Early Universe Models Post Planck

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Outline

Inflation Motivation

- Inflation is the idea that the very early universe experienced a period of accelerated expansion.
- Initially was introduced by Guth to resolve monopole problem.
- He realized that this framework could resolve the horizon and flatness problems.

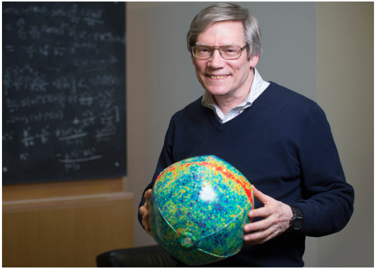


Figure 1: Alan Guth

Monomial Inflation Models

- Guth's initial idea framed solution to the horizon and flatness problems in terms of a mechanism that would "super cool" the universe. Didn't have a clear picture of how to end inflation and generated large inhomogeneities. Now referred to as "old inflation" (Guth, 1981).
- Steinhardt, Albrecht, Linde published work illuminating his initial idea in the years immediately following.
- First instance of simple textbook models we study came from Linde who studied the quartic potential $V = \lambda\phi^4$ (Linde, 1983).
- This class of models have V proportional to ϕ^n and I've seen them referred to as chaotic, monomial, and large field models.

Simple Models

- Simplest models of inflation action
  $S = \int d^4x\sqrt{g}(\frac{R}{2} + \frac{\dot{\phi}^2}{2} - V(\phi))$
- Equation of state
  $w = \frac{p}{\rho} = \frac{\frac{\dot{\phi}^2}{2} - V}{\frac{\dot{\phi}^2}{2} + V}$
- Parameters:

$$\epsilon_v(\phi) = \frac{M_{pl}^2}{2}(\frac{V'}{V})^2 < 1$$

$$\eta_v(\phi) = \frac{M_{pl}^2}{2}(\frac{V''}{V})^2 < 1$$

Key Predictions

1. Inflation drives the universe to flatness.
2. Nearly scale invariant density perturbations.
3. Ratio between tensor and scalar perturbations.

Connecting Predictions to Observables

1. Express the relationship between our inflaton field ϕ and the number of e-folds N

$$N(\phi) = \int_{\phi_{end}}^{\phi_{cmb}} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_V}}$$

2. Relate to n_s and r

   $n_s = 1 - 6\epsilon + 2\eta$ and $r = 16\epsilon$

3. Obtain predictions for different models of inflation

Case: $\frac{m^2\phi^2}{2}$

- Consider $V = \frac{m^2\phi^2}{2}$ and $\epsilon_v(\phi) = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{2M_{pl}^2}{\phi^2}$

$$N(\phi) = \int_{\phi_{end}}^{\phi_{cmb}} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_v}} = \left(\frac{\phi^2}{4M_{pl}^2}\right) \approx 60$$

- Can now say at horizon crossing $\epsilon_v = \frac{1}{2N}$
- For this model $n_s = 1 - \frac{2}{N} \approx .96$ and $r = \frac{8}{N} \approx .10$

Case: $\lambda\phi^4$

- Quartic potential is similar to quadratic case.
- Can do the same analysis with $V = \lambda\phi^4$

$$N(\phi) = \int_{\phi_{end}}^{\phi_{cmb}} \frac{\mathrm{d}\phi}{\sqrt{2\epsilon_v}} = \left(\frac{\phi^2}{8M_{pl}^2}\right) \approx 60$$

- Can now say at horizon crossing $\epsilon_v = \frac{1}{N}$
- For this model $r = \frac{16}{N} \approx .20$

Lyth Bound

- Previous examples demonstrate that there is a relationship between how the field evolves during e-folds of inflation and the predicted tensor to scalar ratio.
- This is relationship is referred to as the Lyth Bound.

$$\frac{\Delta\phi}{M_{pl}} = \int_{N_{cmb}}^{N_{end}} dN \sqrt{\frac{r}{8}}$$

- The relationship can be approximated in the following way which leads to classifying large and small field models.

$$\frac{\Delta\phi}{M_{pl}} \approx \mathcal{O}(1)x\left(\frac{r}{.01}\right)^{\frac{1}{2}}$$

Large Field

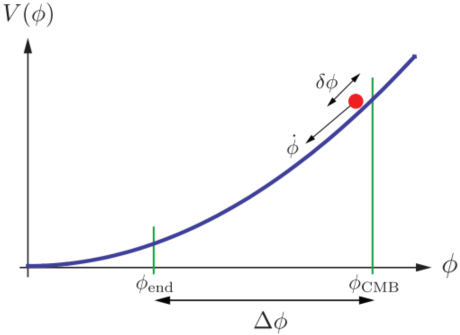


Figure 2: credit Baumann (2012)

Small Field

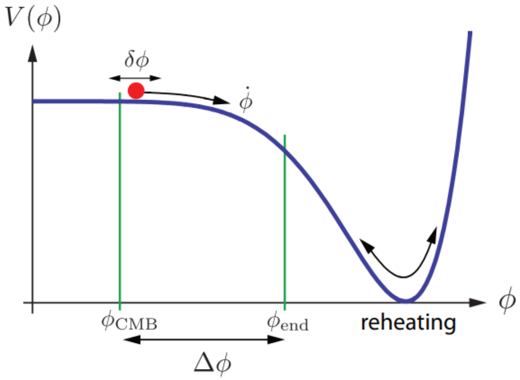
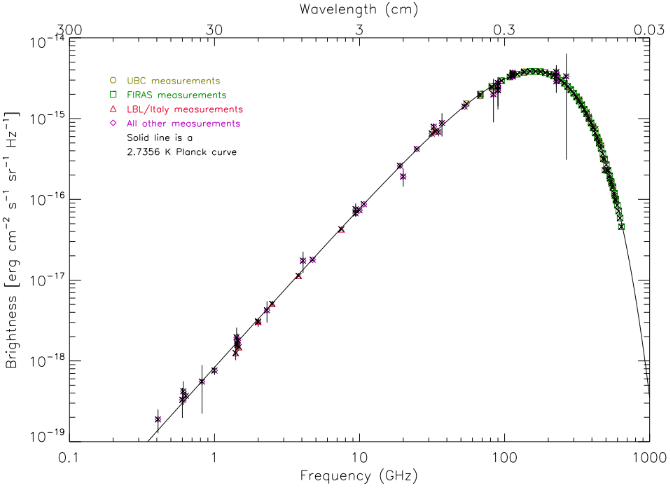


Figure 3: credit Baumann (2012)

COBE Key Results (1999)



COBE Key Results (1999)

- CMB is a near perfect blackbody with temperature $T = 2.725 \pm .02K$
- Detected faint anisotropies including the CMB's dipole
- Placed upper limits on CMB polarization
- Measured spectral index $n_s = 1.2 \pm .3$

WMAP (2008)

Table 7. Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+SN
Parameters for Standard ΛCDM Model [a]			
Age of universe	t_0	13.69 ± 0.13 Gyr	13.72 ± 0.12 Gyr
Hubble constant	H_0	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.5 ± 1.3 km/s/Mpc
Baryon density	Ω_b	0.0441 ± 0.0030	0.0456 ± 0.0015
Physical baryon density	$\Omega_b h^2$	0.02273 ± 0.00062	$0.02267^{+0.00058}_{-0.00059}$
Dark matter density	Ω_c	0.214 ± 0.027	0.228 ± 0.013
Physical dark matter density	$\Omega_c h^2$	0.1099 ± 0.0062	0.1131 ± 0.0034
Dark energy density	Ω_Λ	0.742 ± 0.030	0.726 ± 0.015
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1} [b]	$\Delta^2_{\mathcal{R}}$	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.445 \pm 0.096) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.796 ± 0.036	0.812 ± 0.026
$l(l+1)C^{TT}_{220}/2\pi$	C_{220}	$5756 \pm 42\ \mu\mathrm{K}^2$	$5751^{+42}_{-43}\ \mu\mathrm{K}^2$
Scalar spectral index	n_s	$0.963^{+0.014}_{-0.015}$	0.960 ± 0.013
Redshift of matter-radiation equality	z_{eq}	3176^{+151}_{-150}	3253^{+89}_{-87}
Angular diameter distance to matter-radiation eq. [c]	$d_A(z_{\mathrm{eq}})$	14279^{+186}_{-189} Mpc	14200^{+137}_{-140} Mpc
Redshift of decoupling	z_*	1090.51 ± 0.95	1090.88 ± 0.72
Age at decoupling	t_*	380081^{+5843}_{-5841} yr	376971^{+3162}_{-3167} yr
Angular diameter distance to decoupling [c,d]	$d_A(z_*)$	14115^{+188}_{-191} Mpc	14034^{+138}_{-142} Mpc
Sound horizon at decoupling [d]	$r_s(z_*)$	146.8 ± 1.8 Mpc	$145.9^{+1.1}_{-1.2}$ Mpc
Acoustic scale at decoupling [d]	$l_A(z_*)$	$302.08^{+0.83}_{-0.84}$	302.13 ± 0.84
Reionization optical depth	τ	0.087 ± 0.017	0.084 ± 0.016
Redshift of reionization	z_{reion}	11.0 ± 1.4	10.9 ± 1.4
Age at reionization	t_{reion}	427^{+88}_{-65} Myr	432^{+90}_{-67} Myr
Parameters for Extended Models [e]			
Total density [f]	Ω_{tot}	$1.099^{+0.100}_{-0.085}$	$1.0050^{+0.0060}_{-0.0061}$
Equation of state [g]	w	$-1.06^{+0.41}_{-0.42}$	$-0.992^{+0.061}_{-0.062}$
Tensor to scalar ratio, $k_0 = 0.002$ Mpc^{-1} [b,h]	r	< 0.43 (95% CL)	< 0.22 (95% CL)
Running of spectral index, $k_0 = 0.002$ Mpc^{-1} [b,i]	$dn_s/d\ln k$	-0.037 ± 0.028	-0.028 ± 0.020

Figure 5: 2008 WMAP Cosmological Parameters

Planck 2018 Constraints

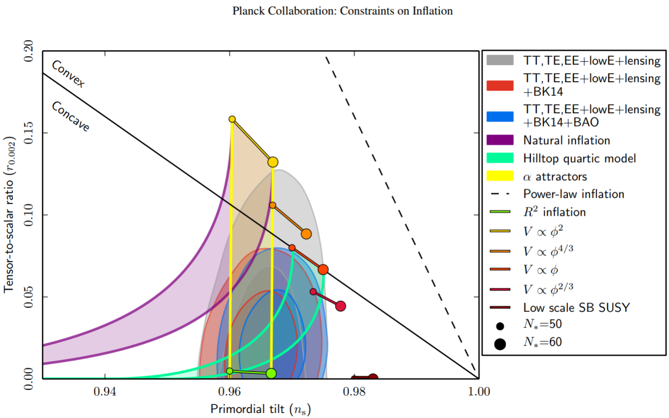


Figure 6: 2018 Planck results

Hilltop Models

- Falls under the category of "small field models".
- Idea is that inflation takes place near a maximum of potential.
- Can write the potential in the following way.

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^n\right] + ...$$

Hilltop Models

- Thinking back to the Lyth bound we expect this inflaton potential to produce a smaller tensor to scalar ratio r.
- $\Delta\phi$ will be smaller than in monomial models because V' will become important as V approaches the drop off.
- Generic bound on tensor to scalar ratio (Lyth, 2005).

$$r < .002 \left(\frac{60}{N}\right)^2 \left(\frac{\phi_{end}}{M_{pl}}\right)^2$$

Advantages

1. Obviously this class of models is consistent with recent constraints from Planck.
2. Additionally they have some motivation from particle physics (Baumann, 2012)
   - Can cast this as a Higgs-like potential.

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu} \right)^2 \right]^2$$

   - Or Coleman-Weinberg potential which is important in GUTs.

$$V(\phi) = V_0 \left[\left(\frac{\phi}{\mu} \right)^4 \left(\ln \frac{\phi}{\mu} - \frac{1}{4} \right) + \frac{1}{4} \right]$$

Disadvantages

1. These require more parameters and thus it can be argued that they require more "tuning".
   - height V_0, width μ, and power n as opposed to a mass m and power n in the monomial case.
2. They are arguably more favorable to the idea of eternal inflation.
   - "The eternal nature of new inflation depends crucially on the scalar field lingering at the top of the plateau (Guth, 2007)".
   - Probability of field remaining near the maximum is non zero.
   - Can still happen in monomial models, but more natural in hilltop models.

Eternal Inflation

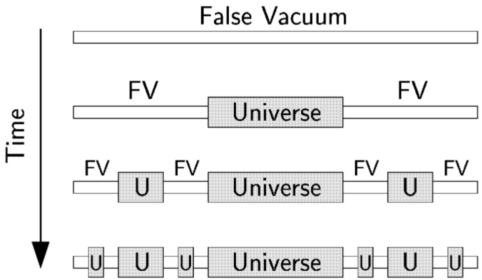


Figure 3. A schematic illustration of eternal inflation.

Figure 7: Credit Guth (2007)

Perspectives on Eternal Inflation

1. Guth, Linde, and others find eternal inflation intriguing especially as a mechanism to populate the landscape of string vacua. (Guth, 2007) and (Linde, 2007)
   - "In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times. Thus, the question of what is possible becomes trivial—anything is possible."
2. Steinhardt dislikes these implications and worries that the inflationary paradigm as currently constrained doesn't actually predict anything (Steinhardt, Loeb, Ijjas 2013)
   - An eternally inflating multiverse renders "inflationary theory totally unpredictive."

Are There Any Viable Alternatives?

- Depends on who you ask.
- Any alternative would have to at minimum reproduce the successes of inflation.
- Steinhardt, Turok, and collaborators have worked on various bouncing/cyclic scenarios to try to do this with varying degrees of success.
- Will discuss how a contracting phase with a bounce can reproduce some of the desired features.

Bouncing Models

- Recall
  1. Friedmann equation $H^2 = \frac{8\pi G}{3}\rho$
  2. The horizon size is $\propto (H)^{-1}$
  3. density ρ can be expressed in terms of the scale factor $\rho \propto \frac{1}{a^{2\epsilon}}$ with $\epsilon = \frac{3}{2}\left(1 + \frac{p}{\rho}\right)$
  4. Curvature parameter $\Omega_k = \frac{k}{a^2}H^{-2}$
- This allows the following
  1. Horizon size $H^{-1} \propto a^{\epsilon}$
  2. Curvature parameter $\Omega_k \propto \frac{a^{2\epsilon}}{a^2}$

Bouncing Models

- Consider a contracting phase and the ratio between horizon size to "patch size" $\frac{a^{\epsilon}}{a}$
- As the universe contracts the horizon shrinks more and can bring previously causally connected regions out of contact with each other.
- Similarly the curvature parameter $\Omega_k \propto \frac{a^{2\epsilon}}{a^2}$ will approach zero.
- Framework can resolve the horizon and flatness problems.

Bouncing Models and the Equation of State

- Important to have $w = \frac{p}{\rho} \geq 1$ which means $\epsilon \geq 6$
- Can be seen from argument made earlier by Steinhardt and Turok in proposing a cyclic model where V flips from positive to negative (2004).

$$w = \frac{p}{\rho} = \frac{\frac{\dot{\phi}^2}{2} - V}{\frac{\dot{\phi}^2}{2} + V} \geq 1$$

- Also important for avoiding large inhomogeneities as this allows scalar energy density to dominate over all other terms during contraction.

Bouncing Cosmologies

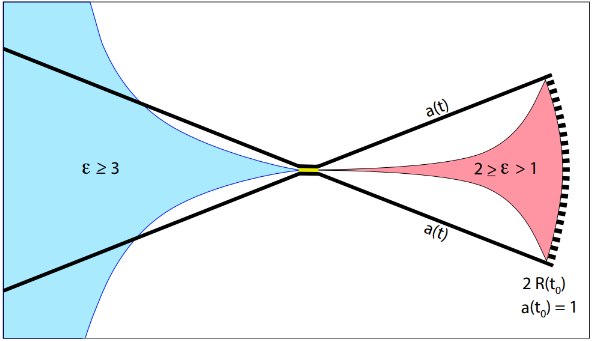


Figure 3: Wedge diagram for non-singular bouncing cosmology. A period of contraction (lhs) is followed by a bounce (small shaded connector, middle) and the current period of expansion (rhs). The figure illustrates that the patch size (thick solid lines) is larger than the horizon size (shaded region) during the expanding phase, but was smaller than the horizon size during most of the contracting phase (lhs of diagram). Note that the evolution of the horizon size ($\propto a^{\epsilon}$) is different before and after the bounce (shaded regions) because $\epsilon \geq 3$ leading up to the bounce and, after conversion of energy to ordinary matter and radiation, $2 \geq \epsilon$ after the bounce.

Figure 8: Credit Ijjas and Steinhardt (2018)

Perturbations in Contracting Universe

- Can use a potential of the form $V = -V_0 exp(-\sqrt{3(1+w)}\phi)$ in describing contracting state with the requirements we have discussed.
- Equation describing fluctuations is

$$\delta\ddot{\phi}_k + (k^2 + V'')\delta\phi_k = 0$$

- Very similar to the inflation scenario, but difference is in how the metric changes.
- Can get scale invariant scalar perturbations, but not GW (Steinhardt and Turok 2002).

Spectral Tilt

- Inflation with $w = -1$ gives

$$n_s - 1 = -2\epsilon + \frac{dlog(\epsilon)}{dN}$$

- Contraction with $w > 1$ gives

$$n_s - 1 = \frac{-2}{\epsilon} - \frac{dlog(\epsilon)}{dN}$$

- Big Takeaway: Contracting phase can reproduce many of the successes of inflation.

Biggest Problem

- How do we reverse contraction as energy density is increasing?
- $\dot{H} \propto -(\rho + p)$
- This requires a Null Energy Condition (NEC) violating phase.
- Attempts to do this often result in ghost/gradient instabilities.

NEC Violating Bounce

$$\mathcal{L} = \frac{M_{pl}^2 R}{2} - \frac{k(\phi)(\partial\phi)^2}{2} + \frac{M_{pl}^{-4} q(\phi)(\partial\phi)^4}{4} + \frac{M_{pl}^{-3} b(\phi)(\partial\phi)^2(\partial_\mu\partial^\mu\phi)}{2} - V$$

$$3H^2 = \rho = \frac{k\dot{\phi}^2}{2} + \frac{(3q - 2b')\dot{\phi}^4}{4} + 3Hb\dot{\phi}^3 + V$$

$$-2\dot{H} = \rho + p = k\dot{\phi}^2 + (q - b')\dot{\phi}^4 + 3Hb\dot{\phi}^3 - b\ddot{\phi}\dot{\phi}^2$$

NEC Violating Bounce

$$S^{(2)} = \int d^4 x a^3(t) \left(A(t)\dot{\xi}^2 - \frac{B(t)}{a^2(t)} (\nabla \xi)^2 \right)$$

where A(t) and B(t) determine stability.

$$A(t) = \frac{k\dot{\phi}^2 + (3q - 2b')\dot{\phi}^4 + 6Hb\dot{\phi}^3 + \frac{3}{2}b^2\dot{\phi}^6}{2\left(H - \frac{1}{2}b\dot{\phi}^3\right)^2}$$

$$B(t) = \frac{k\dot{\phi}^2 + q\dot{\phi}^4 + 4Hb\dot{\phi}^3 - \frac{1}{2}b^2\dot{\phi}^6 + 2b\ddot{\phi}\dot{\phi}^2}{2\left(H - \frac{1}{2}b\dot{\phi}^3\right)^2}$$

Stable Solution

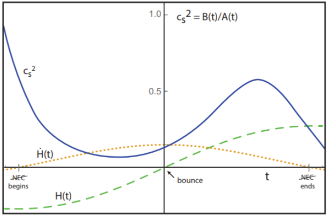


FIG. 1. A plot of the sound speed c_S^2 (solid blue curve) for co-moving curvature perturbations as a function of time t. The time coordinate is given in Planck units and the value of c_S^2 is given in units where the speed of light is unity. Superimposed for illustration purposes are the shapes of the background solutions for $H(t)$ (dashed green curve; also shown in Figs. 2 and 3) and $\dot{H}(t)$ (dotted red curve). More specifically, the results correspond to $H(t) = H_0 t\, e^{-F(t-t^*)^2}$ and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ with the parameter values $H_0 = 3\times10^{-5}, t^* = 0.5, F = 9\times10^{-5}, \gamma_0 = -0.0044, \Theta = 0.0046$. Notably, throughout, the sound speed is real ($A(t), B(t) > 0$) and sub-luminal, with $0 < c_S^2 < 1$. The characteristic energy scale $\sim H^2$ is well below the Planck scale, and the NEC violating phase lasts ~ 150 Planck times; it starts when $\dot{H}$ becomes positive at $t_{\rm beg} \simeq -74 M_{\rm Pl}^{-1}$ and ends when $\dot{H}$ becomes negative at $t_{\rm end} \simeq 75 M_{\rm Pl}^{-1}$; the bounce ($H(t) = 0$) occurs at $t = 0$. Note that the bounce stage occurs well within the classical regime.

Figure 9: Credit Ijjas and Steinhardt (2016)

Conclusions

1. Post Planck it is hard to make simplest models with fewest parameters work.
2. Still many viable inflation models, but these are arguably less satisfying.
3. Important to look at alternatives because it is possible to reproduce some of the successes with different paradigms.

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