Set 2:

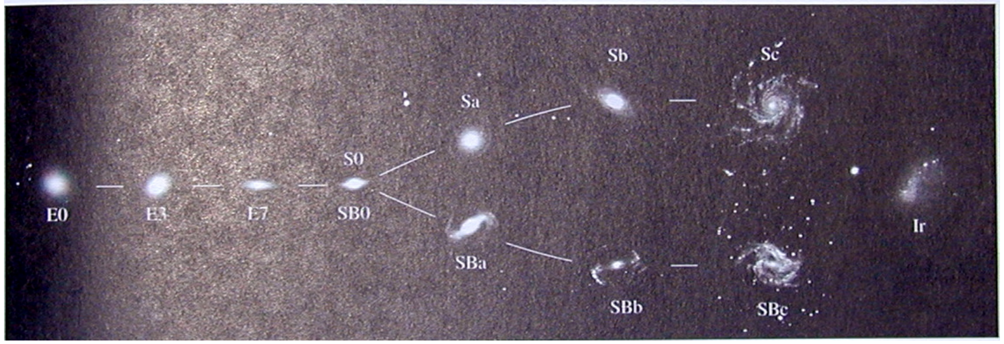
Nature of Galaxies

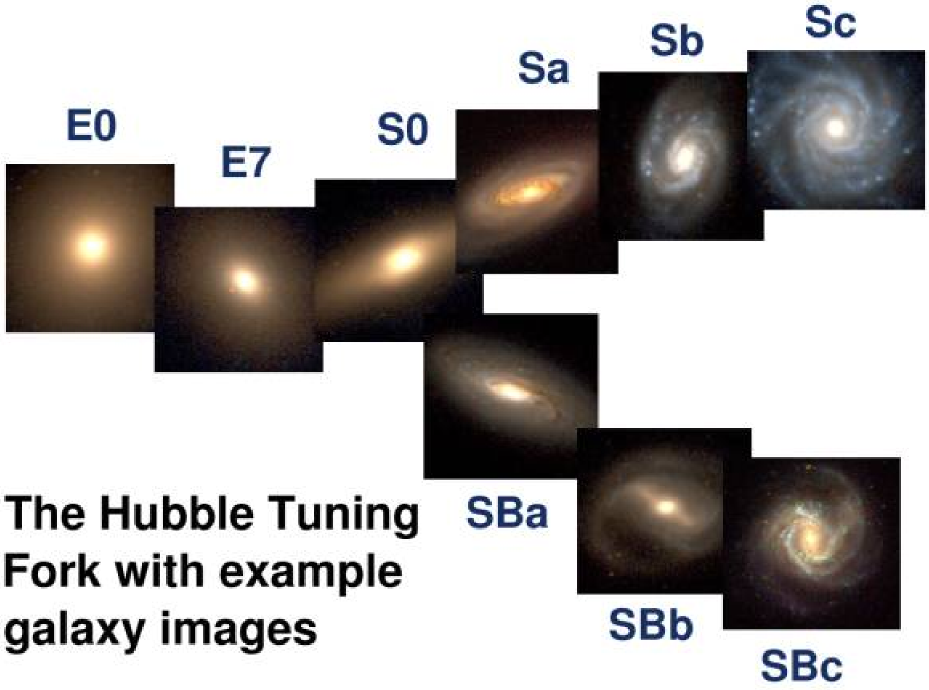
Great Shapley-Curtis Debate

- History: as late as the early 1920’s it was not known that the “spiral nebula” were galaxies like ours
- Debate between Shapley (galactic objects) and Curtis (extragalactic, or galaxies) in 1920 highlighted the difficulties - distances in astrophysics difficult to measure - Shapley’s inferences based on star counts without extinction and too large a galaxy, novae as standard candles, proper motion
- Hubble in 1923 used Cepheids to establish that Andromeda (M31) is extragalactic at 285kpc - modern measurements say it is 770kpc from the sun.
- Our galaxy is just one of many. Copernican principle in cosmology - we do not occupy a special place in the universe

Galaxy Zoology

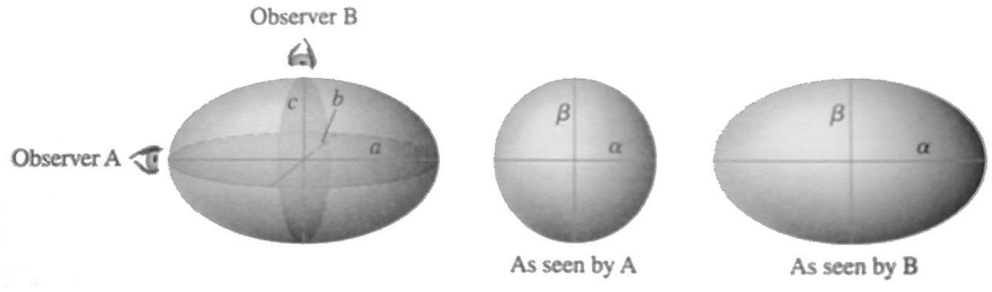
- Hubble’s tuning fork classification of galaxies
- A sequence going from ellipticals En, through regular $S0$ and barred $SB0$ lenticulars, to normal S and barred spirals SB ending in irregulars





The Hubble Tuning Fork with example galaxy images

Galaxy Zoology



- Ellipticals
  are further distinguished
  by the degree of projected
  ellipticity: the projected
  major α and minor β axes

$$\frac{n}{10} = \epsilon \equiv 1 - \beta/\alpha$$

- Classification does not necessarily correspond to physical distinctions!

Galaxy Zoology: Ellipticals

- The actual ellipticity is 3 dimensional and the three axes ordered as $a \geq b \geq c$ determine the degree of oblateness: a sphere has $a = b = c$, perfectly oblate has $a = b$, perfectly prolate $b = c$
- In projection, a strongly prolate or oblate elliptical can have vanishingly small ellipticities
- Ellipticals are often called "early type" and spirals "late type" despite the fact that mergers of spirals can result in ellipticals

Galaxy Zoology: Ellipticals

- Ellipticals vary widely in physical properties from giants to dwarfs
- Absolute B magnitude from -8 to -23
- Total mass from $10^7 M_\odot$ to $10^{13} M_\odot$
- Diameters from few tenths of kpc to hundreds of kpc
- Further classication

cD: high mass, high luminosity, high mass to light, in clusters

Normal elliptical: $B = -15$ to -23, $M = 10^8 - 10^{13} M_\odot$

Dwarf ellipticals: low surface brightness for a given $B = -13$ to -19, $M = 10^7 - 10^9 M_\odot$

Dwarf spheroidal: extremely low luminosity $B = -8$ to -15 and surface brightness can only be detected locally

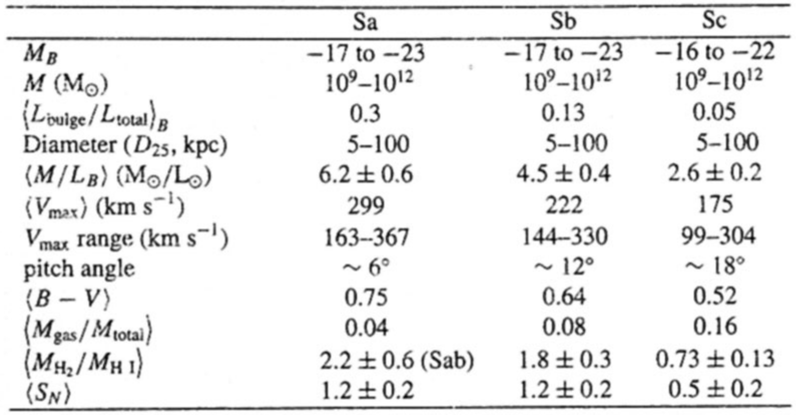
Blue compact dwarf: small with vigorous star formation $B = -14$ to -17 and $M \sim 10^9$.

Galaxy Zoology: Spiral NGC4414

Galaxy Zoology: Spirals

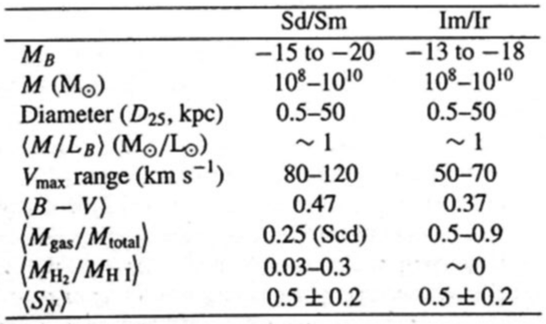
- Spirals are subdivided a, ab, b, bc, c in order of bulge prominance, tightly wound spiral arms, smoothest distribution of stars
- The presence of a central bar is indicated with B
- Milky Way is a $SBbc$, M31 is an Sb
- $S(B)a - c$ smaller range of physical properties compared with ellipticals (table)

TABLE 25.1 Characteristics of Early Spiral Galaxies.



	Sa	Sb	Sc
M_B	-17 to -23	-17 to -23	-16 to -22
M ($M_\odot$)	10^9–10^{12}	10^9–10^{12}	10^9–10^{12}
$\langle L_{\rm bulge}/L_{\rm total}\rangle_B$	0.3	0.13	0.05
Diameter (D_{25}, kpc)	5–100	5–100	5–100
$\langle M/L_B\rangle$ ($M_\odot/L_\odot$)	6.2 ± 0.6	4.5 ± 0.4	2.6 ± 0.2
$\langle V_{\rm max}\rangle$ (km s^{-1})	299	222	175
$V_{\rm max}$ range (km s^{-1})	163–367	144–330	99–304
pitch angle	$\sim 6°$	$\sim 12°$	$\sim 18°$
$\langle B - V\rangle$	0.75	0.64	0.52
$\langle M_{\rm gas}/M_{\rm total}\rangle$	0.04	0.08	0.16
$\langle M_{\rm H_2}/M_{\rm H\,I}\rangle$	2.2 ± 0.6 (Sab)	1.8 ± 0.3	0.73 ± 0.13
$\langle S_N\rangle$	1.2 ± 0.2	1.2 ± 0.2	0.5 ± 0.2

TABLE 25.2 Characteristics of Late Spiral and Irregular Galaxies.



	Sd/Sm	Im/Ir
M_B	-15 to -20	-13 to -18
M ($M_\odot$)	10^8–10^{10}	10^8–10^{10}
Diameter (D_{25}, kpc)	0.5–50	0.5–50
$\langle M/L_B\rangle$ ($M_\odot/L_\odot$)	~ 1	~ 1
$V_{\rm max}$ range (km s^{-1})	80–120	50–70
$\langle B - V\rangle$	0.47	0.37
$\langle M_{\rm gas}/M_{\rm total}\rangle$	0.25 (Scd)	0.5–0.9
$\langle M_{\rm H_2}/M_{\rm H\,I}\rangle$	0.03–0.3	~ 0
$\langle S_N\rangle$	0.5 ± 0.2	0.5 ± 0.2

Galaxy Zoology: Irregulars

- Irregulars classed as $IrrI$ if there is any organized structure such as spiral arms
- Otherwise $IrrII$ otherwise
- Examples: Large Magellanic Clouds (LMC) is $IrrI$ and Small Magellanic Clouds (SMC) is $IrrII$
- Physical properties: tend to be small and faint
- Absolute B magnitude from -13 to -20
- Masses from $10^8 M_\odot$ to $10^{10} M_\odot$

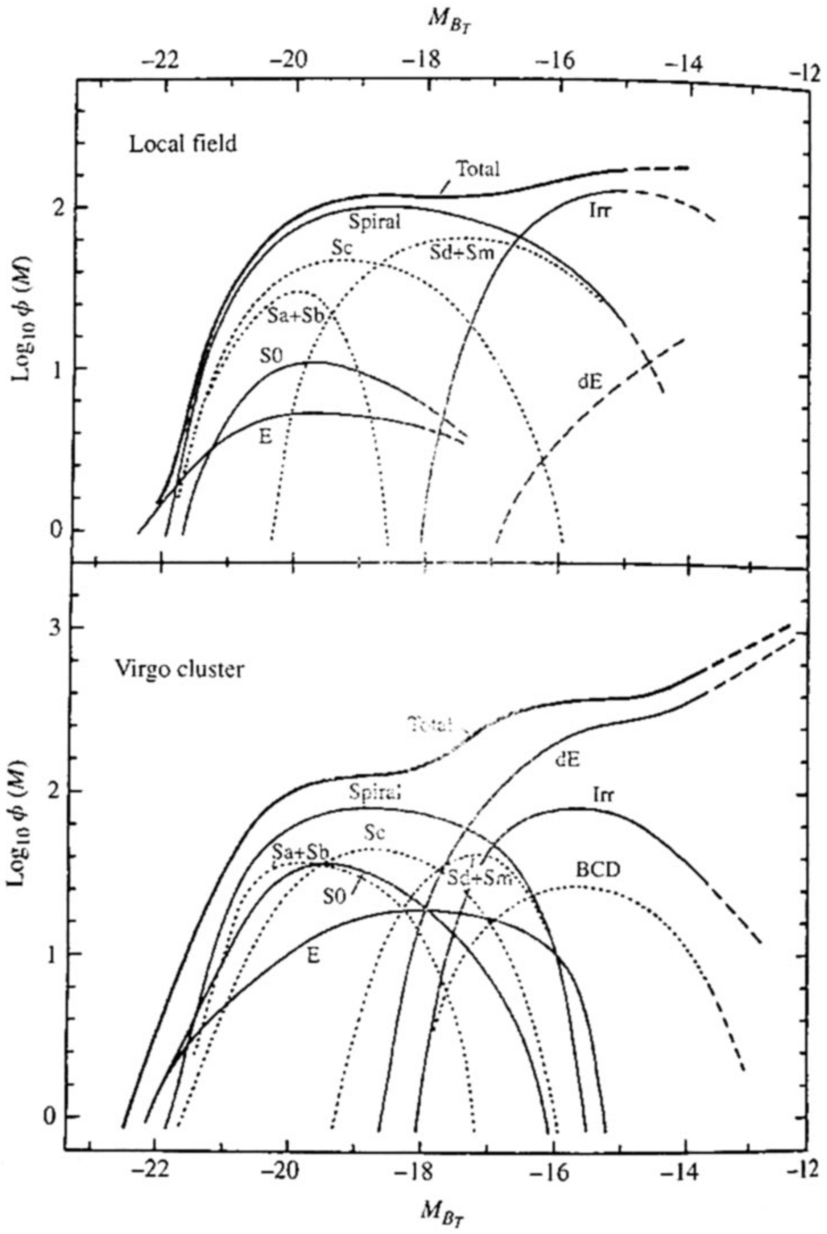
Galaxy Properties: Luminosity Function

- Abundance as a function of luminosity is called the “luminosity function”. Number of galaxies in dL around L and has a rough shape of a “Schechter function”

$$\phi_L dL \propto L^{\alpha} e^{-L/L_*} dL$$

$$\phi_M dM \propto 10^{-0.4(\alpha+1)M} e^{-10^{0.4(M_* - M)}}$$

with $\alpha \approx -1$
and L_* from $M_* = -21$ in B



Galaxy Properties: Luminosity Function

- Luminosity function is to galaxies what the distribution in magnitudes of stars is to star counts
- Galaxy counts probe the galaxy number density as a function of angular position (and redshift) to a limiting magnitude (a “redshift” survey)
- Luminosity function (determined locally) tells you how to interpret the observed counts in terms of a 3D distribution of galaxies

Galaxy Properties: Surface Brightness

- Surface brightness profile defines the effective scale of the bulge and disk components
- Surface brightness μ measured in B-mag arcsec^{-2}
- Define r_e as the radius within which $1/2$ the light emitted.
- Bulges of spirals and ellipticals follow a Sersic profile where the surface brightness in mag scales as a power law at $r \gg r_e$

$$\mu(r) = \mu_e + 8.3268 \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right]$$

where $n = 4$ is the de Vaucouleurs profile and μ_e is the surface brightness at r_e

Galaxy Properties: Surface Brightness

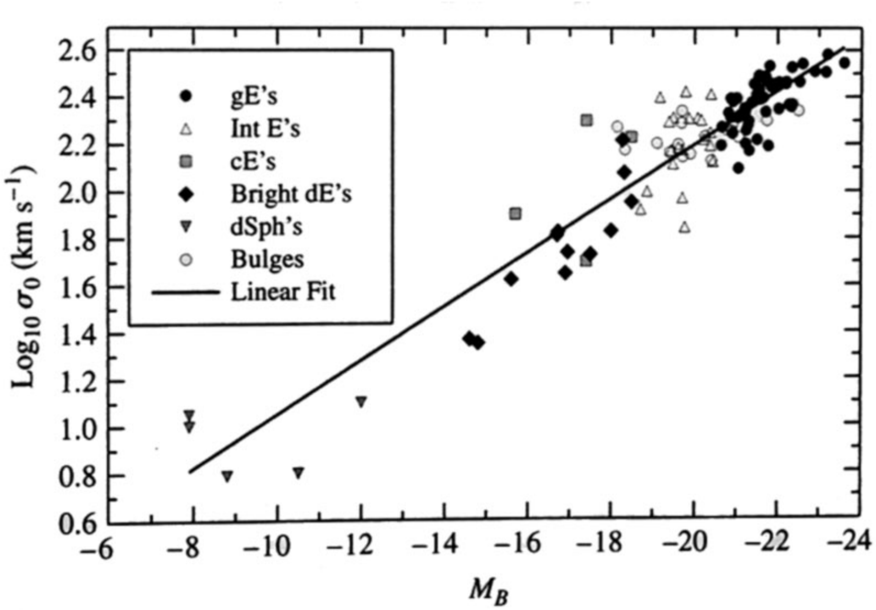
- Disks follow an exponential which in mag scales as

$$\mu(r) = \mu_0 + 1.09 \left(\frac{r}{h_r} \right)$$

where h_r is the characteristic scale length

Galaxy Properties: Fundamental Plane

- Faber Jackson correlation between luminosity and velocity dispersion of stars (measured from the width of lines from aggregate unresolved stars) $L \propto \sigma_0^4$



- Expected if mass to light and surface brightness a constant. Consider virial theorem

$$-2\langle K\rangle = \langle U\rangle, \qquad -2\sum_i^N \frac{1}{2} m_i v_i^2 = U$$

Galaxy Properties: Fundamental Plane

- Simplify as equal mass objects composing M

$$-\frac{m}{N}\sum_{i}^{N} v_i^2 = \frac{U}{N}$$

- Sum is the average v^2 and is an observable assuming that radial velocities reflect total $\langle v^2\rangle = 3\langle v_r^2\rangle \equiv 3\sigma_r^2$

$$-3m\sigma_r^2 = \frac{U}{N}$$

- Potential energy for a constant density spherical distribution of mass $M = Nm$ and radius R

$$\frac{U}{N} = -\frac{3}{5}\frac{GM^2}{NR}, \qquad M_{\rm vir} = \frac{5R\sigma_r^2}{G}$$

Galaxy Properties: Fundamental Plane

- Eliminate R by assuming constant surface brightness $L/R^2 = C_{SB}$ eliminate $R = (L/C_{SB})^{1/2}$

$$M_{\rm vir} = \frac{5\sigma_r^2}{G}\,(L/C_{SB})^{1/2}$$

- Eliminate $M_{\rm vir}$ by assuming constant mass to light $M/L = 1/C_{ML}$

$$L = C_{ML}\frac{5\sigma_r^2}{G}\,(L/C_{SB})^{1/2}$$

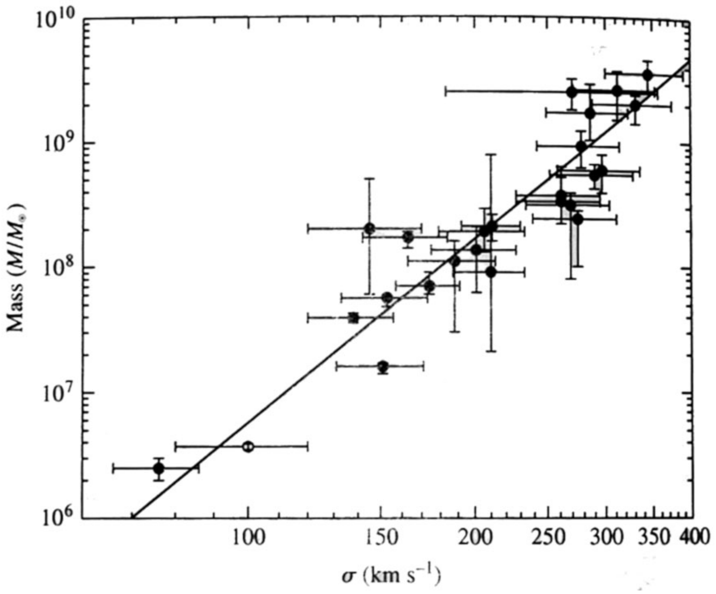
$$L \propto \sigma_r^4$$

- A tighter relation is obtained by introducing a second observable - either the effective radius

$$L \propto \sigma_r^{2.65} r_e^{0.65}$$

which defines the fundamental plane of ellipticals

Galaxy Properties: SMBH

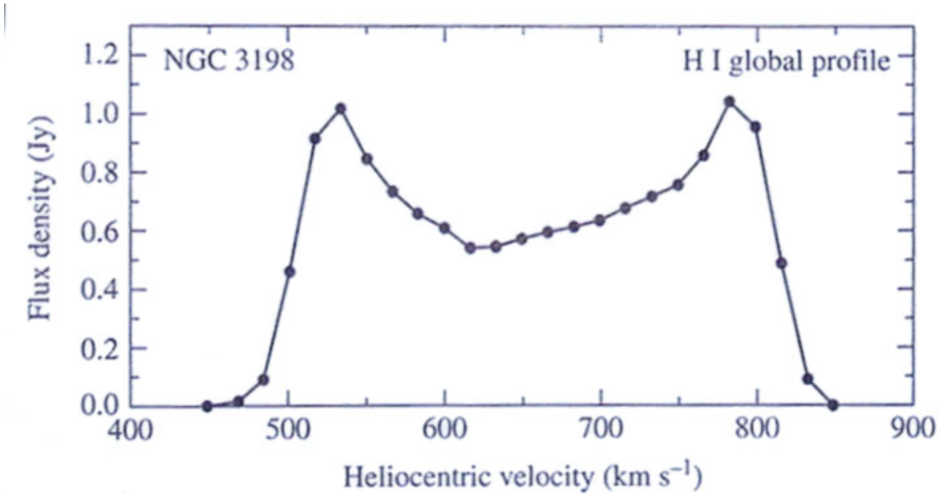


- A similar argument is used to measure the mass of the central black hole from the velocity dispersion of stars around it in both spirals and ellipticals
- The inferred mass is also correlated with the velocity dispersion much further out in the bulge

$$M_{bh} \propto \sigma^{\beta} \qquad (\beta = 4.86 \pm 0.43)$$

- Assembly of the bulge must be linked to the SMBH formation

Galaxy Properties: v_{max}



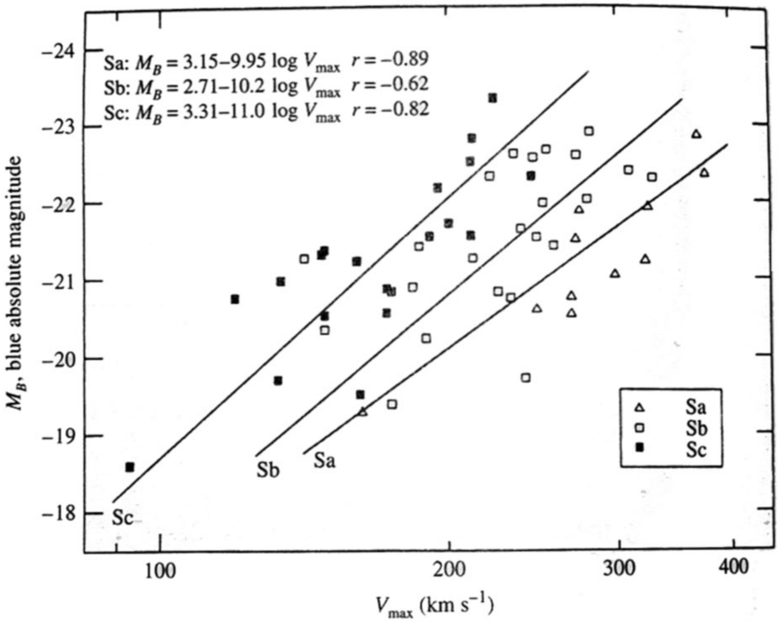
- The maximum velocity in a rotation curve is a robust observable
- The 21 cm line of the disk as a whole reflects the Doppler shifts of the HI participating in the rotation
- Line has a double peaked profile with the peaks near v_{max} since much of the gas is in the flat part of the rotation curve near the peak

Galaxy Properties: Tully Fisher relation

- Correcting
  for the inclination from
  the observed radial velocity v_r

$$\frac{\Delta\lambda}{\lambda_{\rm rest}} = \frac{v_r}{c} = \frac{v_{\rm max}}{c}\sin i$$

- Tully and Fisher established
  that $v_{\rm max}$ is correlated
  with B band luminosity
  as approximately $L_B \propto v_{\rm max}^4$



Galaxy Properties: Tully Fisher relation

- Tully-Fisher relationship is expected if galaxies have a constant mass to light ratio and constant surface brightness
- Enclosed mass

$$M = \frac{v_{\max}^2 R}{G}$$

- Mass to light ratio $M/L = 1/C_{ML}$

$$L = C_{ML} \frac{v_{\max}^2 R}{G}$$

- Surface brightness $L/R^2 = C_{SB}$ eliminate $R = (L/C_{SB})^{1/2}$

$$L = C_{ML} \frac{v_{\max}^2}{G} \left(\frac{L}{C_{SB}} \right)^{1/2}$$

$$L \propto v_{\max}^4$$

Galaxy Properties: Tully Fisher relation

- In absolute magnitude

$$M_B = -2.5\log_{10} L_B + \mathrm{const}$$

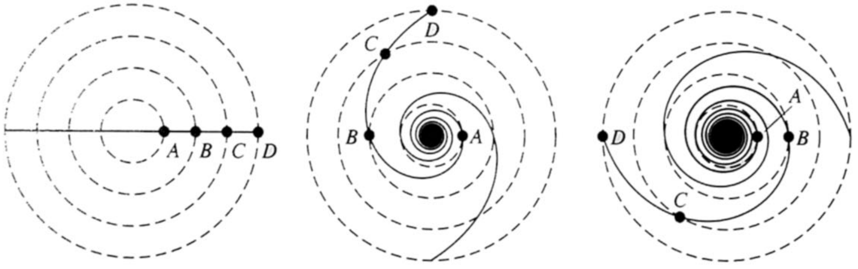
$$M_B = -2.5\log_{10} v_{\mathrm{max}}^4 + \mathrm{const}$$

$$M_B = -10\log_{10} v_{\mathrm{max}} + \mathrm{const}$$

- Tully Fisher relation is even tighter in IR bands such as H band - less extinction and late type giant stars are better tracers of overall luminoisity
- Tully Fisher relation can be used to measure distances: measure v_{max}, infer absolute magnitude and compare to apparent magnitude

Galaxy Properties: Spiral Structure

- Winding problem: if spiral structure were physical structures, a flat rotation curve would cause the arms to wind up tightly



(a) (b) (c)

- Lin-Shu density wave theory: spiral arms are quasistatic density waves - bunching is like cars in a traffic jam
- Stars pass through the wave/jam and do not cause a winding problem

Galaxy Properties: Spiral Structure

- Consider the orbital motion of a star in cylindrical coordinates (R, ϕ, z) where z is the coordinate out of the disk

  $$\frac{d^2\mathbf{r}}{dt^2} = -\nabla\Phi$$

  where Φ is the gravitational potential.

- Assuming axial symmetry for the potential this yields 3 equations for the three directions

  $$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial\Phi}{\partial R}$$

  $$\frac{1}{R}\frac{\partial(R^2\dot{\phi})}{\partial t} = 0$$

  $$\ddot{z} = -\frac{\partial\Phi}{\partial z}$$

Galaxy Properties: Spiral Structure

- Second equation says that there is no force in the azimuthal direction or torque $\tau = \mathbf{r} \times \mathbf{F}$

$$L_z = MRv_\phi = MR^2\dot{\phi} = \text{const}$$

where M is the mass of the star. Defining $J_z = L_z/M = R^2\dot{\phi}$ the angular momentum per unit mass

$$R\dot{\phi}^2 = \frac{J_z^2}{R^3}$$

- Radial equation becomes

$$\ddot{R} = -\frac{\partial\Phi}{\partial R} + \frac{J_z^2}{R^3}$$

Galaxy Properties: Spiral Structure

- The second term is an angular momentum barrier against radial infall or equivalently the centripetal acceleration required to keep R constant v_ϕ^2. It can be absorbed into an effective potential

  $$\Phi_{\rm eff} = \Phi + \frac{J_z^2}{2R^2}$$

  so that the equations of motion becomes (J_z is a constant in z

  $$\ddot{R} = -\frac{\partial \Phi_{\rm eff}}{\partial R}$$

  $$\ddot{z} = -\frac{\partial \Phi_{\rm eff}}{\partial z}$$

- Structure of $\Phi_{\rm eff}(R, z)$ determines motion. In z minimum is at the midplane. In R, minimum forms from the competition of gravity and angular momentum

Galaxy Properties: Spiral Structure

- Minimum found by seeing where slope vanishes (or equivalently where the gravitational and centripetal acceleration match)

$$\frac{\partial \Phi_{\text{eff}}}{\partial R} = \frac{\partial \Phi}{\partial R} - \frac{J_z^2}{R^3} = 0$$

- Orbits near this stable minimum m oscillate around it:

  $\rho \equiv R - R_m$

$$\Phi_{\text{eff}} \approx \Phi_{\text{eff},m} + \frac{1}{2}\kappa^2\rho^2 + \frac{1}{2}\nu^2 z^2$$

  where $\kappa^2 = \partial^2\Phi_{\text{eff}}/\partial R^2|_m$ and $\nu^2 = \partial^2\Phi_{\text{eff}}/\partial z^2|_m$

- Equations of motion

$$\ddot{\rho} = -\kappa^2\rho$$

$$\ddot{z} = -\nu^2 z$$

Galaxy Properties: Spiral Structure

- Star executes simple harmonic motion around minimum:

$$\rho(t) = A_R \sin \kappa t$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

where ζ is a phase factor and we have defined $t = 0$ to eliminate the other phase factor

- Azimuthal coordinate given in terms of radial motion

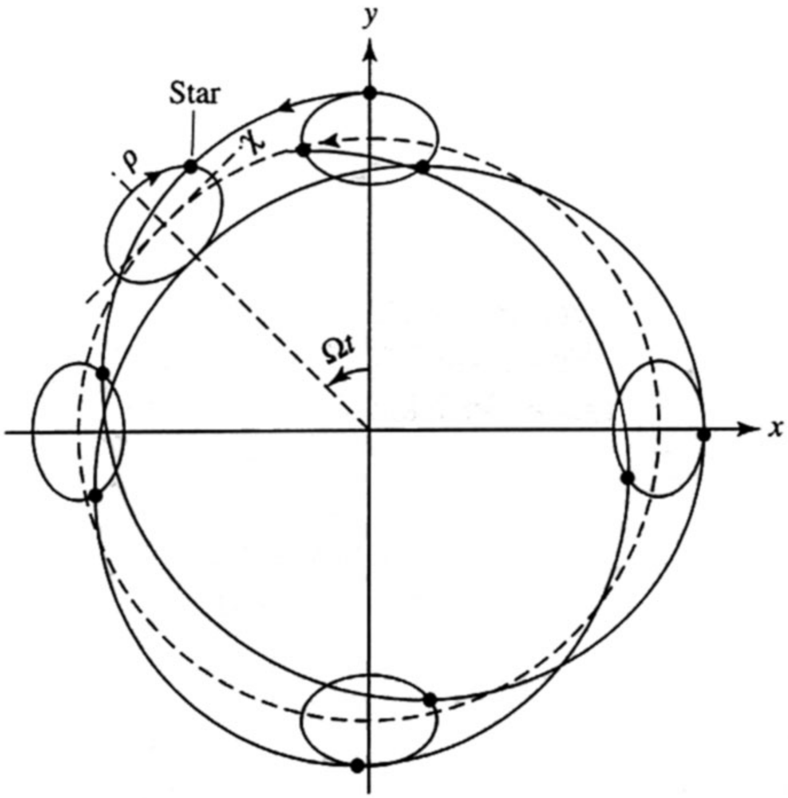
$$\dot{\phi} = \frac{J_z}{R^2} \approx \frac{J_z}{R_m^2}\left(1 - 2\frac{\rho(t)}{R_m}\right)$$

$$\phi(t) = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t$$

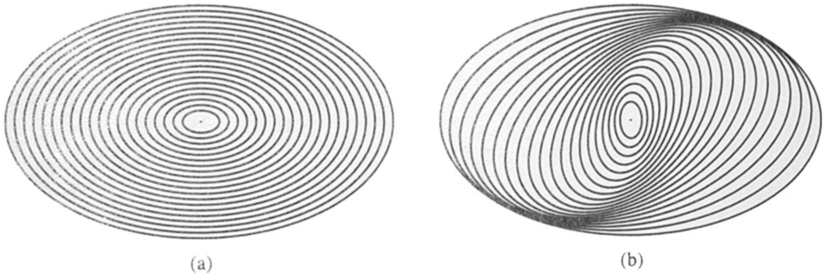
where the unperturbed angular frequency $\Omega = J_z/R_m^2$

Galaxy Properties: Spiral Structure

- Star executes epicyclic motion or rosette
- κ also known as epicyclic frequency
- Relative to the unperturbed orbit (corrotating with the local angular speed Ω, star executes a simple retrograde closed orbit around R_m



Galaxy Properties: Spiral Structure



- If the epicyclic frequency $\kappa/\Omega = m/n$ integer ratio then the orbit is closed in the fixed frame: star executes m epicycles during n orbits
- More generally, can always go into a rotating frame “local pattern speed” Ω_{lp} where this condition is true and orbits are closed

$$m(\Omega - \Omega_{lp}) = n\kappa$$

- An $(n = 1, m = 2)$ is shown for a case where Ω_{lp} is independent of R: if axis of orbit ovals are aligned then bar structure, if rotated then a two armed bar.

Galaxy Properties: Spiral Structure

- Only pattern is stationary - stars are continuously orbiting and piling up in the arms
- Non-constancy of the Ω_{lp} will still cause winding but of the pattern and typically at a slower rate for $(1, 2)$.
- Where the local pattern speed matches the global pattern speed Lindblad resonances occur where the epicyclic amplitude increases due to forcing from the local density enhancement - can destroy spiral pattern.
- N-body simulations show formation of transient $m = 2$ arm patterns and long lived bar instability.