Astro 321: Problem Set 8

Again this can be a group submission. You may want to break your team up to do the two parts separately.

Spline Interpolation

- Go to `http://www.nr.com`, download chapter 3.3 and read the section on spline interpolation.
- Familiarize yourself with the use of their "spline" and "splint" routines.
- Take the $\eta(a)$ output of the Runge-Kutta test and spline interpolate to find the value at some a that is not at an output time, e.g. $a = 0.001234$. "Spline invert" the relation and find $a(\eta)$ at some η say 100 Mpc. Check your answers against the analytic result.
- Write these spline interpolation routines as a piece of modular code so that your Boltzmann code can efficiently convert between a and η. This is not strictly necessary for a matter-radiation universe where analytic solutions are available but will come in handy if you want to generalize your code.

Initial Conditions

Take the initial conditions for a given k-mode to be $\zeta(\eta_i) = 1$ where η_i is the initial time step for the integration (we will choose this to be sufficiently early that the mode is well outside of the horizon and the time is well before matter radiation equality). We will scale this back to an appropriate scale invariant spectrum later. We will take a system of CMB photons γ, baryons b and cold dark matter c. Since we will ignore neutrinos, the anisotropic stress π is negligible in the initial conditions.

- From the relation
$$\Phi = \frac{3+3w}{5+3w}\zeta\,, \tag{1}$$
$w = 1/3$, the Poisson equation
$$k^2\Phi = 4\pi G a^2 \sum_i \rho_i \Delta_i \tag{2}$$
and the adiabatic condition
$$\Delta_c = \Delta_b = \frac{3}{4}\Delta_\gamma \tag{3}$$
find the initial conditions at $\eta_i \ll \eta_{\rm eq}$ for the three comoving density perturbations. [You should find a simple expression in powers of $(k\eta_i)$.]
- Again with the adiabatic condition $v_c = v_b = v_\gamma$ and the continuity equation for $\dot{\Delta}_\gamma = -4kv_\gamma/3 - 4\dot{\zeta}$ derived in the previous problem set. Find the initial conditions for the velocities at η_i. [Recall $\dot{\zeta}$ can be expressed in terms of Δ_γ. Again you should find a simple expression in powers of $(k\eta_i)$.]
- Since the anisotropic stress vanishes, the initial conditions for the Newtonian potential $\Psi = -\Phi$.

These are the IC's for your Runge-Kutta integration.