Astro 321: Problem Set 1
Due Jan 14

1 Problem 1: Units

Convert the following quantitites by inserting the appropriate factors of c, $\hbar$, k_B and unit conversions. You may find Peacock chapter 9 helpful.

- $H_0 = 100h$km s^{-1} Mpc^{-1} into (a) eV, (b) Mpc^{-1}, (c) Gyr^{-1}. [Corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age of Universe.]
- $\rho_{\rm crit} = 3H_0^2/8\pi G$ into (a) g cm^{-3}, (b) GeV^4, (c) eV cm^{-3}, (d) protons cm^{-3}, (e) $M_\odot$ Mpc^{-3}. If the cosmological constant, has $\rho_\Lambda = 2\rho_{\rm crit}/3$, what is its energy scale in eV (i.e. $\rho_\Lambda^{1/4}$). Compare that to the Planck mass; that these numbers are so different is the cosmological constant problem.
- $T_{\rm CMB} = 2.728$K to (a) eV. Assuming a black body distribution, convert this to number density n_γ in photons cm^{-3} and energy density ρ_γ in (a) eV cm^{-3} (b) g cm^{-3}, and $\Omega_\gamma = \rho_\gamma/\rho_{\rm crit}$.
- $T_\nu = (4/11)^{1/3} T_{\rm CMB}$. Use this to express n_ν, ρ_ν and Ω_ν in the above units assuming that the neutrinos are relativistic (and fermions and have three species!).
- With the above relic number density, now consider the case where one out of three neutrino species has a mass of 1 eV and the rest are massless. What is the density of relic neutrinos in units of the critical density $\Omega_{\nu,\rm massive}$. For what mass is the density at the critical value.

2 Problem 2: Conformal Time

- Assume the universe today is flat with both matter (Ω_m) and a cosmological constant (Ω_Λ). (a) Compute the conformal age or horizon of the universe and plot your result for $H_0\eta_0$ as a function of Ω_m. (b) What is the current horizon size for a universe with $\Omega_m = 1/3$ and $h = 1/\sqrt{2}$? (c) What is the mass contained within the current horizon in solar masses. If all objects were $10^{13}h^{-1}$ $M_\odot$ in mass, how many are in the observable universe.
- Evaluate the conformal age as a function of the scale factor in the above cosmology. What happens when $a \to \infty$. Comment on the implications for establishing causal contact between observers currently separated by much more than a Hubble length.