle = 2A_1 A_2 \sin(\phi_2 - \phi_1)$$

so that the Stokes parameters define the state up to an unobservable overall phase of the wave

Detection

- This suggests that abstractly there are two different ways to detect polarization: separate and difference orthogonal modes (bolometers I, Q) or correlate the separated components (U, V).



- In the correlator example the natural output would be U but one can recover V by introducing a phase lag $\phi = \pi/2$ on one arm, and Q by having the OMT pick out directions rotated by $\pi/4$.
- Likewise, in the bolometer example, one can rotate the polarizer and also introduce a coherent front end to change V to U.

Detection

- Techniques also differ in the systematics that can convert unpolarized sky to fake polarization
- Differencing detectors are sensitive to relative gain fluctuations
- Correlation detectors are sensitive to cross coupling between the arms
- More generally, the intended block diagram and systematic problems map components of the polarization matrix onto others and are kept track of through "Jones" or instrumental response matrices $\mathbf{E}_{\rm det} = \mathbf{J}\mathbf{E}_{\rm in}$

$$\mathbf{P}_{\rm det} = \mathbf{J}\mathbf{P}_{\rm in}\mathbf{J}^\dagger$$

where the end result is either a differencing or a correlation of the $\mathbf{P}_{\rm det}$.

Polarization

- Radiation field involves a directed quantity, the electric field vector, which defines the polarization
- Consider a general plane wave solution

$$\mathbf{E}(t,z) = E_1(t,z)\hat{\mathbf{e}}_1 + E_2(t,z)\hat{\mathbf{e}}_2$$

$$E_1(t,z) = \mathrm{Re} A_1 e^{i\phi_1} e^{i(kz-\omega t)}$$

$$E_2(t,z) = \mathrm{Re} A_2 e^{i\phi_2} e^{i(kz-\omega t)}$$

or at $z = 0$ the field vector traces out an ellipse

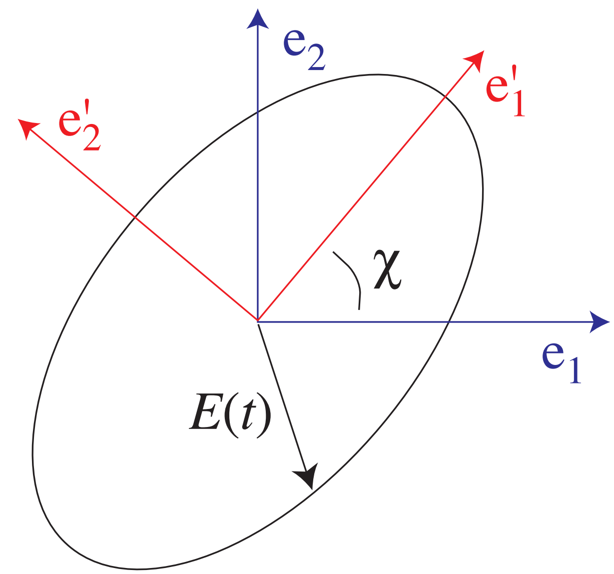
$$\mathbf{E}(t,0) = A_1 \cos(\omega t - \phi_1)\hat{\mathbf{e}}_1 + A_2 \cos(\omega t - \phi_2)\hat{\mathbf{e}}_2$$

with principal axes defined by

$$\mathbf{E}(t,0) = A_1' \cos(\omega t)\hat{\mathbf{e}}_1' - A_2' \sin(\omega t)\hat{\mathbf{e}}_2'$$

so as to trace out a clockwise rotation for $A_1', A_2' > 0$

Polarization



- Define polarization angle

$$\hat{\mathbf{e}}'_1 = \cos\chi\hat{\mathbf{e}}_1 + \sin\chi\hat{\mathbf{e}}_2$$
$$\hat{\mathbf{e}}'_2 = -\sin\chi\hat{\mathbf{e}}_1 + \cos\chi\hat{\mathbf{e}}_2$$

- Match

$$\begin{aligned}
\mathbf{E}(t,0) &= A'_1\cos\omega t[\cos\chi\hat{\mathbf{e}}_1 + \sin\chi\hat{\mathbf{e}}_2] \\
&\quad - A'_2\cos\omega t[-\sin\chi\hat{\mathbf{e}}_1 + \cos\chi\hat{\mathbf{e}}_2] \\
&= A_1[\cos\phi_1\cos\omega t + \sin\phi_1\sin\omega t]\hat{\mathbf{e}}_1 \\
&\quad + A_2[\cos\phi_2\cos\omega t + \sin\phi_2\sin\omega t]\hat{\mathbf{e}}_2
\end{aligned}$$

Polarization

- Define relative strength of two principal states

$$A_1' = E_0 \cos\beta \quad A_2' = E_0 \sin\beta$$

- Characterize the polarization by two angles

$$\begin{aligned} A_1 \cos\phi_1 &= E_0 \cos\beta \cos\chi, & A_1 \sin\phi_1 &= E_0 \sin\beta \sin\chi, \\ A_2 \cos\phi_2 &= E_0 \cos\beta \sin\chi, & A_2 \sin\phi_2 &= -E_0 \sin\beta \cos\chi \end{aligned}$$

Or Stokes parameters by

$$I = E_0^2\,, \quad Q = E_0^2 \cos 2\beta \cos 2\chi$$
$$U = E_0^2 \cos 2\beta \sin 2\chi\,, \quad V = E_0^2 \sin 2\beta$$

- So $I^2 = Q^2 + U^2 + V^2$, double angles reflect the spin 2 field or headless vector nature of polarization

Polarization

Special cases

- If $\beta = 0, \pi/2, \pi$ then only one principal axis, ellipse collapses to a line and $V = 0 \rightarrow$ linear polarization oriented at angle χ

  If $\chi = 0, \pi/2, \pi$ then $I = \pm Q$ and $U = 0$

  If $\chi = \pi/4, 3\pi/4...$ then $I = \pm U$ and $Q = 0$ - so U is Q in a frame rotated by 45 degrees

- If $\beta = \pi/4, 3\pi/4$, then principal components have equal strength and E field rotates on a circle: $I = \pm V$ and $Q = U = 0 \rightarrow$ circular polarization

- $U/Q = \tan 2\chi$ defines angle of linear polarization and $V/I = \sin 2\beta$ defines degree of circular polarization

Natural Light

- A monochromatic plane wave is completely polarized
  $I^2 = Q^2 + U^2 + V^2$
- Polarization matrix is like a density matrix in quantum mechanics and allows for pure (coherent) states and mixed states
- Suppose the total $\mathbf{E}_{\rm tot}$ field is composed of different (frequency) components

$$\mathbf{E}_{\rm tot} = \sum_i \mathbf{E}_i$$

- Then components decorrelate in time average

$$\left\langle \mathbf{E}_{\rm tot}\mathbf{E}_{\rm tot}^\dagger \right\rangle = \sum_{ij} \left\langle \mathbf{E}_i \mathbf{E}_j^\dagger \right\rangle = \sum_i \left\langle \mathbf{E}_i \mathbf{E}_i^\dagger \right\rangle$$

Natural Light

- So Stokes parameters of incoherent contributions add

$$I = \sum_i I_i \quad Q = \sum_i Q_i \quad U = \sum_i U_i \quad V = \sum_i V_i$$

  and since individual Q, U and V can have either sign:
  $I^2 \geq Q^2 + U^2 + V^2$, all 4 Stokes parameters needed

Linear Polarization

- $Q \propto \langle E_1 E_1^* \rangle - \langle E_2 E_2^* \rangle$, $U \propto \langle E_1 E_2^* \rangle + \langle E_2 E_1^* \rangle$.
- Counterclockwise rotation of axes by $\theta = 45^\circ$

$$E_1 = (E_1' - E_2')/\sqrt{2}\,, \quad E_2 = (E_1' + E_2')/\sqrt{2}$$

- $U \propto \langle E_1' E_1'^* \rangle - \langle E_2' E_2'^* \rangle$, difference of intensities at 45° or Q'
- More generally, $\mathbf{P}$ transforms as a tensor under rotations and

$$\begin{aligned} Q' &= \cos(2\theta)Q + \sin(2\theta)U \\ U' &= -\sin(2\theta)Q + \cos(2\theta)U \end{aligned}$$

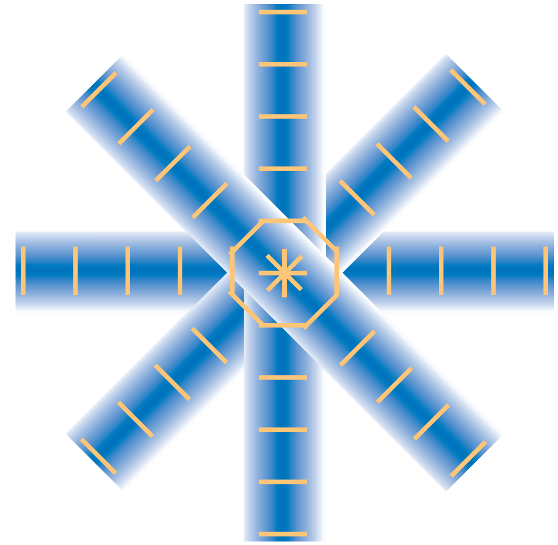
or

$$Q' \pm iU' = e^{\mp 2i\theta}[Q \pm iU]$$

acquires a phase under rotation and is a spin ± 2 object

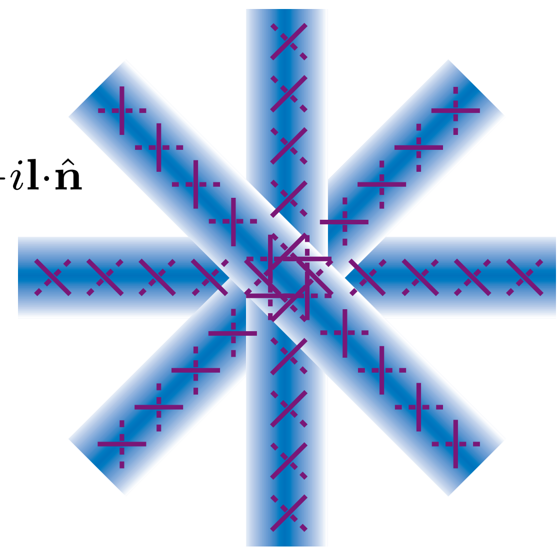
Coordinate Independent Representation

- Two directions: orientation of polarization and change in amplitude, i.e. Q and U in the basis of the Fourier wavevector (pointing with angle ϕ_l) for small sections of sky are called E and B components



$$E(\mathbf{l}) \pm iB(\mathbf{l}) = -\int d\hat{\mathbf{n}}[Q'(\hat{\mathbf{n}}) \pm iU'(\hat{\mathbf{n}})]e^{-i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

$$= -e^{\mp 2i\phi_l}\int d\hat{\mathbf{n}}[Q(\hat{\mathbf{n}}) \pm iU(\hat{\mathbf{n}})]e^{-i\mathbf{l}\cdot\hat{\mathbf{n}}}$$



- For the B-mode to not vanish, the polarization must point in a direction not related to the wavevector - not possible for density fluctuations in linear theory
- Generalize to all-sky: eigenmodes of Laplace operator of tensor

Spin Harmonics

- Laplace Eigenfunctions

$$\nabla^2{}_{\pm 2}Y_{\ell m}[\boldsymbol{\sigma}_3 \mp i\boldsymbol{\sigma}_1] = -[l(l+1)-4]_{\pm 2}Y_{\ell m}[\boldsymbol{\sigma}_3 \mp i\boldsymbol{\sigma}_1]$$

- Spin s spherical harmonics: orthogonal and complete

$$\int d\hat{\mathbf{n}}\, {}_sY^*_{\ell m}(\hat{\mathbf{n}})\, {}_sY_{\ell' m'}(\hat{\mathbf{n}}) = \delta_{\ell\ell'}\delta_{mm'}$$

$$\sum_{\ell m} {}_sY^*_{\ell m}(\hat{\mathbf{n}})\, {}_sY_{\ell m}(\hat{\mathbf{n}}') = \delta(\phi-\phi')\delta(\cos\theta-\cos\theta')$$

where the ordinary spherical harmonics are $Y_{\ell m} = {}_0Y_{\ell m}$

- Given in terms of the rotation matrix

$${}_sY_{\ell m}(\beta\alpha) = (-1)^m\sqrt{\frac{2\ell+1}{4\pi}}D^{\ell}_{-ms}(\alpha\beta 0)$$

Statistical Representation

- All-sky decomposition

$$[Q(\hat{\mathbf{n}}) \pm iU(\hat{\mathbf{n}})] = \sum_{\ell m} [E_{\ell m} \pm iB_{\ell m}]_{\pm 2}Y_{\ell m}(\hat{\mathbf{n}})$$

- Power spectra

$$\langle E^*_{\ell m} E_{\ell m} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{EE}$$

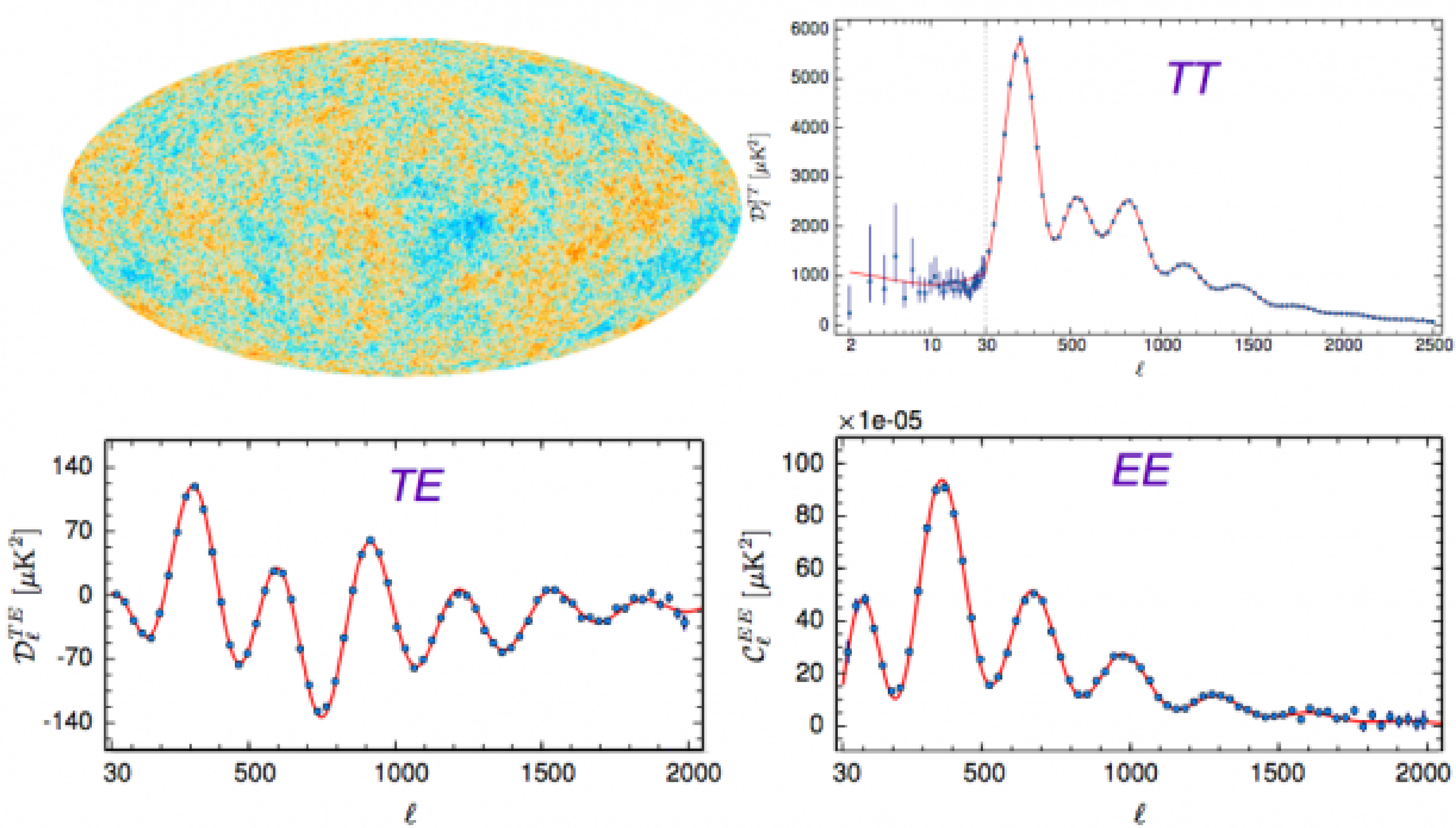
$$\langle B^*_{\ell m} B_{\ell m} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{BB}$$

- Cross correlation

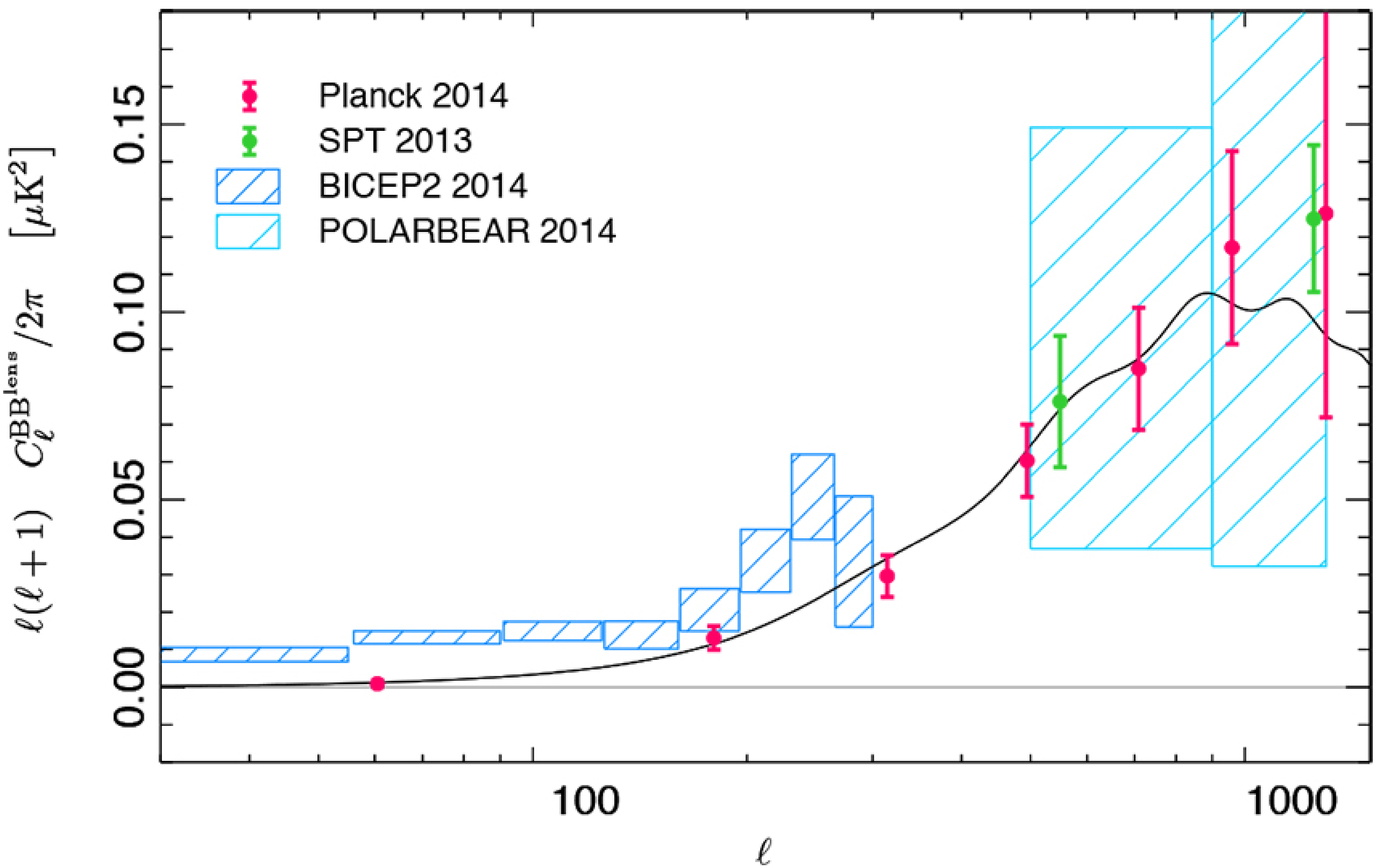
$$\langle \Theta^*_{\ell m} E_{\ell m} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{\Theta E}$$

others vanish if parity is conserved

Planck Power Spectrum



B-modes: Auto & Cross



Thomson Scattering

- Polarization state of radiation in direction $\hat{\mathbf{n}}$ described by the intensity matrix $\langle E_i(\hat{\mathbf{n}})E_j^*(\hat{\mathbf{n}})\rangle$, where $\mathbf{E}$ is the electric field vector and the brackets denote time averaging.
- Differential cross section

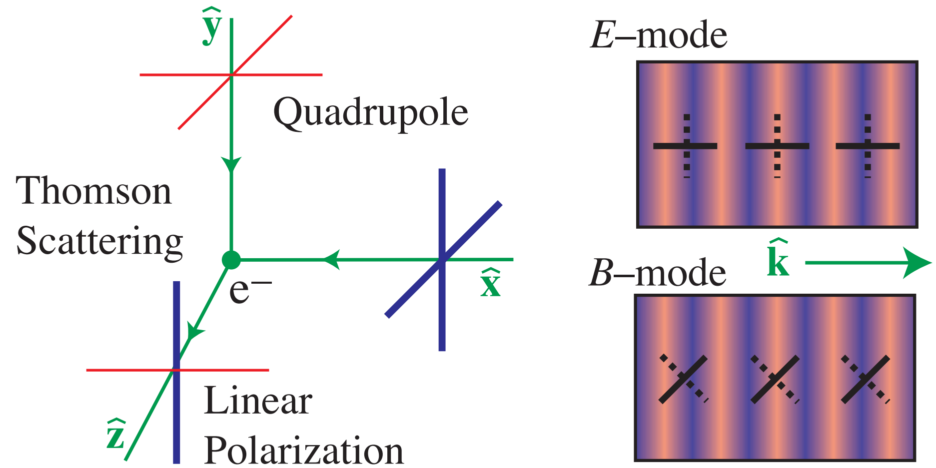
$$\frac{d\sigma}{d\Omega} = \frac{3}{8\pi}|\hat{\mathbf{E}}' \cdot \hat{\mathbf{E}}|^2 \sigma_T \, ,$$

  where $\sigma_T = 8\pi\alpha^2/3m_e$ is the Thomson cross section, $\hat{\mathbf{E}}'$ and $\hat{\mathbf{E}}$ denote the incoming and outgoing directions of the electric field or polarization vector.
- Summed over angle and incoming polarization

$$\sum_{i=1,2} \int d\hat{\mathbf{n}}' \frac{d\sigma}{d\Omega} = \sigma_T$$

Polarization Generation

- Heuristic:
  incoming radiation shakes
  an electron in direction
  of electric field vector $\hat{\mathbf{E}}'$



- Radiates photon with
  polarization also in direction $\hat{\mathbf{E}}'$
- But photon cannot be longitudinally polarized so that scattering into $90°$ can only pass one polarization
- Linearly polarized radiation like polarization by reflection
- Unlike reflection of sunlight, incoming radiation is nearly isotropic
- Missing from direction orthogonal to original incoming direction
- Only quadrupole anisotropy generates polarization by Thomson scattering

Acoustic Polarization

- Break down of tight-coupling leads to quadrupole anisotropy of

$$\pi_\gamma \approx \frac{k}{\dot\tau} v_\gamma$$

- Scaling $k_D = (\dot\tau/\eta_*)^{1/2} \rightarrow \dot\tau = k_D^2 \eta_*$
- Know: $k_D s_* \approx k_D \eta_* \approx 10$
- So:

$$\pi_\gamma \approx \frac{k}{k_D}\frac{1}{10} v_\gamma$$

$$\Delta_P \approx \frac{\ell}{\ell_D}\frac{1}{10}\Delta_T$$

Acoustic Polarization

- Gradient of velocity is along direction of wavevector, so polarization is pure E-mode
- Velocity is 90° out of phase with temperature – turning points of oscillator are zero points of velocity:

$$\Theta + \Psi \propto \cos(ks); \quad v_\gamma \propto \sin(ks)$$

- Polarization peaks are at troughs of temperature power

Cross Correlation

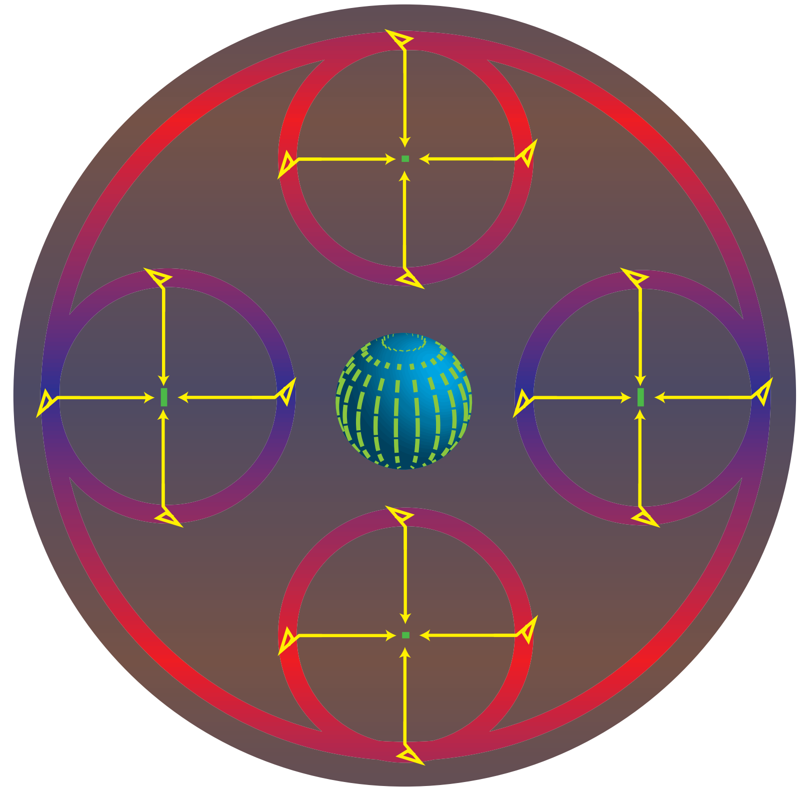
- Cross correlation of temperature and polarization

$$(\Theta + \Psi)(v_\gamma) \propto \cos(ks)\sin(ks) \propto \sin(2ks)$$

- Oscillation at twice the frequency
- Correlation: radial or tangential around hot spots
- Partial correlation: easier to measure if polarization data is noisy, harder to measure if polarization data is high S/N or if bands do not resolve oscillations
- Good check for systematics and foregrounds
- Comparison of temperature and polarization is proof against features in initial conditions mimicking acoustic features

Reionization

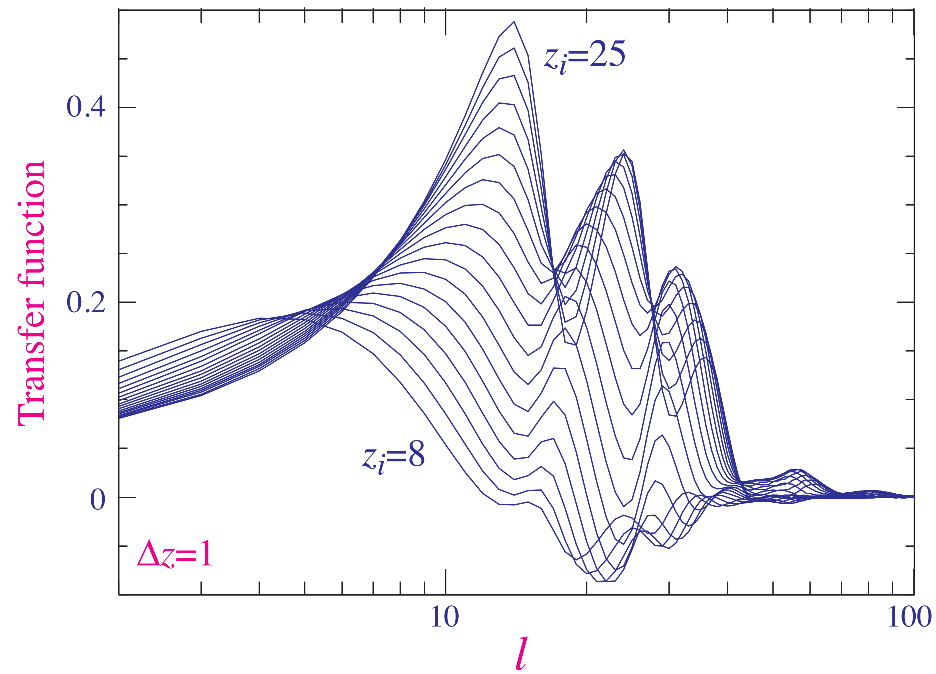
- Reionization causes rescattering of radiation
- Suppresses temperature anisotopy as $e^{-\tau}$ and changes interpretation of amplitude to $A_s e^{-2\tau}$
- Electron sees temperature anisotropy on its recombination surface



- For wavelengths that are comparable to the horizon at reionization, a quadrupole moment
- Rescatters to a linear polarization that is correlated with the Sachs-Wolfe temperature anisotropy

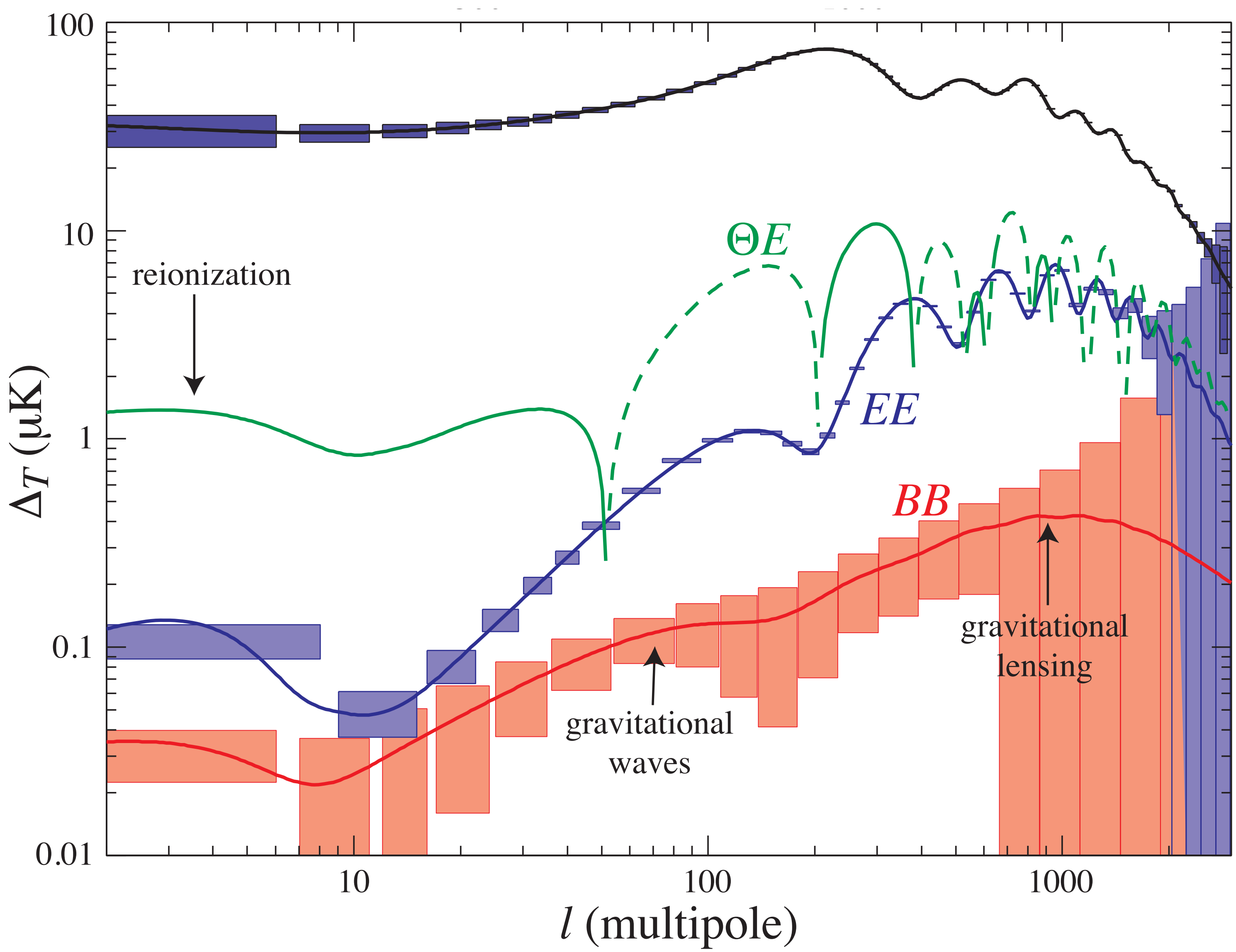
Reionization

- Amplitude of C_ℓ^{EE} depends mainly on τ
- Shape of C_ℓ^{EE} depends on reionization history
- Horizon at earlier epochs subtends a smaller angle, higher multipole peak



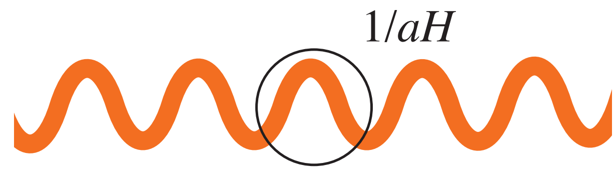
- Precision measurements can constrain the reionization history to be either low or high z dominated

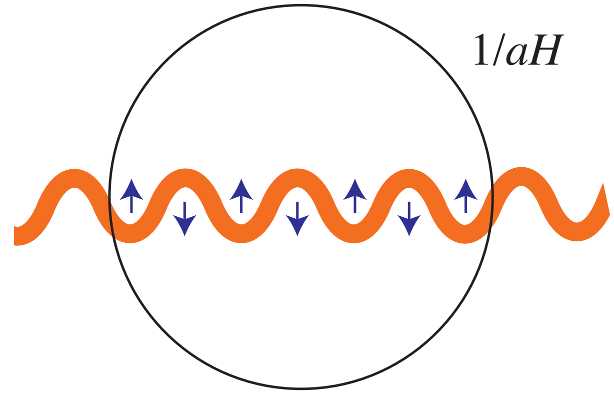
Polarization Power



Tensor Power

- Gravitational
  waves obey a Klein-Gordon like equation
- Like inflation, perturbations generated
  by quantum fluctuations during inflation
- Freeze out at horizon crossing
  during inflation an amplitude
  that reflects the energy scale of inflation

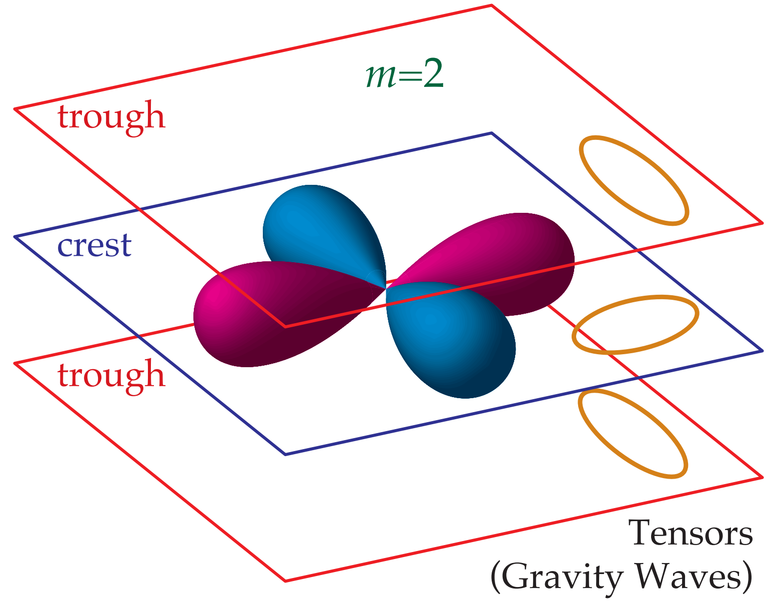




$$\Delta^2_{+,\times} = \frac{H^2}{2\pi^2 M^2_{\rm Pl}} \propto E_i^4$$

- Gravitational waves remain frozen outside the horizon at constant amplitude
- Oscillate inside the horizon and decay or redshift as radiation

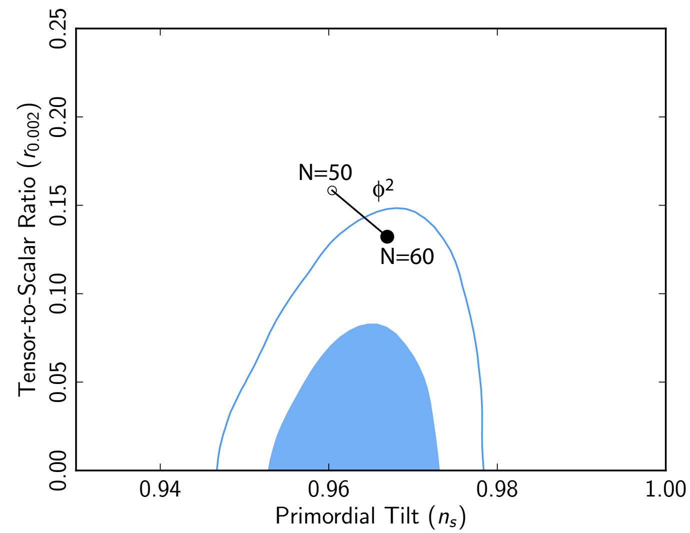
Tensor Quadrupoles



- Changing transverse-traceless distortion of space creates a quadrupole CMB anisotropy much like the distortion of test ring of particles
- As the tensor mode enters the horizon it imprints a quadrupole temperature distortion: $\dot{H}_T^{\pm 2}$ is source to $S_2^{\pm 2}$
- Modes that cross before recombination: effect erased by rescattering $e^{-\tau}$ in the integral solution
- Modes that cross after recombination: integrate contributions along the line of sight - tensor ISW effect

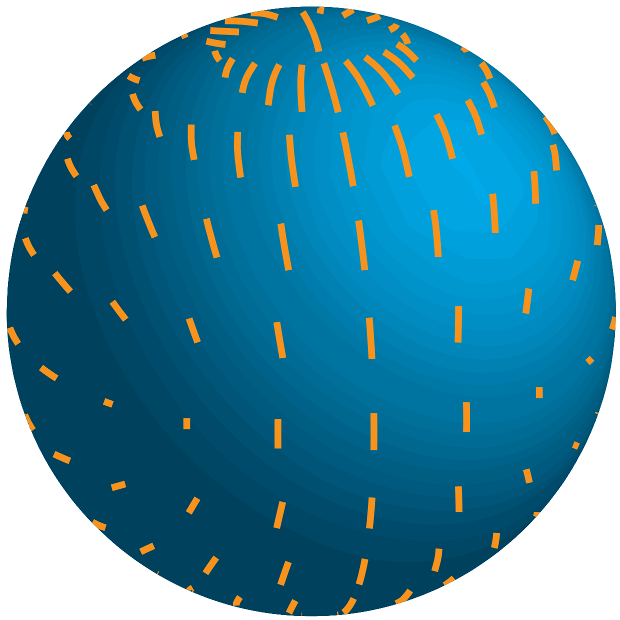
Tensor Temperature Power Spectrum

- Resulting spectum,
  near scale invariant out to
  horizon at recombination $\ell < 100$



- Suppressed on smaller scales or higher multipoles $\ell > 100$, weakly degenerate with tilt
- When added to scalar spectrum, enhances large scale anisotropy over small scale
- Shape of total temperature spectrum can place tight limit $r < 0.1$, for power law curvature spectrum
- Smaller tensor-scalar ratios cannot be constrained by temperature alone due the high cosmic variance of the low multipole specrum

Tensor Polarization Power Spectrum



- Polarization
  of gravitational wave determines the
  quadrupole temperature anisotropy
- Scattering of quadrupole
  temperature anisotropy generates
  linear
  polarization aligned with cold lobe
- Direction of CMB polarization is therefore determined by gravitational wave polarization rather than direction of wavevector
- B-mode polarization when the amplitude is modulated by the plane wave
- Requires scattering: two peaks - horizon at recombination and reionization

Tensor Polarization Power Spectrum

- Measuring B-modes from gravitational waves determines the energy scale of inflation

$$\Delta B_{\rm peak} \approx 0.024 \left(\frac{E_i}{10^{16}{\rm GeV}} \right)^2 \mu{\rm K}$$

- Also generates E-mode polarization which, like temperature, is a consistency check for $r \sim 0.1$
- Projection is less sharp than for scalar E, so evading temperature bounds by adding features to the curvature spectrum can be tested

Gravitational Lensing

- Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int_{\eta_*}^{\eta_0} d\eta \, \frac{(D_* - D)}{D \, D_*} \Phi(D\hat{\mathbf{n}}, \eta) \, .$$

  such that the fields are remapped as

$$x(\hat{\mathbf{n}}) \to x(\hat{\mathbf{n}} + \nabla\phi) \, ,$$

  where $x \in \{\Theta, Q, U\}$ temperature and polarization.

- Taylor expansion leads to product of fields and Fourier mode-coupling

Flat-sky Treatment

- Talyor expand

$$\Theta(\hat{\mathbf{n}}) = \tilde{\Theta}(\hat{\mathbf{n}} + \nabla\phi)$$

$$= \tilde{\Theta}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{\Theta}(\hat{\mathbf{n}}) + \frac{1}{2}\nabla_i\phi(\hat{\mathbf{n}})\nabla_j\phi(\hat{\mathbf{n}})\nabla^i\nabla^j\tilde{\Theta}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition

$$\phi(\hat{\mathbf{n}}) = \int \frac{d^2 l}{(2\pi)^2}\phi(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

$$\tilde{\Theta}(\hat{\mathbf{n}}) = \int \frac{d^2 l}{(2\pi)^2}\tilde{\Theta}(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

Flat-sky Treatment

- Mode coupling of harmonics

$$
\begin{aligned}
\Theta(\mathbf{l}) &= \int d\hat{\mathbf{n}}\, \Theta(\hat{\mathbf{n}}) e^{-il\cdot\hat{\mathbf{n}}} \\
&= \tilde{\Theta}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} \tilde{\Theta}(\mathbf{l}_1) L(\mathbf{l}, \mathbf{l}_1)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
L(\mathbf{l}, \mathbf{l}_1) &= \phi(\mathbf{l} - \mathbf{l}_1)\,(\mathbf{l} - \mathbf{l}_1)\cdot \mathbf{l}_1 \\
&\quad + \frac{1}{2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2} \phi(\mathbf{l}_2)\phi^*(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\,(\mathbf{l}_2\cdot\mathbf{l}_1)(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\cdot\mathbf{l}_1\,.
\end{aligned}
$$

- Represents a coupling of harmonics separated by $L \approx 60$ peak of deflection power

Power Spectrum

- Power spectra

$$\langle \Theta^*(\mathbf{l})\Theta(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\; C_l\,,$$

$$\langle \phi^*(\mathbf{l})\phi(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\; C_l^{\phi\phi}\,,$$

becomes

$$C_l = \left(1 - l^2 R\right)\tilde{C}_l + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\tilde{C}_{|\mathbf{l}-\mathbf{l}_1|}C_{l_1}^{\phi\phi}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2\,,$$

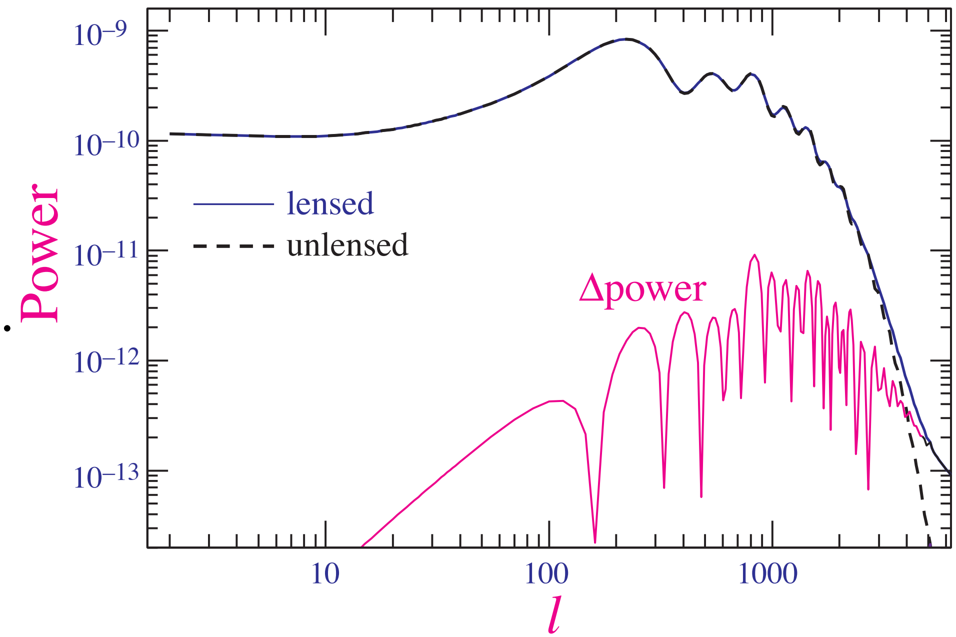
where

$$R = \frac{1}{4\pi}\int \frac{dl}{l}\, l^4 C_l^{\phi\phi}\,.$$

Smoothing Power Spectrum

- If $\tilde{C}_l$ slowly varying then two term cancel

$$\tilde{C}_l \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} C_l^{\phi\phi} (\mathbf{l}\cdot\mathbf{l}_1)^2 \approx l^2 R \tilde{C}_l \,.$$



- So lensing acts to smooth features in the power spectrum. Smoothing kernel is $L \sim 60$ the peak of deflection power spectrum
- Because acoustic feature appear on a scale $l_A \sim 300$, smoothing is a subtle effect in the power spectrum.
- Lensing generates power below the damping scale which directly reflect power in deflections on the same scale

Polarization Lensing

- Polarization field harmonics lensed similarly

$$[Q \pm iU](\hat{\mathbf{n}}) = -\int \frac{d^2 l}{(2\pi)^2} [E \pm iB](\mathbf{l}) e^{\pm 2i\phi_{\mathbf{l}}} e^{\mathbf{l}\cdot\hat{\mathbf{n}}}$$

so that

$$\begin{aligned}
[Q \pm iU](\hat{\mathbf{n}}) &= [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}} + \nabla\phi) \\
&\approx [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) + \nabla_i \phi(\hat{\mathbf{n}}) \nabla^i [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) \\
&\quad + \frac{1}{2} \nabla_i \phi(\hat{\mathbf{n}}) \nabla_j \phi(\hat{\mathbf{n}}) \nabla^i \nabla^j [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}})
\end{aligned}$$

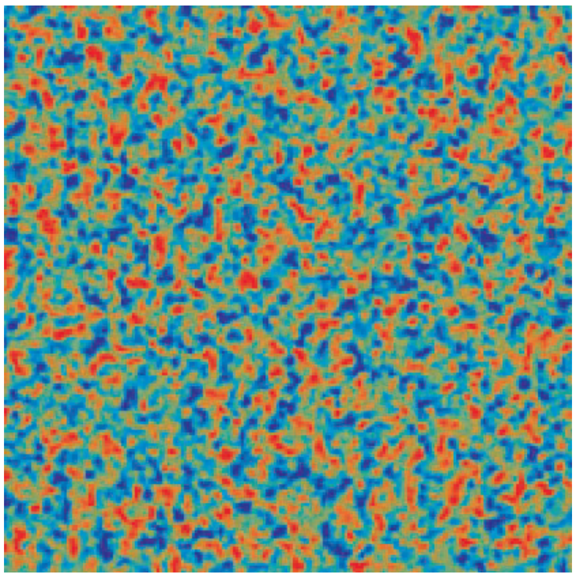
Polarization Power Spectra

- Carrying through the algebra

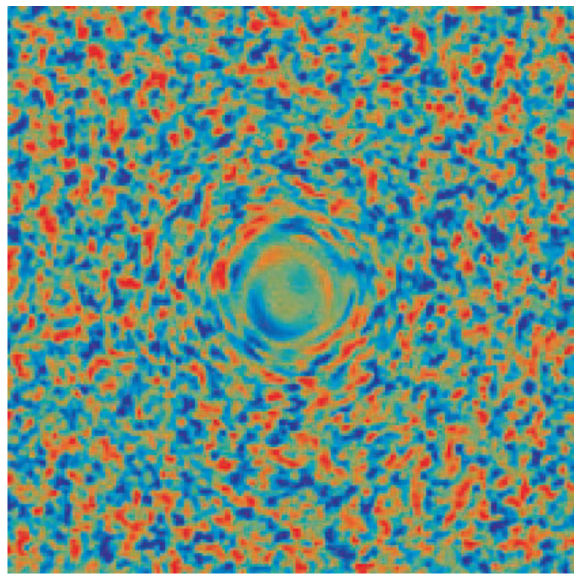
$$
\begin{aligned}
C_l^{EE} &= \left(1 - l^2 R\right) \tilde{C}_l^{EE} + \frac{1}{2}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|} \\
&\quad \times [(\tilde{C}^{EE}_{l_1} + \tilde{C}^{BB}_{l_1}) + \cos(4\varphi_{l_1})(\tilde{C}^{EE}_{l_1} - \tilde{C}^{BB}_{l_1})]\,, \\
C_l^{BB} &= \left(1 - l^2 R\right) \tilde{C}_l^{BB} + \frac{1}{2}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|} \\
&\quad \times [(\tilde{C}^{EE}_{l_1} + \tilde{C}^{BB}_{l_1}) - \cos(4\varphi_{l_1})(\tilde{C}^{EE}_{l_1} - \tilde{C}^{BB}_{l_1})]\,, \\
C_l^{\Theta E} &= \left(1 - l^2 R\right) \tilde{C}_l^{\Theta E} + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|} \\
&\quad \times \tilde{C}^{\Theta E}_{l_1}\cos(2\varphi_{l_1})\,,
\end{aligned}
$$

Polarization Lensing

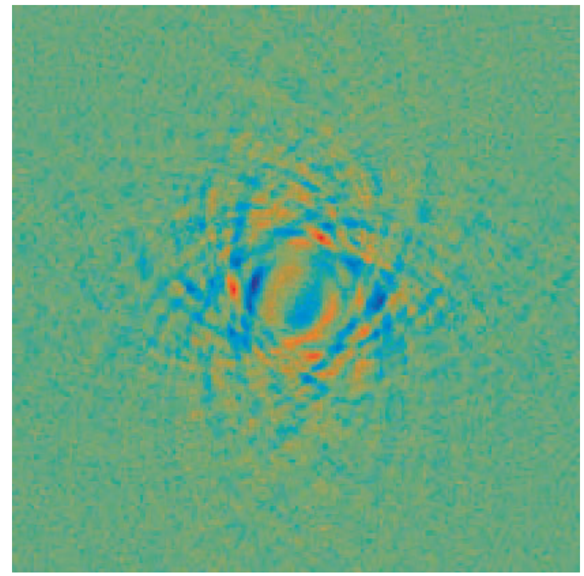
- Lensing generates B-modes out of the acoustic polaraization E-modes contaminates gravitational wave signature if $E_i < 10^{16}$GeV.



Original



Lensed E



Lensed B

Reconstruction from the CMB

- Correlation between Fourier moments reflect lensing potential

$$\langle x(\mathbf{l}) x'(\mathbf{l}') \rangle_{\rm CMB} = f_\alpha(\mathbf{l}, \mathbf{l}') \phi(\mathbf{l} + \mathbf{l}') \,,$$

  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass