Lecture 1

AST 449

Spacetime geometry

- Robertson — Walker metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

- Friedmann equations

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \end{aligned}$$

Distances

- Line-of-sight comoving distance

$$\chi(a) = \int_{t(a)}^{t_0} \frac{dt'}{a(t')} = \int_a^1 \frac{da'}{a'^2 H(a')}$$

$$= \int_a^1 \frac{da'}{a'^2 H_0 \left[\Omega_m a'^{-3} + \Omega_r a'^{-4} + \Omega_k a'^{-2} + \Omega_\Lambda\right]^{1/2}}$$

- Transverse comoving distance:

$$d_M(a) = \begin{cases} \frac{1}{H_0\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k} H_0 \chi(a)\right) & \text{for } \Omega_k > 0 \\ \chi(a) & \text{for } \Omega_k = 0 \\ \frac{1}{H_0\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|} H_0 \chi(a)\right) & \text{for } \Omega_k < 0 \end{cases}$$

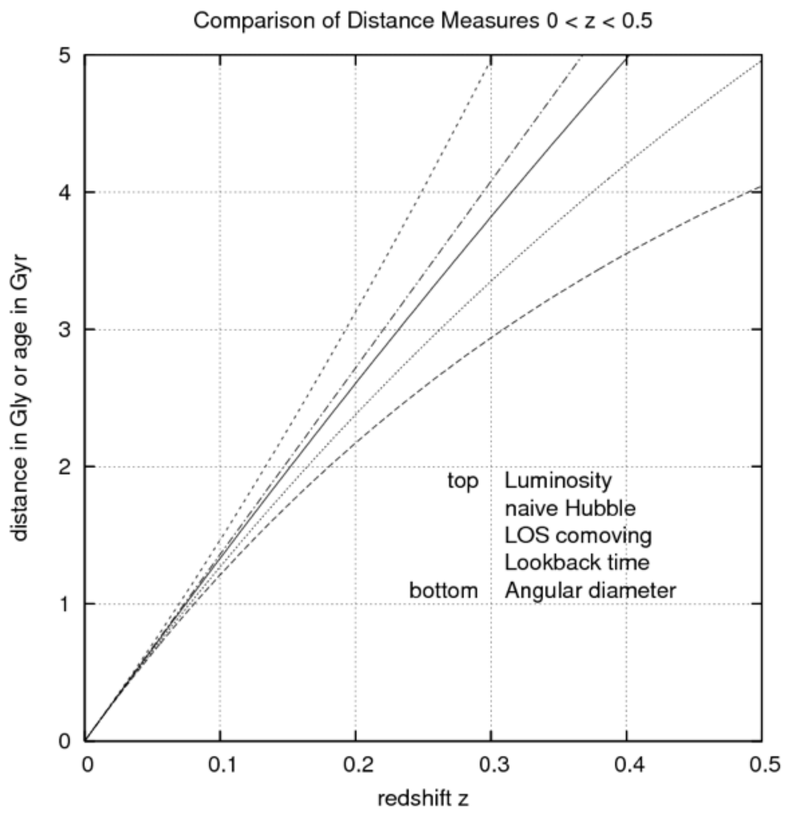
Distances

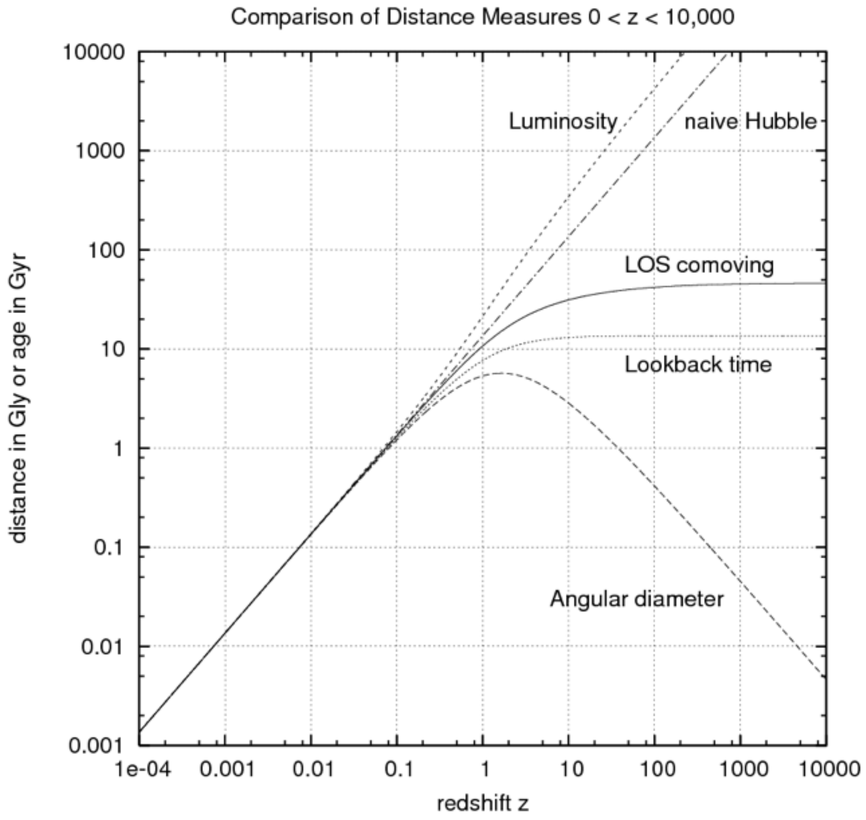
- Luminosity distance

$$d_L(a) = d_M(a)/a$$

- Angular diameter distance:

$$d_A(a) = a d_M(a)$$





Distance at high redshift: Accelerated expansion

- Standard candle
  - brightest galaxies in rich clusters
  - Type Ia supernovae
    - bright
    - physics well known, with variations correctable
    - luminosity distance as a function of redshift

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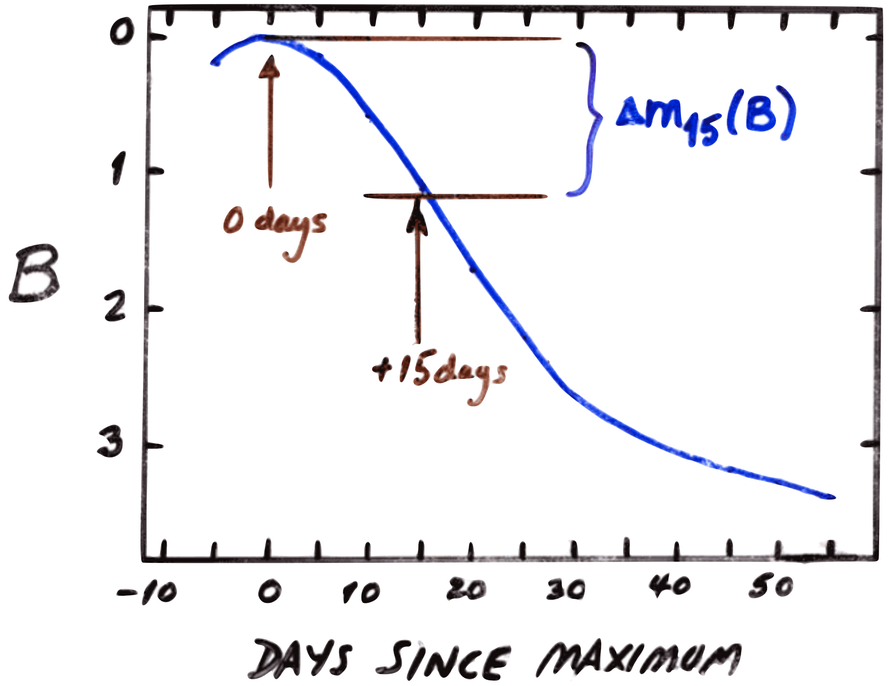
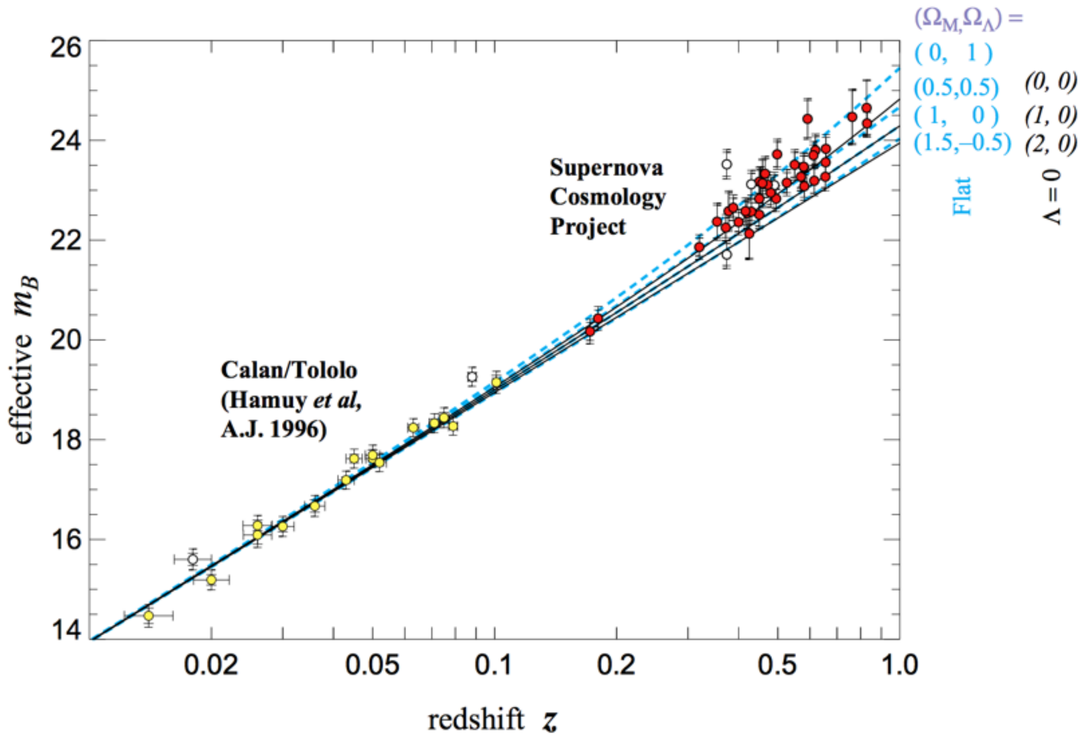
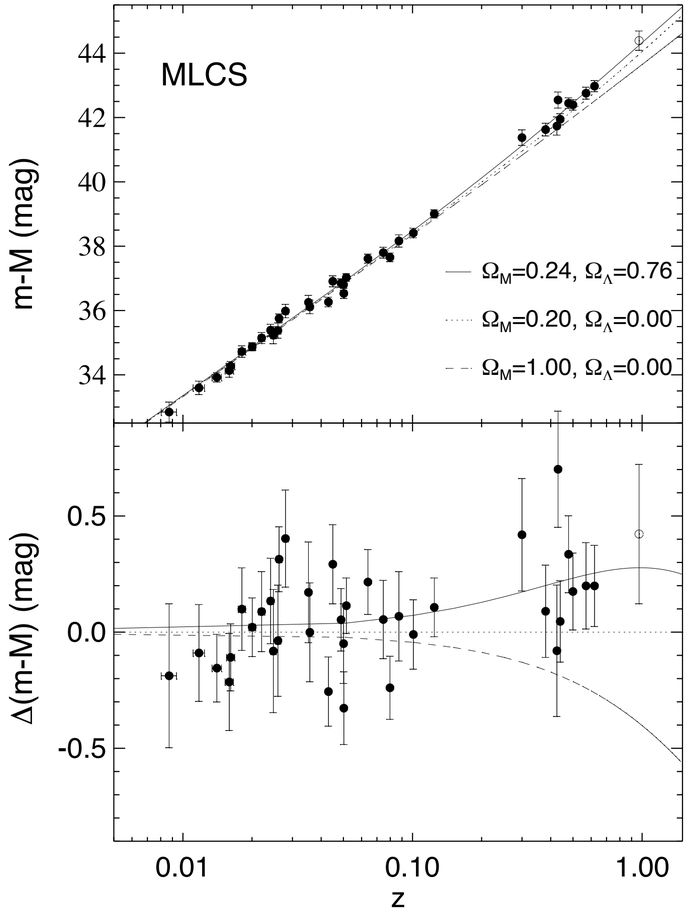


$$M_{\max}(B) = -21.726 + 2.698\Delta m_{15}(B)$$

The Supernova Cosmology Project and The High-z Supernova Search Team



S. Perlmutter et al., 1999, ApJ, 517, 565



A. Riess et al., 1998, AJ, 116, 1009

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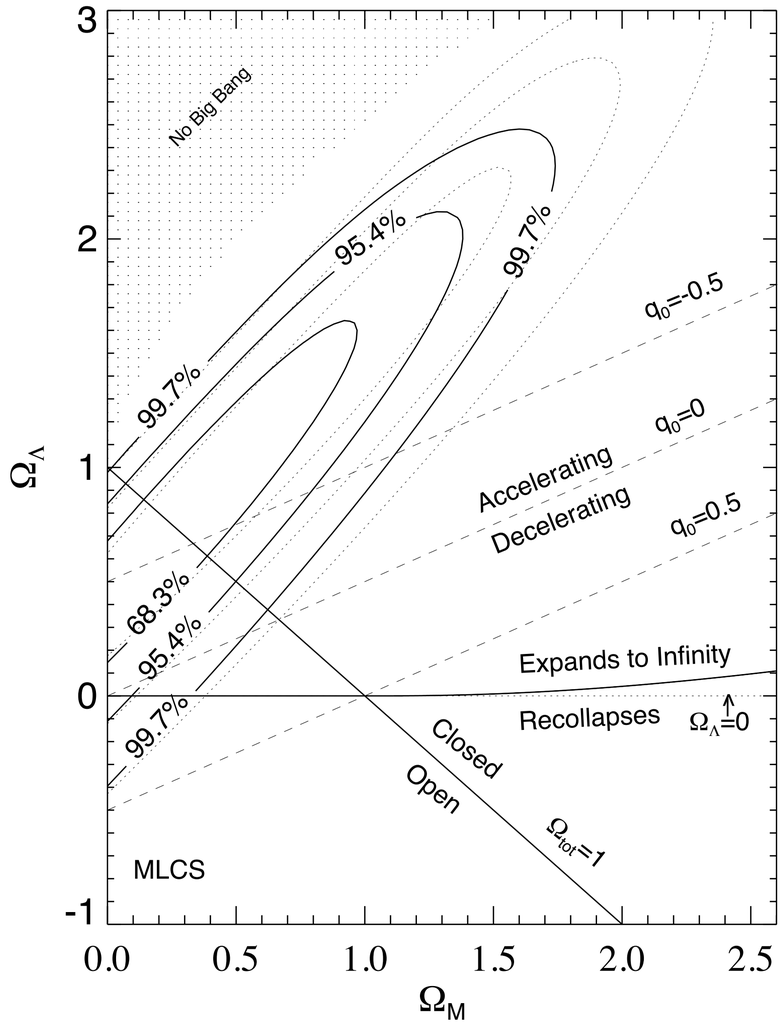
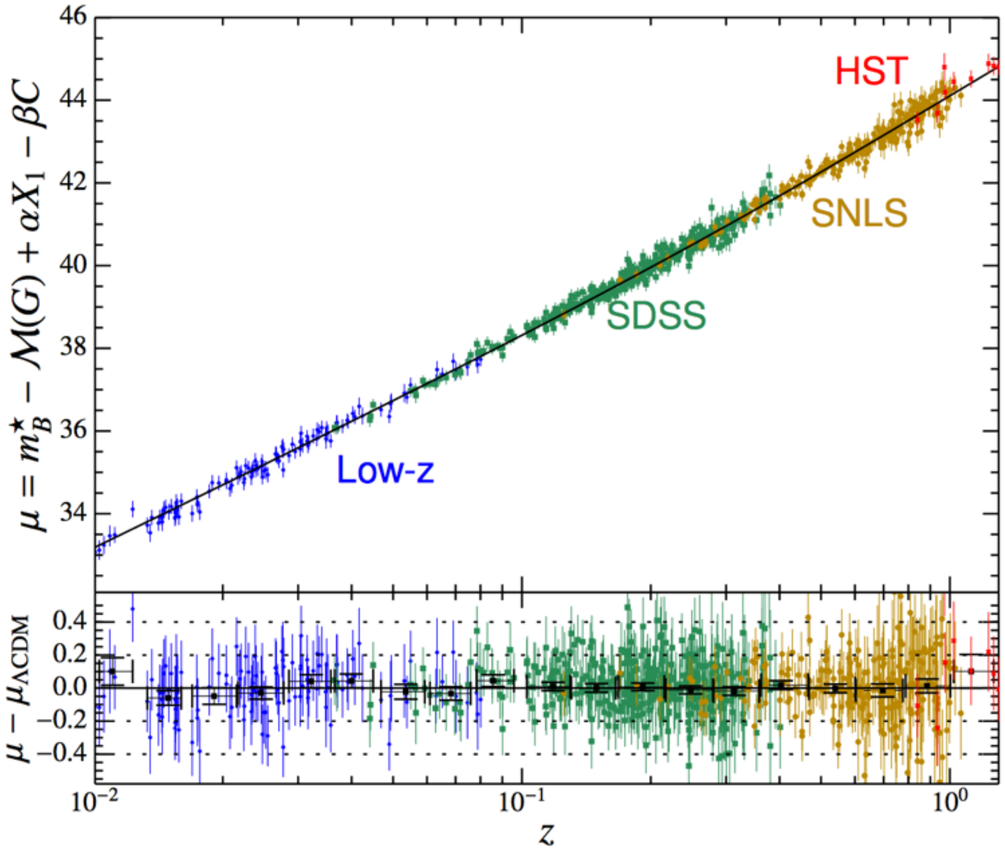


$$\chi^2(H_0, \Omega_m, \Omega_\Lambda) = \sum_i \frac{(\mu_{p,i}(z_i; H_0, \Omega_m, \Omega_\Lambda) - \mu_{0,i})^2}{\sigma^2_{\mu_0,i} + \sigma^2_v}$$

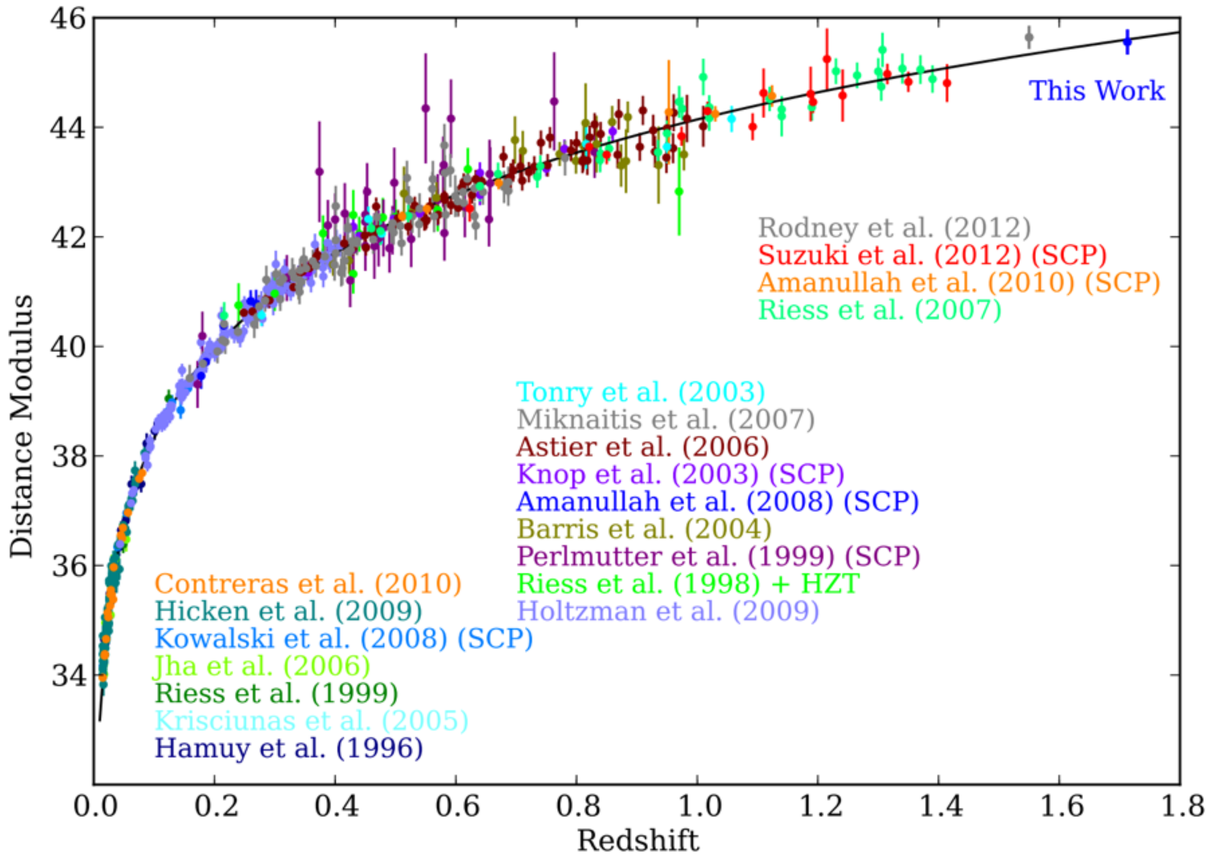
$$p(\mu_0 | H_0, \Omega_m, \Omega_\Lambda) = \left(\prod_i \frac{1}{\sqrt{2\pi(\sigma^2_{\mu_0,i} + \sigma^2_v)}} \right) \exp\left(-\frac{\chi^2}{2}\right)$$

A. Riess et al., 1998, AJ, 116, 1009

Supernovae cosmology today



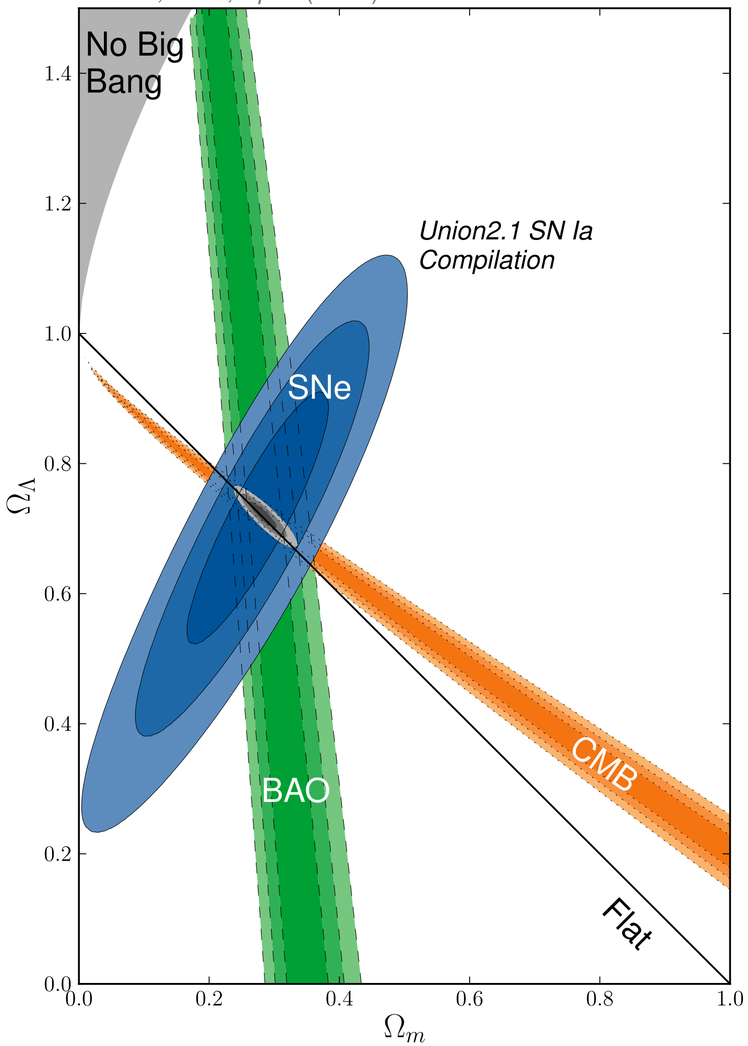
M. Betoule et al. (SNLS+SDSS), 2014, A&A, 568, 22



D. Rubin et al. (Union2.1), 2013, ApJ, 763, 35

Supernovae cosmology today

Supernova Cosmology Project
Suzuki, et al., *Ap.J.* (2011)



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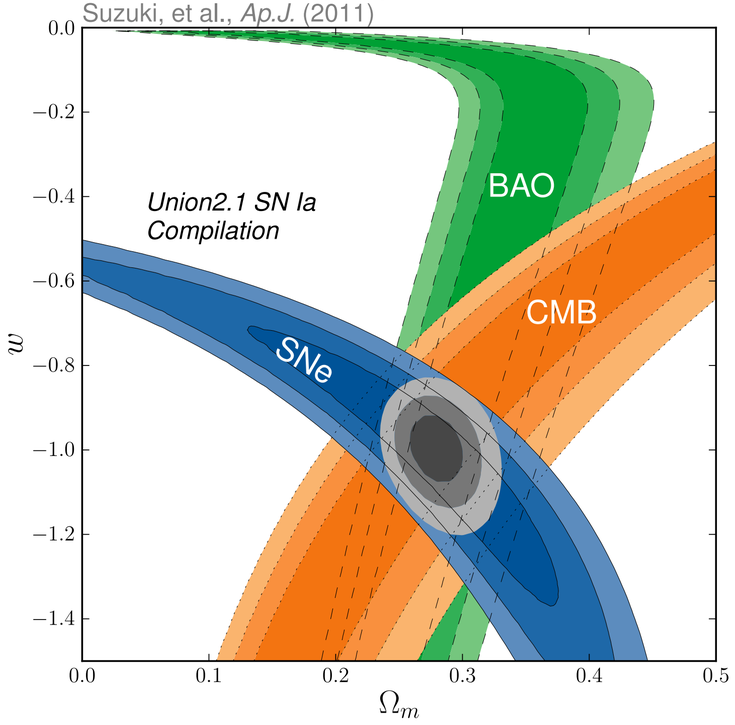

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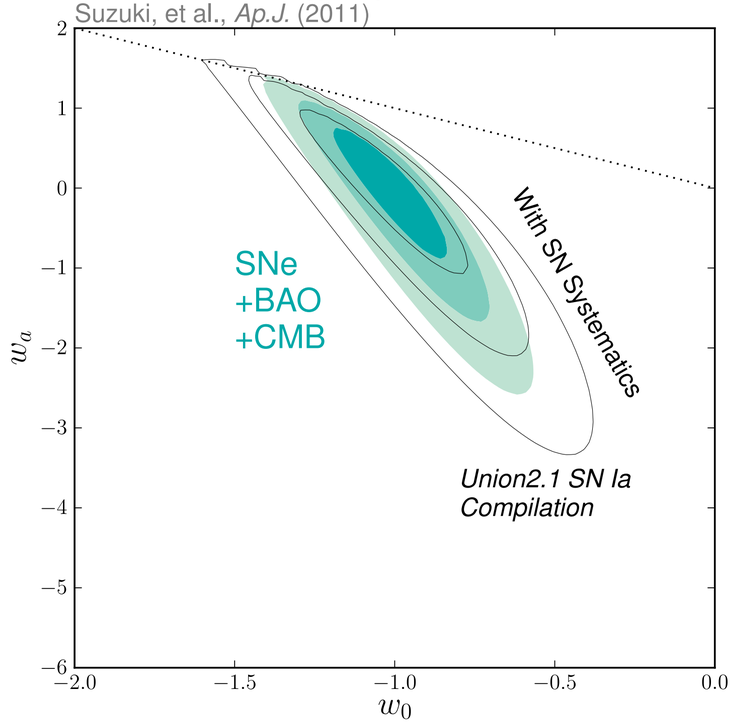


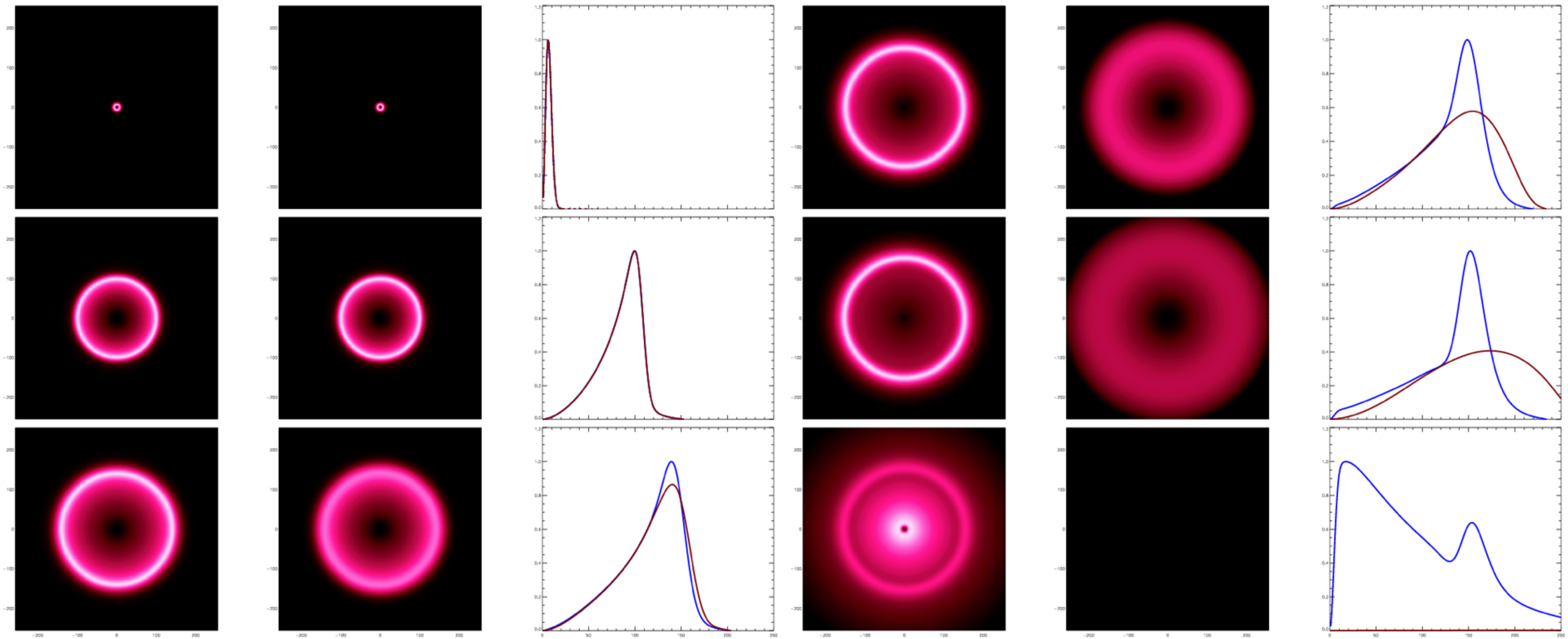
$$\rho_\Lambda \propto \Omega_\Lambda a^{-3(1+w)}$$

$$w(a) = w_0 + w_a(1-a)$$

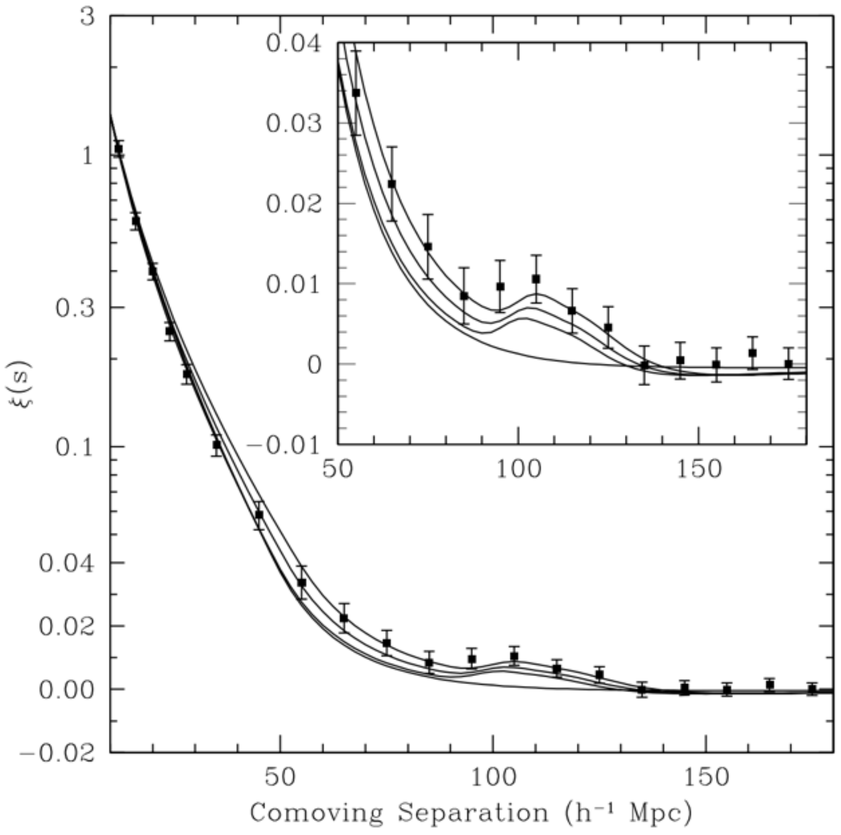
Angular diameter distance

- Definition
- Current measurements of angular diameter distance
  - Baryon acoustic oscillation
  - Cosmic microwave background
  - Weak gravitational lensing

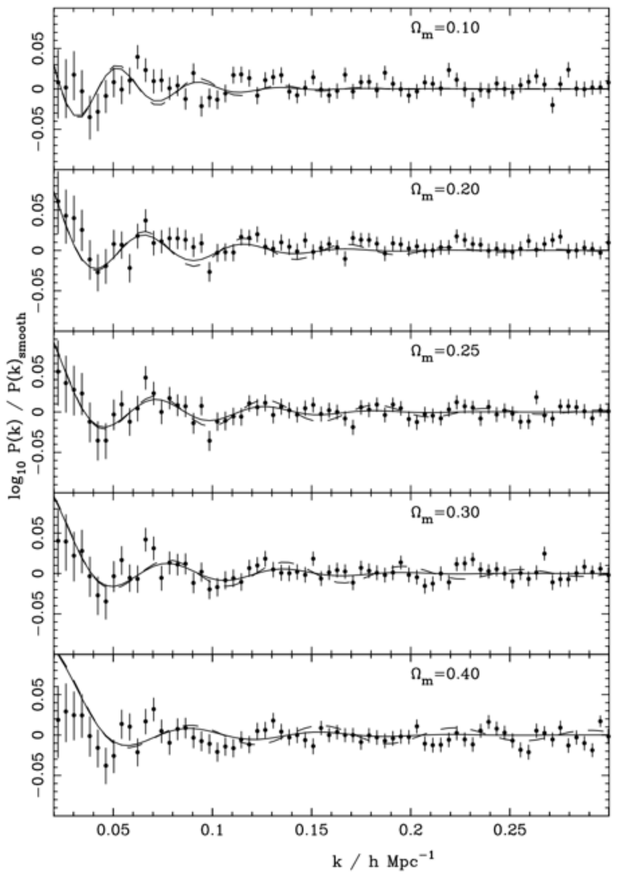
Angular diameter distance — Baryon Acoustic Oscillation (BAO)



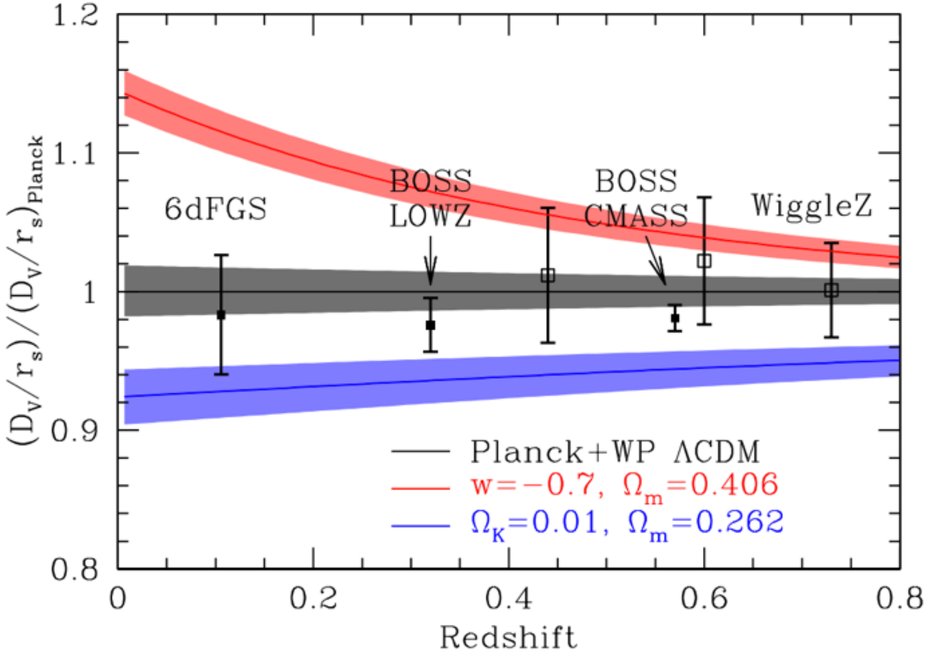
Angular diameter distance — Baryon Acoustic Oscillation (BAO)



D. Eisenstein et al., 2005, ApJ, 633, 560

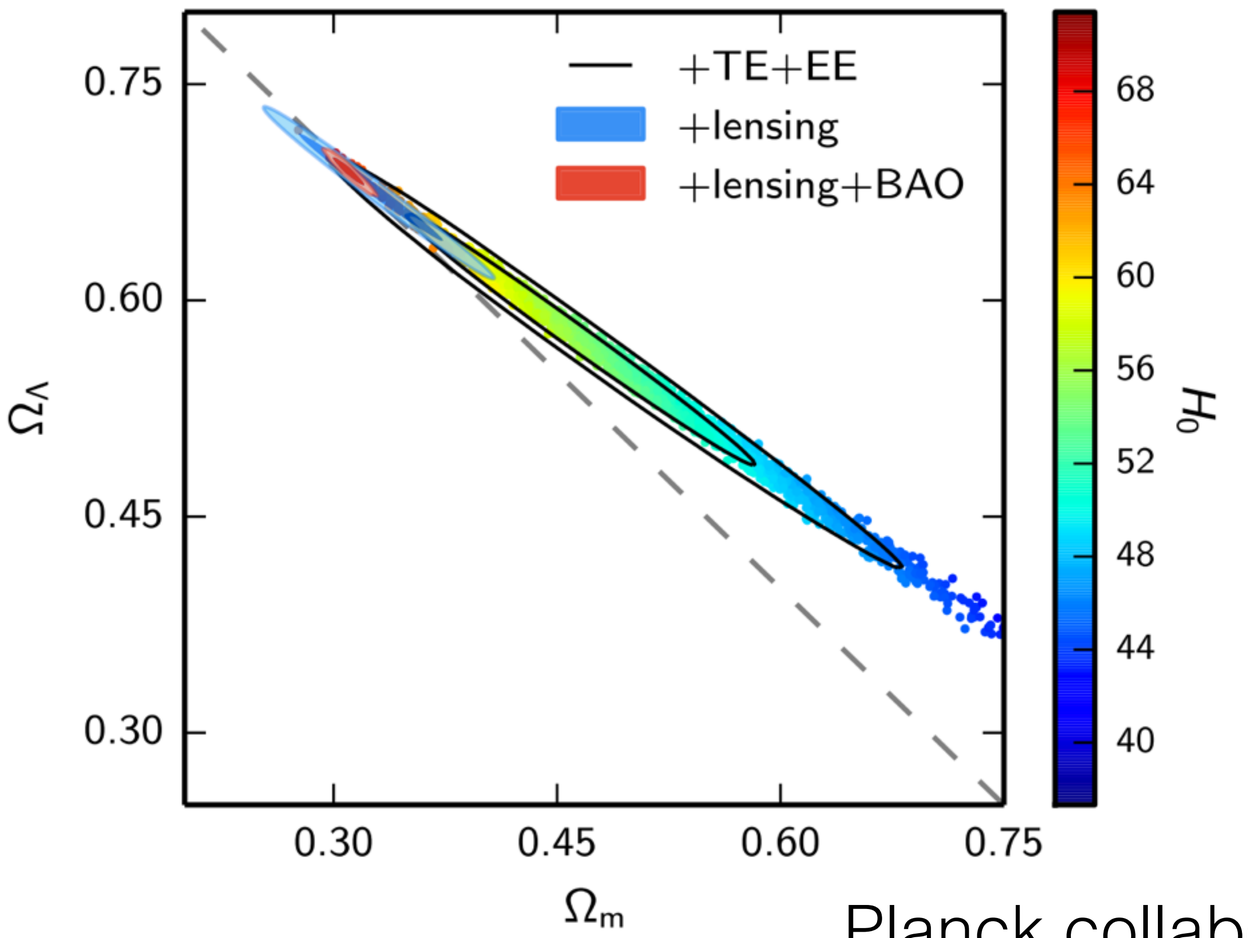


W. Percival et al., 2007, ApJ, 657, 51



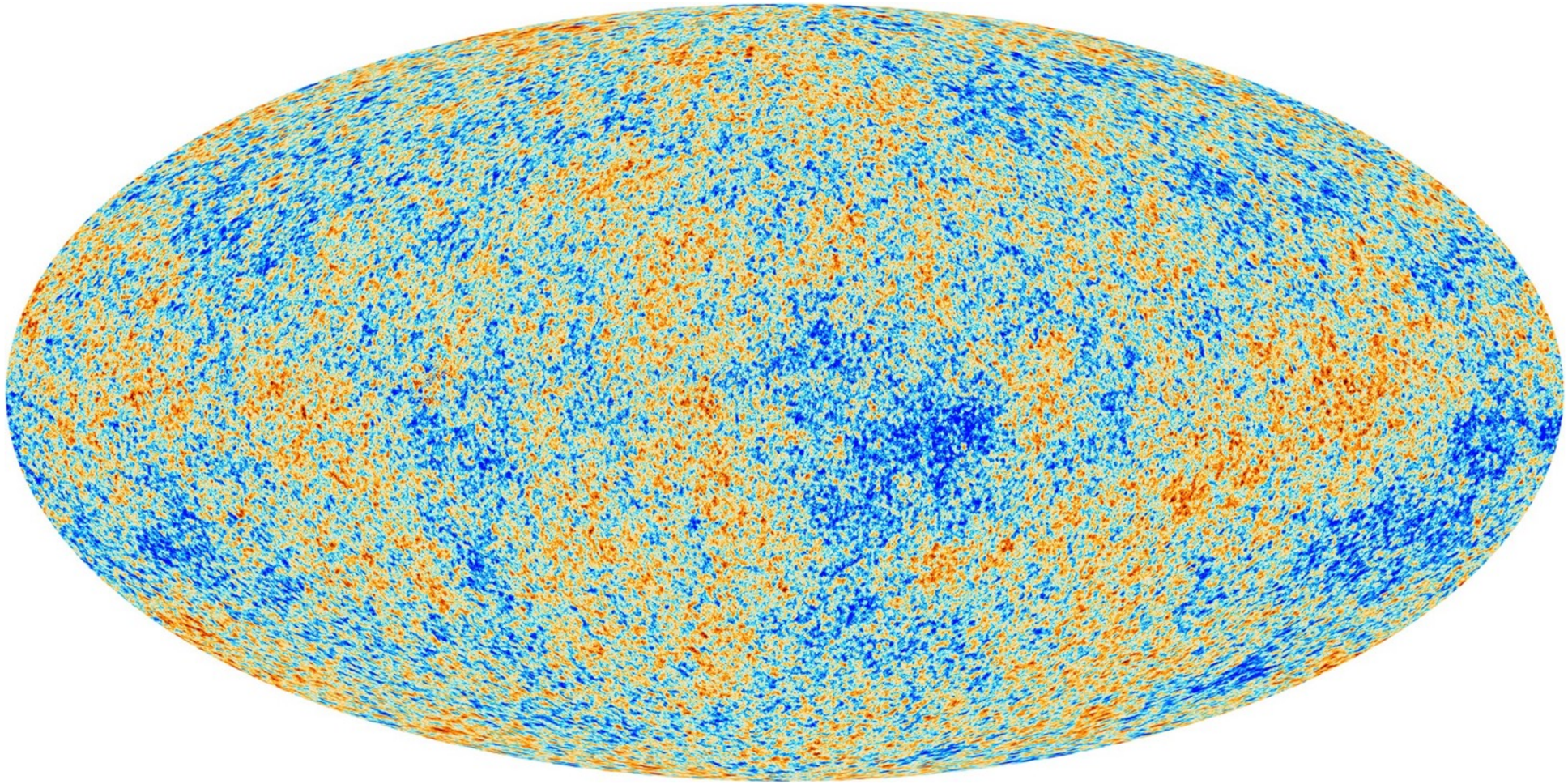
Anderson et al., 2014 MNRAS, 441, 24

Angular diameter distance — Baryon Acoustic Oscillation (BAO)



Planck collaboration, 2015, arXiv:1502.01589

Angular diameter distance — Cosmic Microwave Background (CMB)



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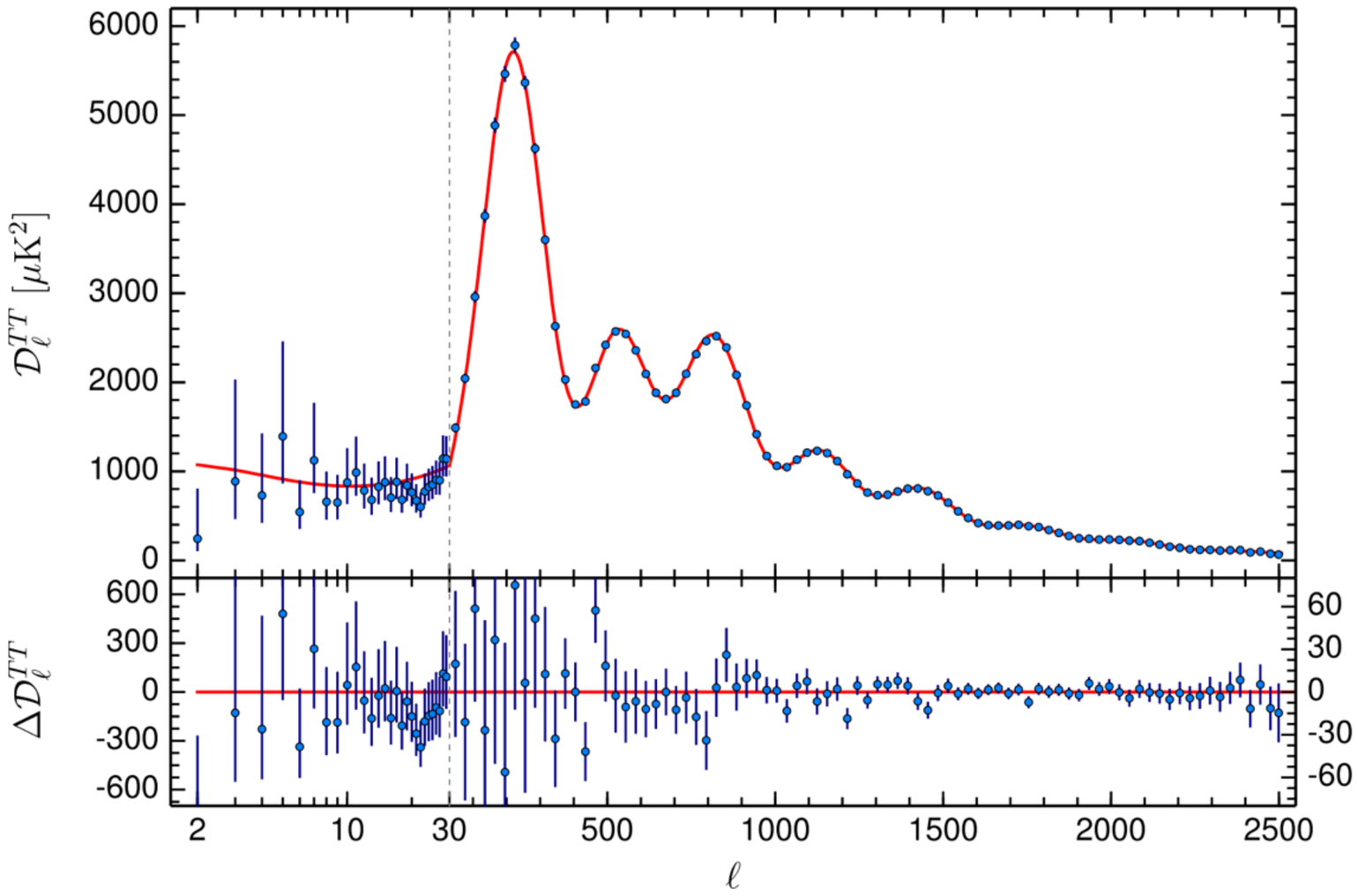
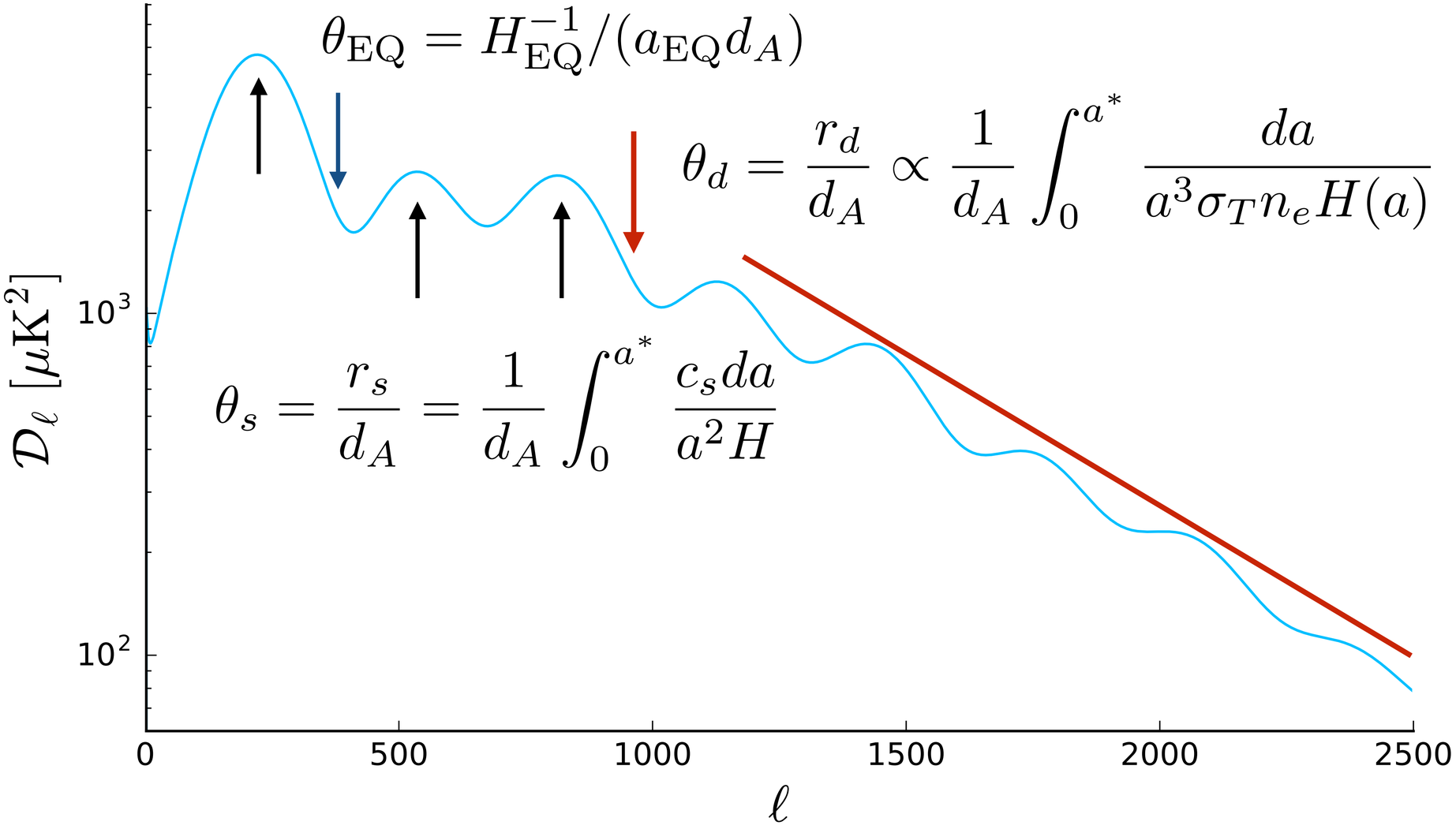
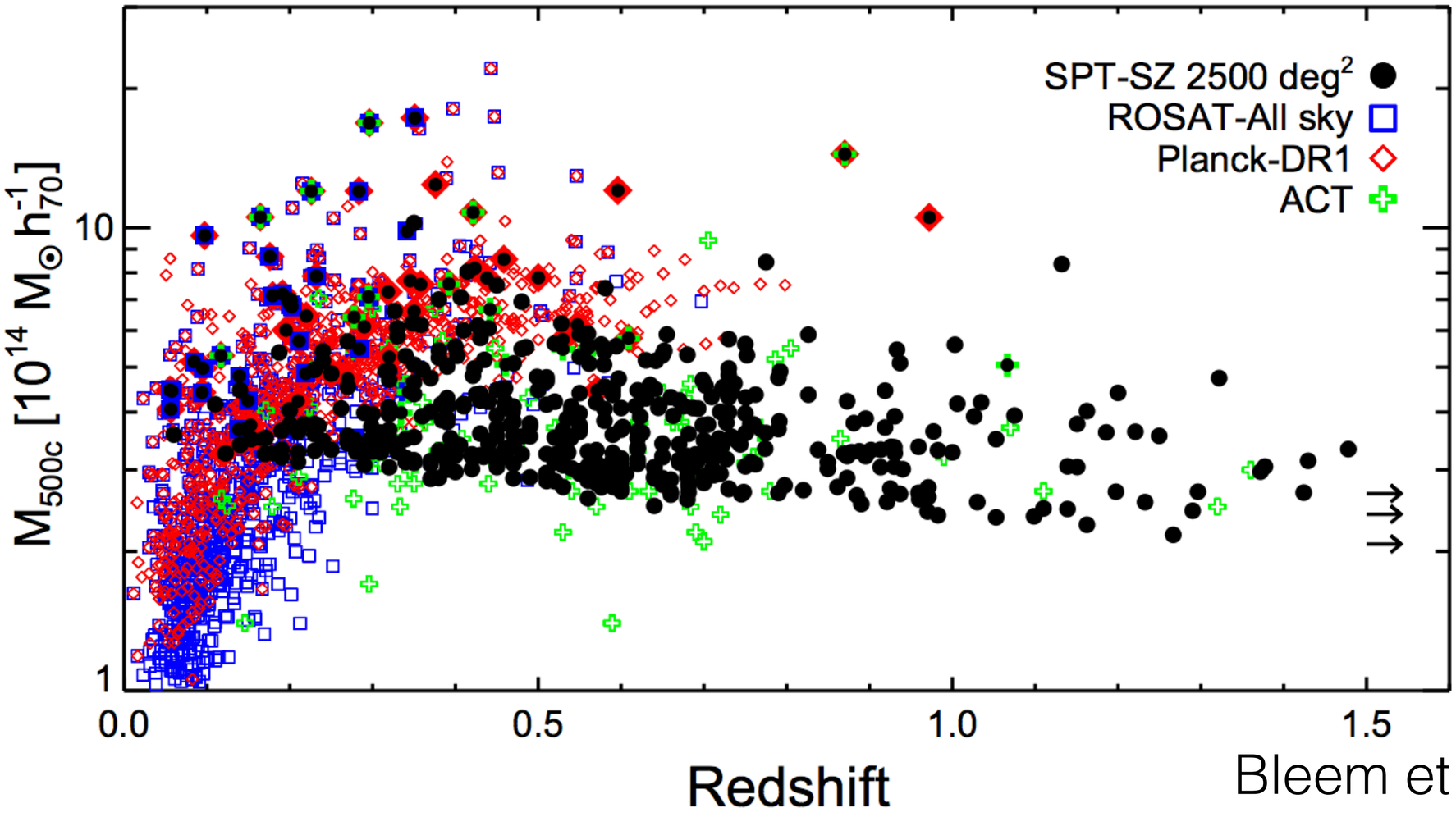


Planck collaboration, 2015, arXiv:1507.02704

Angular diameter distance — Cosmic Microwave Background (CMB)



Further probe of dark energy — Galaxy clusters



Bleem et al., 2015, ApJS, 216, 27