

Astro 242

The Physics of Galaxies and the Universe: Lecture Notes

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Syllabus

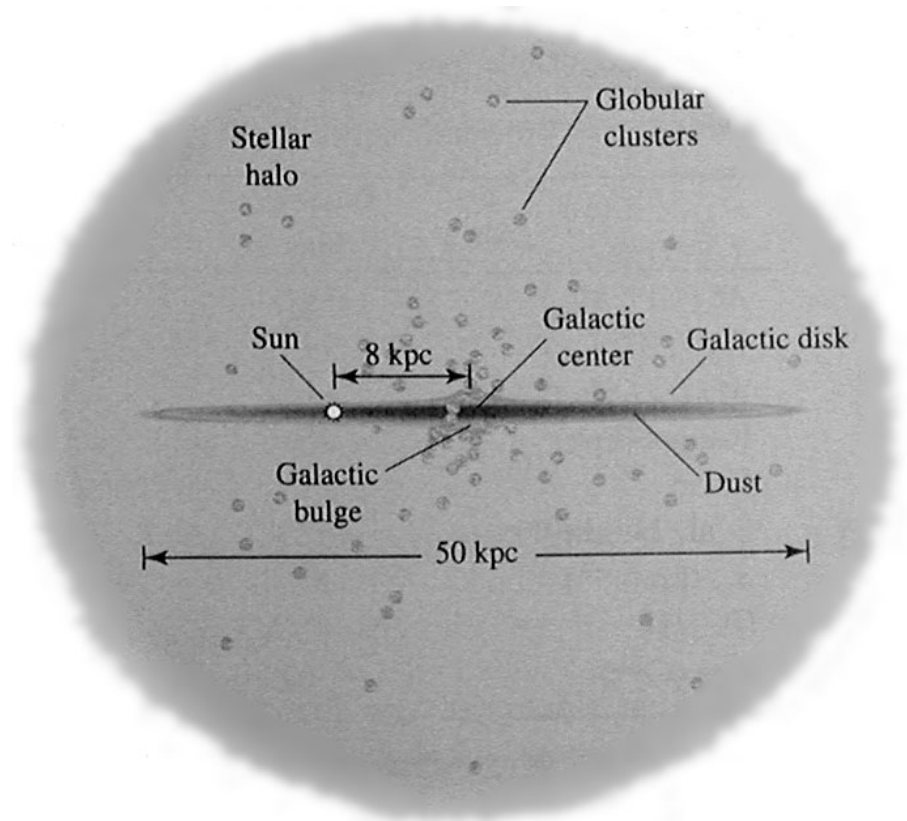
- Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie
- First class Wed Jan 3. Reading period Mar 8-9
- Jan 1: Milky Way Galaxy
- Jan 8: Nature of Galaxies
- Jan 15: Galactic Evolution
- Jan 22: Active Galaxies
- Midterm
- Jan 29: Structure of the Universe
- Feb 5-12: Cosmology
- Feb 19-26: Early Universe

Set 1:

Milky Way Galaxy

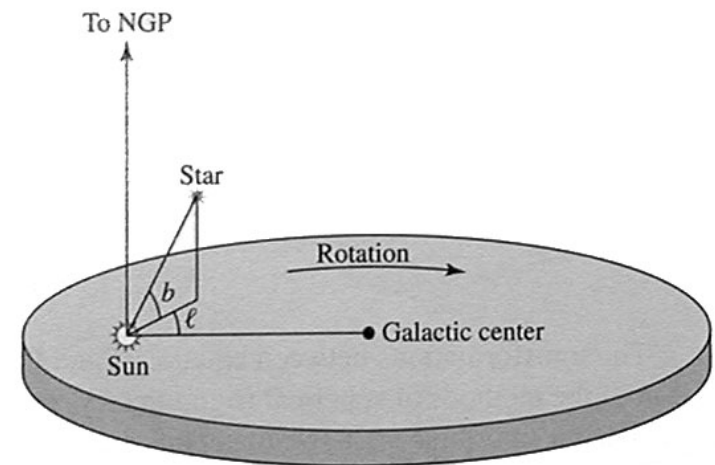
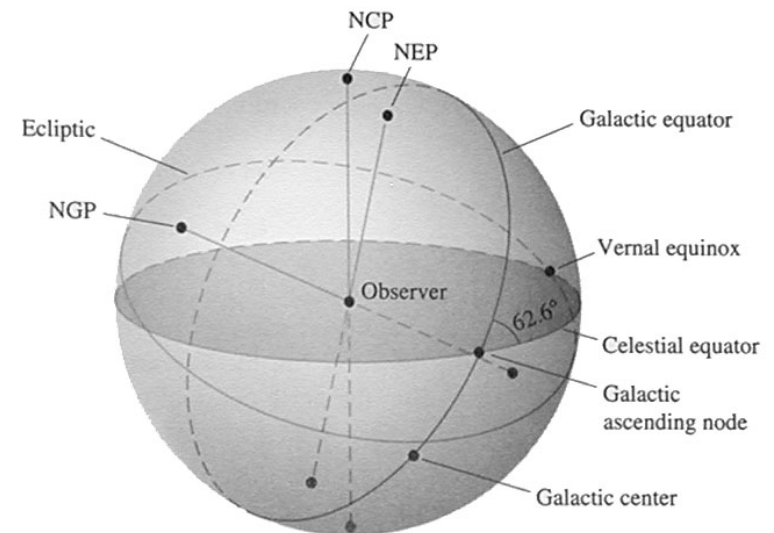
Components and Scale

- Sun is embedded in a stellar disk ~ 8 kpc from the galactic center
- Extent of disk ~ 25 kpc radius, spiral structure
- Thickness of neutral gas disk < 0.1 kpc
- Thickness of thin disk of young stars ~ 0.35 kpc
- Thickness of thick disk ~ 1 kpc



Components and Scale

- Central stellar bulge
radius ~ 4 kpc, with central bar
- Supermassive black hole, inferred
from mass < 120 AU of center
- Extended spherical stellar
halo with globular
clusters, radius > 100 kpc
- Extended dark matter halo,
radius > 200 kpc



Mass and Luminosity

- Neutral gas disk: $M \sim 0.5 \times 10^{10} M_{\odot}$
- Thin disk: $M \sim 6 \times 10^{10} M_{\odot}$, $L_B \sim 1.8 \times 10^{10} L_{\odot}$
- Thick disk: $M \sim 0.2 - 0.4 \times 10^{10} M_{\odot}$, $L_B \sim 0.02 \times 10^{10} L_{\odot}$
- Bulge: $M \sim 1 \times 10^{10} M_{\odot}$, $L_B \sim 0.3 \times 10^{10} L_{\odot}$
- Supermassive black hole mass $3.7 \pm 0.2 \times 10^6 M_{\odot}$
- Stellar halo: $M \sim 0.3 \times 10^{10} M_{\odot}$, $L_B \sim 0.1 \times 10^{10} L_{\odot}$
- Dark matter halo: $M \sim 2 \times 10^{12} M_{\odot}$
- Total $M \sim 2 \times 10^{12} M_{\odot}$, $L_B \sim 3.6 \times 10^{10} L_{\odot}$

Starlight: Optical Image

Optical Map of the Milky Way



Methods: Star Counts

- History: Kapteyn (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy
- Fundamental assumptions
 - Stars have a known distribution in absolute magnitude
 - No obscuration
- Consider a star with known absolute magnitude M (magnitude at 10pc). It's distance can be inferred from the inverse square law from its observed m as

$$\frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5}$$

Combined with the angular position on the sky, the 3d position of the star can be measured - mapping the galaxy

Methods: Star Counts

- Consider the spatial number density within dM of M :
 $n_M(M, \mathbf{r})dM$ with total number density

$$n(\mathbf{r}) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r})dM$$

- Given limiting magnitude m , distance d out to which stars of a given M are included is given by $d(m - M)$
- Total number observed out to in solid angle $d\Omega$ within dM of M

$$N_M dM = \left[\int_0^{d(m-M)} n_M(M, r) r^2 dr \right] d\Omega dM$$

- Differentiating with respect to $d(m - M)$ provides a measurement of $n_M(M, r)$.

Methods: Star Counts

- Similarly replace M with an observable attribute like spectral type.
- Alternately: given a functional form for n_M obtain normalization $n(\mathbf{r})$.
- Kapteyn inferred a flattened spheroidal system of $< 10\text{kpc}$ extent in plane and $< 2\text{kpc}$ out of plane: too small
- Missing: interstellar extinction dims stars dropping them out of the sample at a given limiting magnitude

Methods: Variable Stars

- History: Shapley (1910-1920) used RR Lyrae and W Virginis variable stars - period-luminosity relation
- Measure the period of oscillation, infer a luminosity and hence an absolute magnitude, infer a distance from the observed apparent magnitude
- Inferred a 100kpc scale for the galaxy - overestimate again due to interstellar extinction
- Apparent magnitude is dimmed by extinction leading to a distance inference that is too large

Interstellar Extinction

- Dust in the interstellar medium (see Chap 12) dims stars at visible wavelengths making the true distance less than the apparent distance

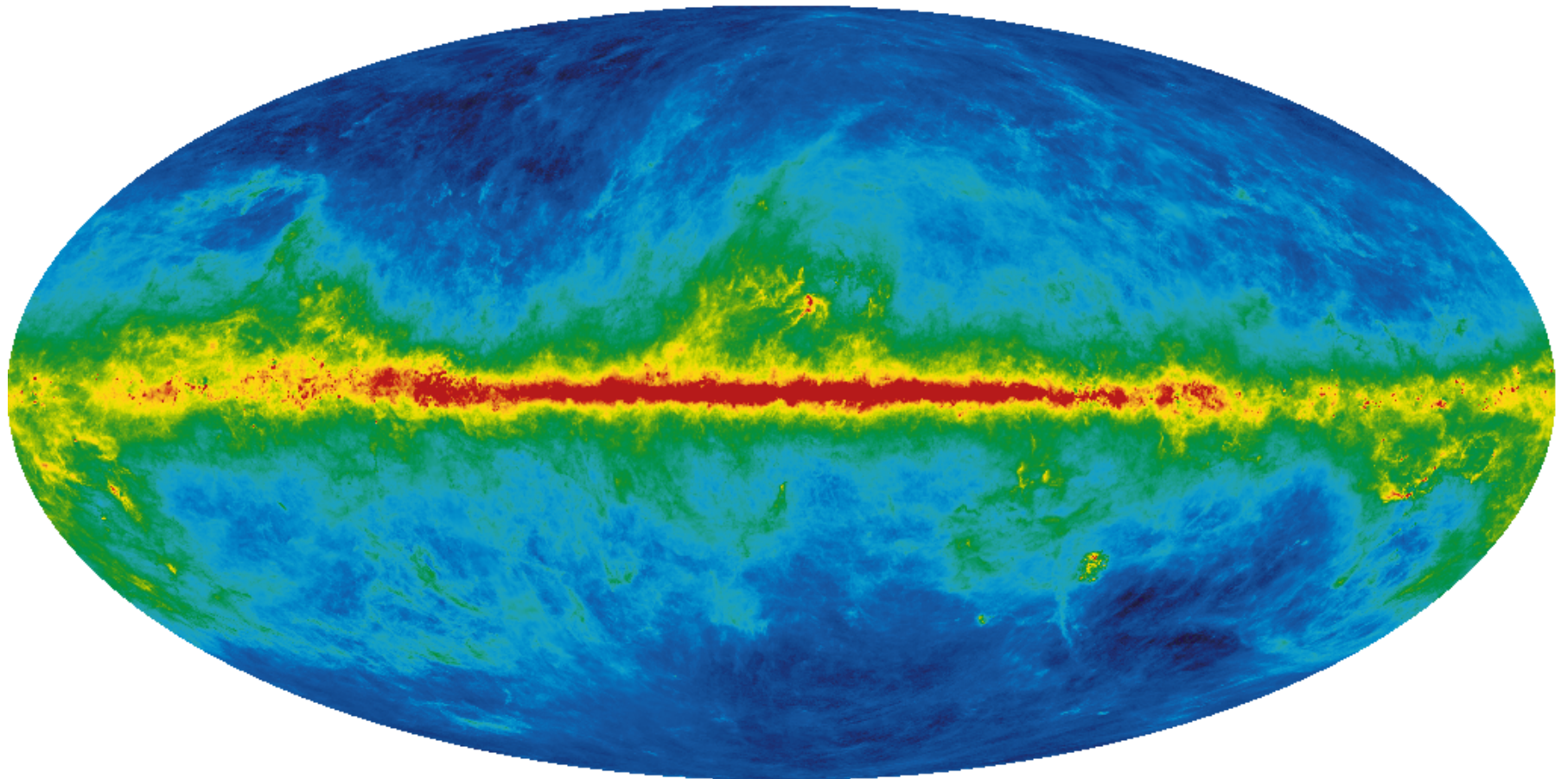
- Distance formula modified to be

$$\frac{d}{10\text{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5}$$

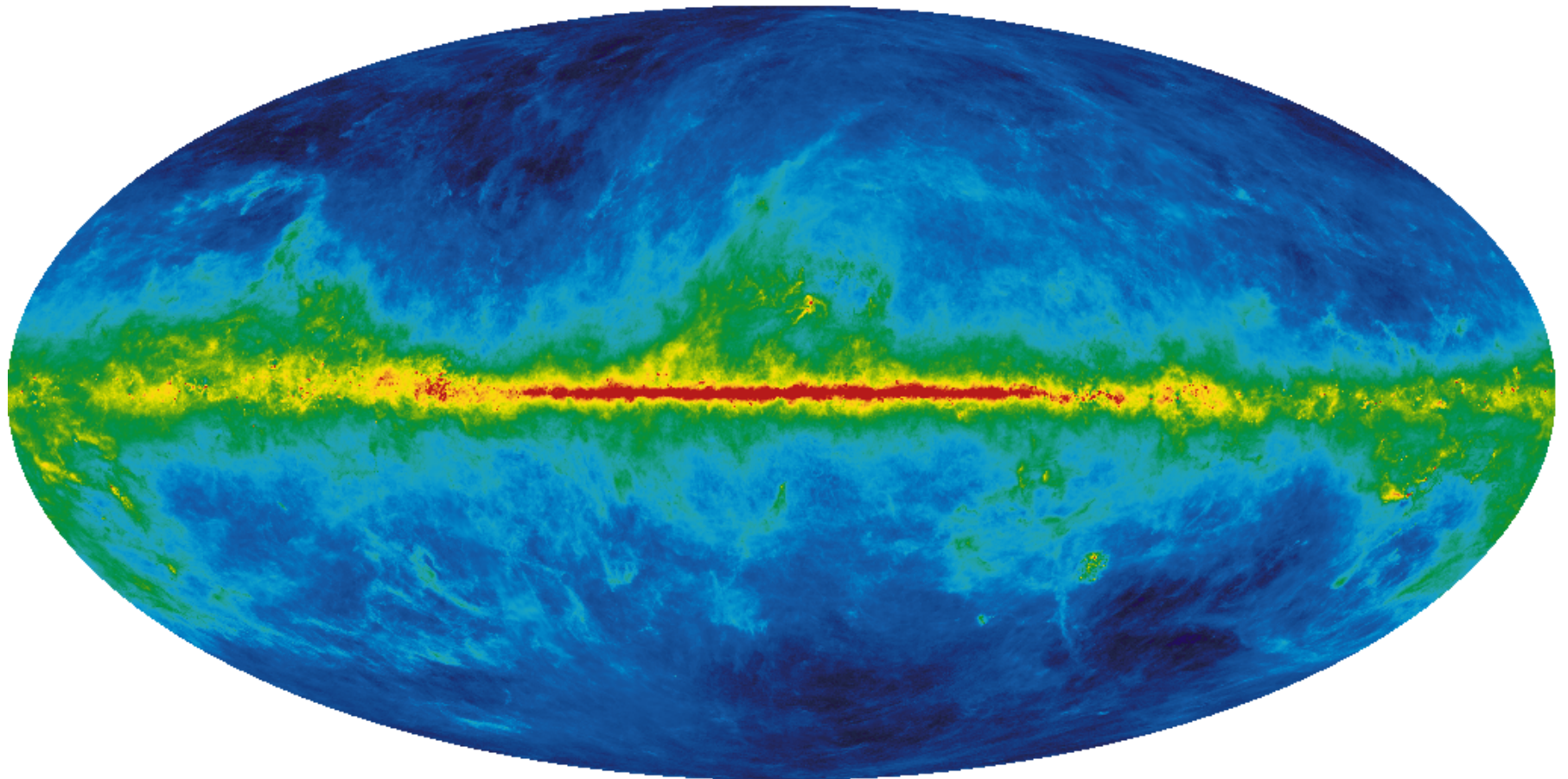
where the extinction coefficient $A_\lambda \geq 0$ depends on wavelength λ

- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc
- Typical value at visible wavelengths and in the disk is 1 mag/kpc
- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE] can be used to calibrate extinction

Dust Emission



Extinction Correction



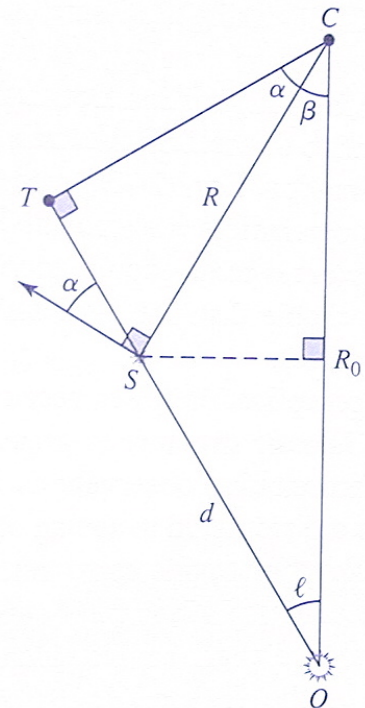
Methods: Stellar Kinematics

- Stars around sun - higher velocity, lower metallicity stars: thick disk, lower velocity higher metallicity stars orbiting with sun in thin disk
- Halo stars vs sun or LSR suggests orbital speed of $\Theta_0 = 220$ km/s

- Differential rotation $\Theta(R) = R\Omega(R)$ where $\Omega(R)$ is the angular velocity curve— observables are radial and tangential motion with respect to LSR

$$v_r = R\Omega \cos \alpha - R_0\Omega_0 \sin \ell$$

$$v_t = R\Omega \sin \alpha - R_0\Omega_0 \cos \ell$$



Methods: Stellar Kinematics

- d (parallax) and R_0 are known observables, R is not - eliminate with trig relations

$$R \cos \alpha = R_0 \sin \ell \quad R \sin \alpha = R_0 \cos \ell - d$$

- Eliminate R

$$v_r = (\Omega - \Omega_0) R_0 \sin \ell$$

$$v_t = (\Omega - \Omega_0) R_0 \cos \ell - \Omega d$$

solve for $\Omega(R)$ locally where

$$\begin{aligned} \Omega - \Omega_0 &\approx \frac{d\Omega}{dR} (R - R_0) \\ &\approx \frac{1}{R_0} \left(\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right) (R - R_0) \quad [\Omega = \Theta/R] \end{aligned}$$

and $d \ll R_0$, $\cos \beta \approx 1$

Methods: Stellar Kinematics

- Reduce with trig identities

$$R_0 = d \cos \ell + R \cos \beta \approx d \cos \ell + R$$

$$R - R_0 \approx -d \cos \ell$$

$$\cos \ell \sin \ell = \frac{1}{2} \sin 2\ell$$

$$\cos^2 \ell = \frac{1}{2} (\cos 2\ell + 1)$$

to obtain

$$v_r \approx Ad \sin 2\ell$$

$$v_t \approx Ad \cos 2\ell + Bd$$

Methods: Stellar Kinematics

- Oort constants

$$A = -\frac{1}{2} \left[\frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right] = -\frac{R_0}{2} \frac{d\Omega}{dR}$$
$$B = -\frac{1}{2} \left[\frac{d\Theta}{dR} + \frac{\Theta_0}{R_0} \right]$$

- Observables v_r , v_t , ℓ , d : solve for Oort's constants. From Hipparcos

$$A = 14.8 \pm 0.8 \text{ km/s/kpc}$$

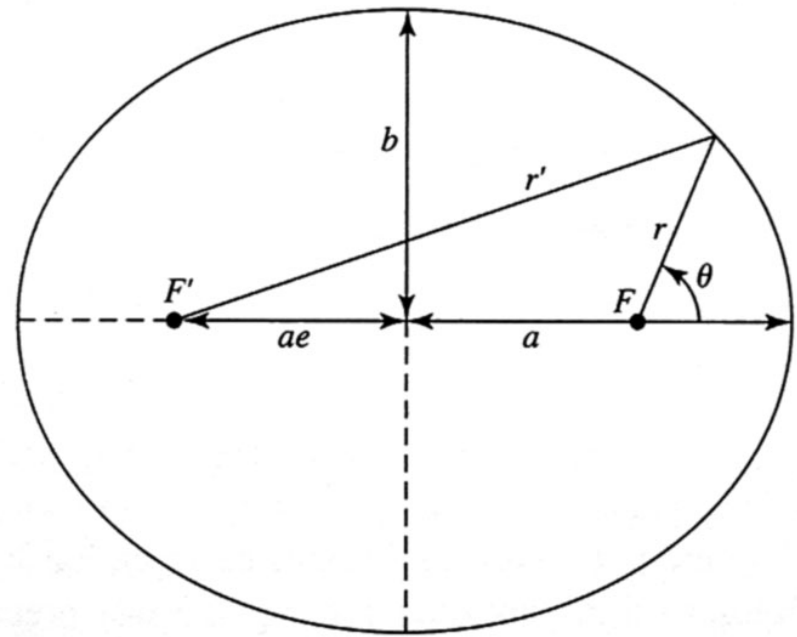
$$B = -12.4 \pm 0.6 \text{ km/s/kpc}$$

- Angular velocity $\Omega = v/r$ decreases with radius: differential rotation. Physical velocity $\Theta(R)$: $d\Theta/dR|_{R_0} = -(A + B) = 2.4$ km/s/kpc decreases slowly compared with 220km/s - flat rotation curve

Methods: Stellar Kinematics

- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2yrs, eccentricity $e = 0.87$, perigalacticon distance 120 AU= 1.8×10^{13} m
- Estimate mass: semimajor axis

$$a = \frac{r_p}{1 - e} = 1.4 \times 10^{14} \text{m}$$



Methods: Stellar Kinematics

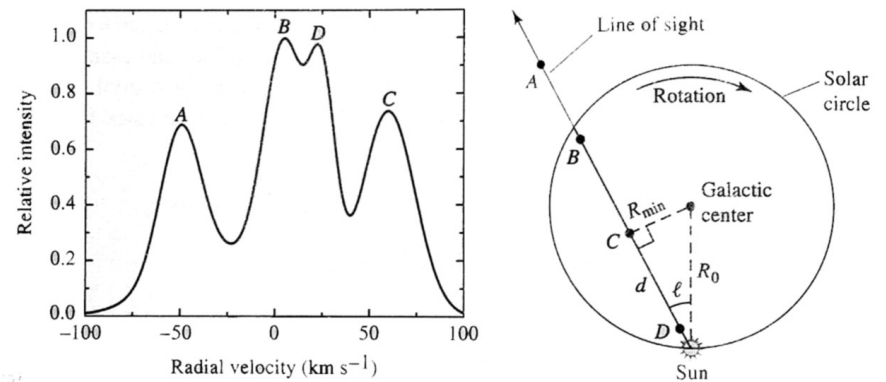
- Kepler's 3rd law

$$M = \frac{4\pi^2 a^3}{GP^2} = 7 \times 10^{36} \text{kg} = 3.5 \times 10^6 M_{\odot}$$

- That much mass in that small a radius can plausibly only be a (supermassive) black hole

Methods: 21 cm

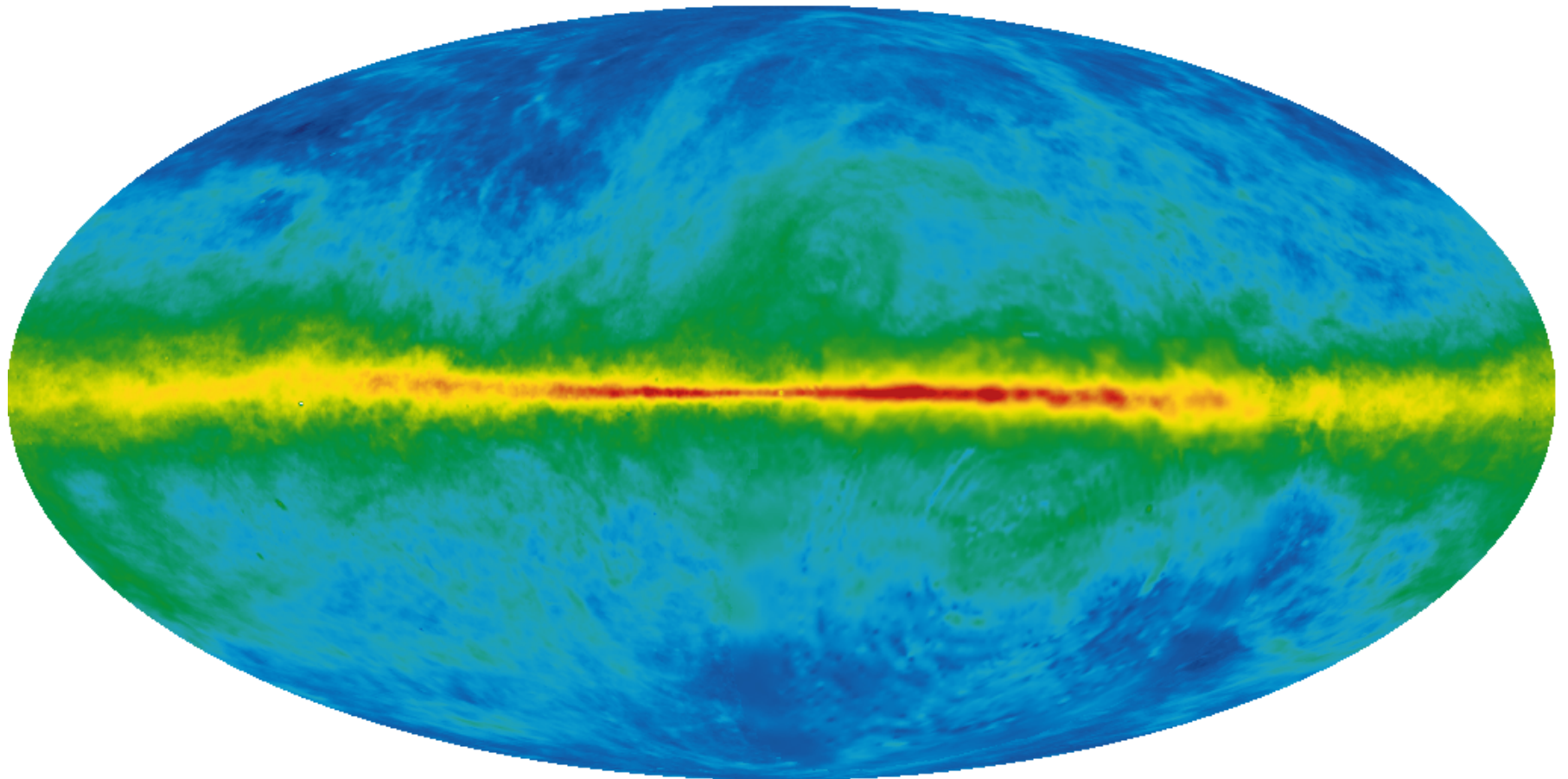
- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21cm
- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy
- No intrinsic distance measure
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line



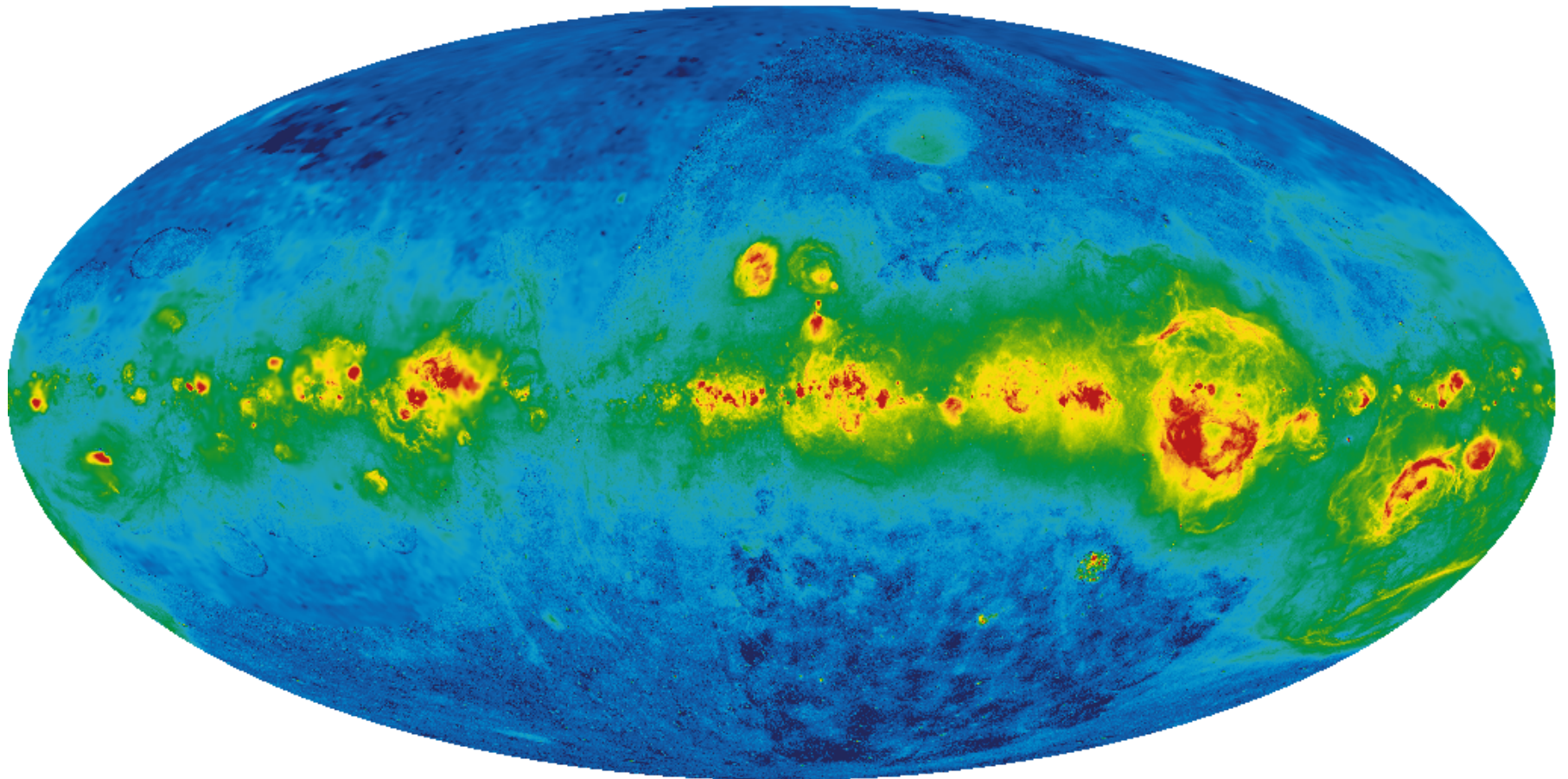
Methods: 21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point
- Build up a rotation curve interior to the solar circle $R < R_0$
- Rotation curve steeply rises in the interior $R < 1\text{kpc}$, consistent with near rigid body rotation and then remains flat out through the solar circle

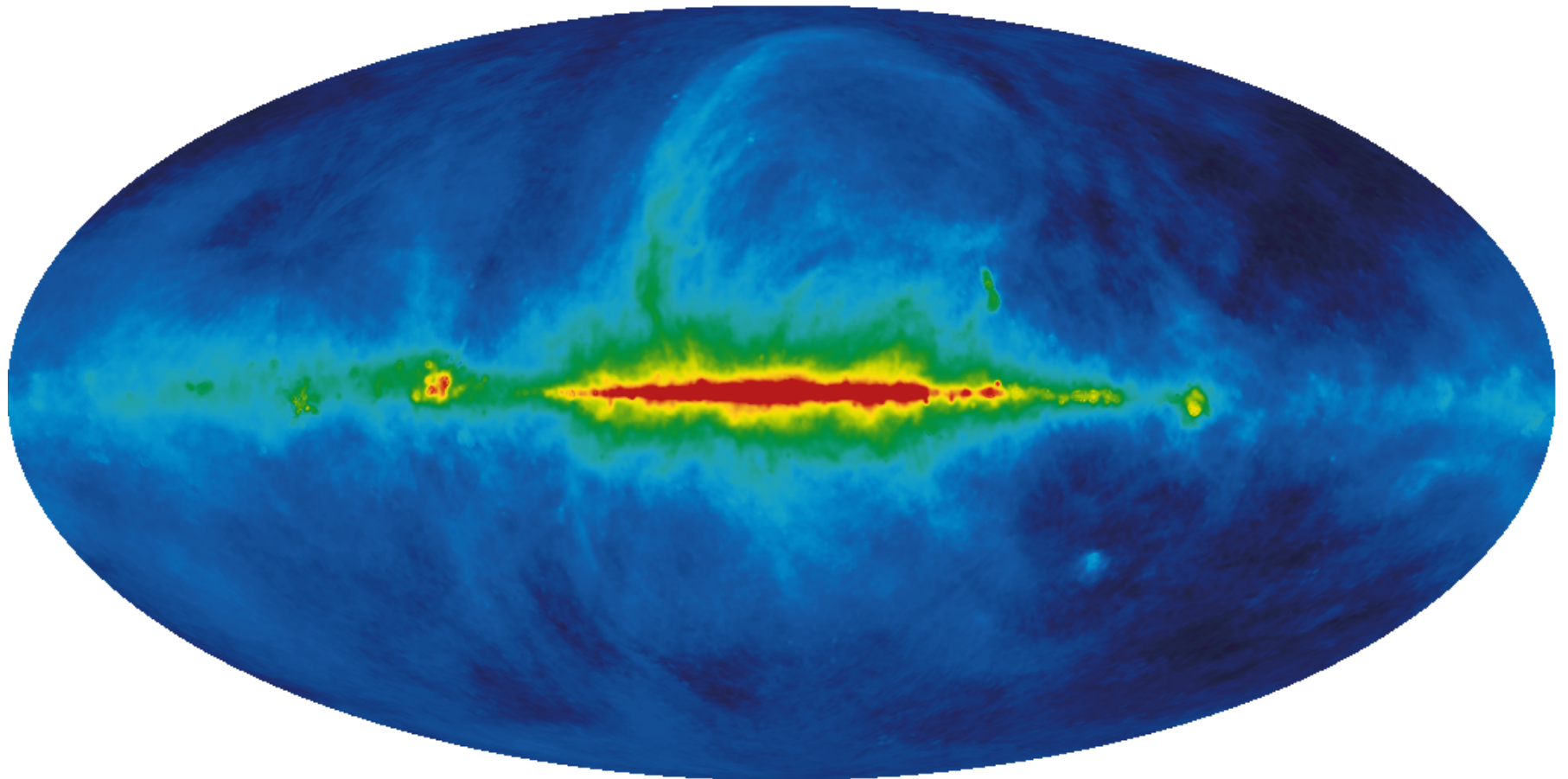
Neutral Gas: 21cm Emission



Ionized Gas: $H\alpha$ Line Emission

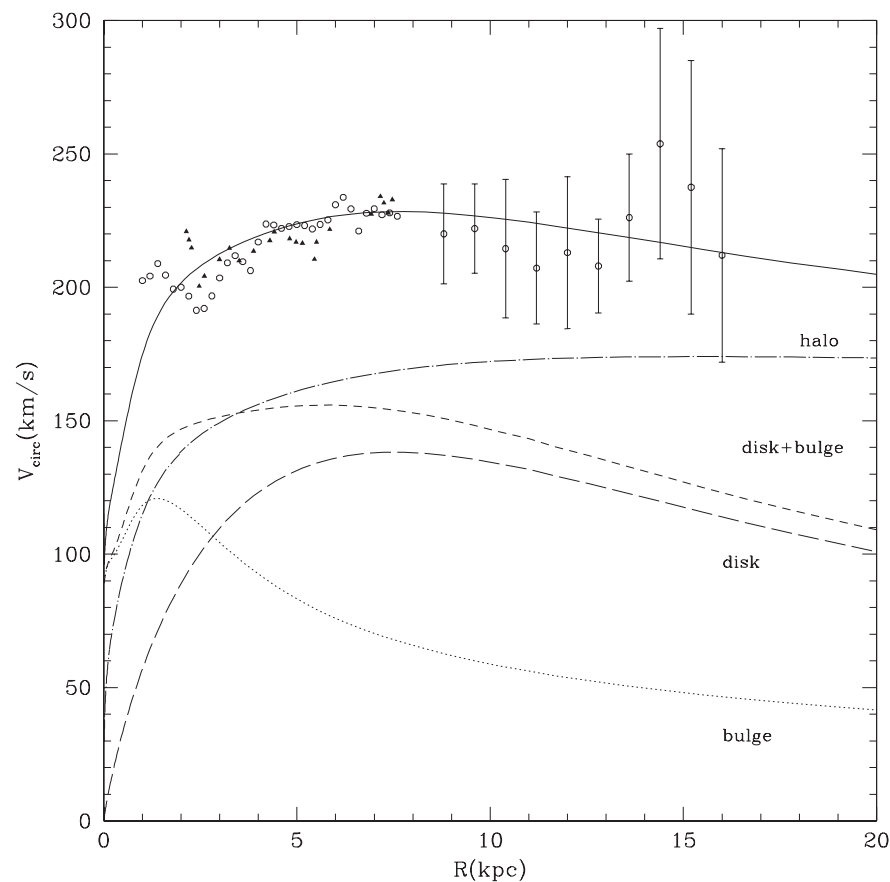


Cosmic Rays in B Field: Synchrotron



Methods: Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to $\sim 20\text{kpc}$
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass m orbiting at a radius r around an enclosed mass $M(r)$



Methods: Rotation Curves

- Setting the centripetal force to the gravitational force

$$\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}$$

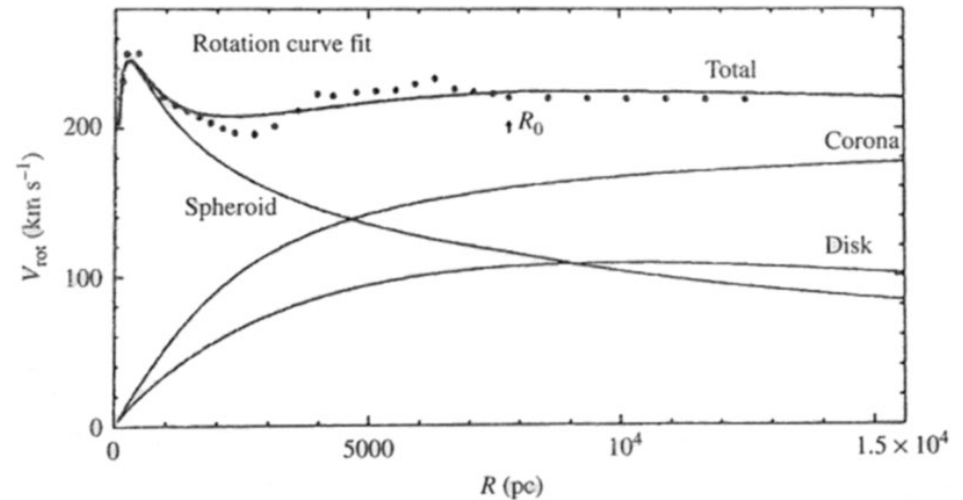
$$v(r) = \left(\frac{GM(r)}{r} \right)^{1/2}$$

Side note: this is the fundamental way masses are measured - balance internal motions of luminous matter with gravitational force - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve $v(r)$ is equivalent to measuring the mass profile $M(r)$ or density profile $\rho(r) \propto M(r)/r^3$

Methods: Rotation Curves

- Flat rotation curve $v(r) = \text{const}$ implies $M \propto r$ - a mass linearly increasing with radius
- Rigid rotation implies $\Omega = v/r = \text{const}$. $v \propto r$ or $M \propto r^3$ or $\rho = \text{const}$
- Rotation curves in other galaxies show the same behavior: evidence that “dark matter” is ubiquitous in galaxies



Methods: Rotation Curves

- Consistent with dark matter density given by

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

Methods: Gravitational Lensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object “MACHO”
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance $u = \theta/\theta_E$ from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where θ_E is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Methods: Gravitational Lensing

- A MACHO would move at a velocity typical of the disk and halo $v \sim 200\text{km/s}$ and so the star behind it would brighten as it crossed the line of sight to a background star. With u_{\min} as the distance of closest approach at $t = 0$

$$u^2(t) = u_{\min}^2 + \left(\frac{vt}{d_L \theta_E} \right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.

Methods: Gravitational Lensing

- In the 1990's large searches measured the rate of microlensing in the halo and bulge and determined that only a small fraction of its mass could be in MACHOs

