Astro 321: Problem Set 6

Due Feb 25

Slow Roll Relations

Recall the equation of motion for the unperturbed scalar field

$$\ddot{\phi}_0 + 2\frac{\dot{a}}{a}\dot{\phi}_0 + a^2 V' = 0\,, \tag{1}$$

the definitions of the slow-roll parameters

$$\epsilon = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2, \tag{2}$$

$$\delta = \epsilon - \frac{1}{8\pi G}\frac{V''}{V}, \tag{3}$$

where primes are derivatives with repect to the argument, ϕ for $V(\phi)$, and the formulae for the curvature and gravity wave power spectra

$$\Delta_{\mathcal{R}}^2 = \left(\frac{H}{m_{\rm pl}}\right)^2 \frac{1}{\pi\epsilon}, \tag{4}$$

$$\Delta_h^2 = \left(\frac{H}{m_{\rm pl}}\right)^2 \frac{4}{\pi}. \tag{5}$$

where $m_{\rm pl} = G^{-1/2}$.

1. Chaotic Inflation

Consider polynomial chaotic inflation where $V = m^2\phi^2/2$.

- Write down ϵ and δ. Inflation will occur if the initial field $\phi_0(0) = \phi_i$ meets what conditions?
- Write down the slow roll equation in coordinate time ($d^2\phi_0/dt^2 = 0$; $\delta \ll 1$) with $H(\phi)$ ($\epsilon \ll 1$) evaluated with the Friedmann equation.
- Solve for $\phi_0(t)$.
- Solve for $a(t)$ using the $H(\phi)$ relation and assume $a(t=0) = a_i$.
- Take $\epsilon = 1$ to define the end of inflation. Show that the number of efoldings of inflation can be written as

  $$N = \ln(a_{\rm end}/a_i) = 2\pi\frac{\phi_i^2}{m_{\rm pl}^2} - \frac{1}{2} \tag{6}$$

  what is the condition on ϕ_i such that sufficient inflation occurs ($N > 70$). Is it compatible with the slow roll conditions?
- Write down the curvature power spectrum $\Delta_{\mathcal{R}}^2$ and gravity wave power spectrum Δ_h^2 for this model in terms of ϕ. Taking $\phi = \phi_i$ defined now as $N = 70$ above, what is the condition on m such that the rms is $\Delta_{\mathcal{R}} = 10^{-5}$. What is tensor-scalar ratio $\Delta_h^2/\Delta_{\mathcal{R}}^2$ for such a model?