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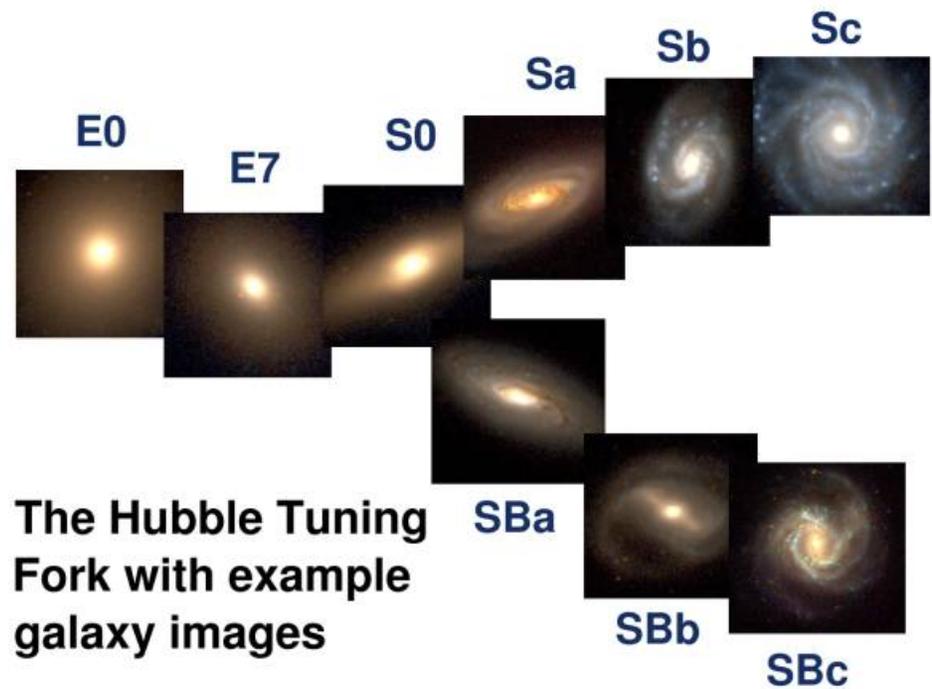
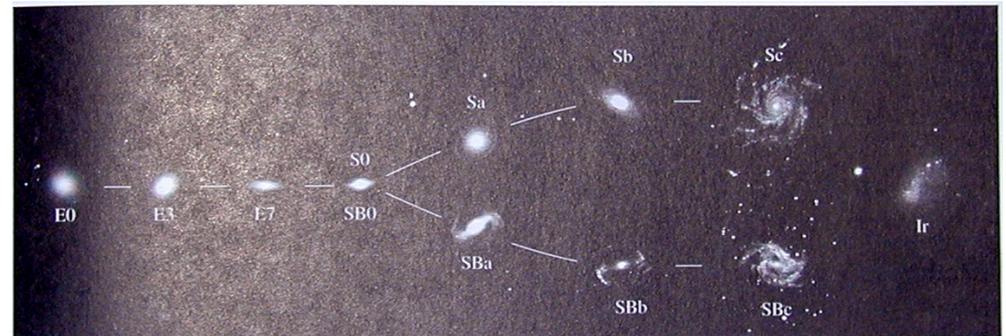
Nature of Galaxies

Great Shapley-Curtis Debate

- History: as late as the early 1920's it was not known that the “spiral nebula” were galaxies like ours
- Debate between Shapley (galactic objects) and Curtis (extragalactic, or galaxies) in 1920 highlighted the difficulties - distances in astrophysics difficult to measure - Shapley's inferences based on star counts without extinction and too large a galaxy, novae as standard candles, proper motion
- Hubble in 1923 used Cepheids to establish that Andromeda (M31) is extragalactic at 285kpc - modern measurements say it is 770kpc from the sun.
- Our galaxy is just one of many. Copernican principle in cosmology - we do not occupy a special place in the universe

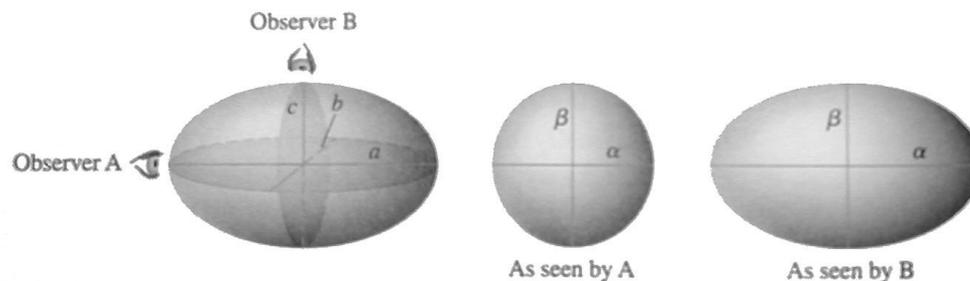
Galaxy Zoology

- Hubble's tuning fork classification of galaxies
- A sequence going from ellipticals E_n , through regular $S0$ and barred $SB0$ lenticulars, to normal S and barred spirals SB ending in irregulars



Galaxy Zoology

- Ellipticals are further distinguished by the degree of projected ellipticity: the projected major α and minor β axes



$$\frac{n}{10} = \epsilon \equiv 1 - \beta/\alpha$$

- Classification does not necessarily correspond to physical distinctions!

Galaxy Zoology: Ellipticals

- The actual ellipticity is 3 dimensional and the three axes ordered as $a \geq b \geq c$ determine the degree of oblateness: a sphere has $a = b = c$, perfectly oblate has $a = b$, perfectly prolate $b = c$
- In projection, a strongly prolate or oblate elliptical can have vanishingly small ellipticities
- Ellipticals are often called “early type” and spirals “late type” despite the fact that mergers of spirals can result in ellipticals

Galaxy Zoology: Ellipticals

- Ellipticals vary widely in physical properties from giants to dwarfs
- Absolute B magnitude from -8 to -23
- Total mass from $10^7 M_{\odot}$ to $10^{13} M_{\odot}$
- Diameters from few tenths of kpc to hundreds of kpc
- Further classification

cD: high mass, high luminosity, high mass to light, in clusters

Normal elliptical: $B = -15$ to -23 , $M = 10^8 - 10^{13} M_{\odot}$

Dwarf ellipticals: low surface brightness for a given $B = -13$ to -19 , $M = 10^7 - 10^9 M_{\odot}$

Dwarf spheroidal: extremely low luminosity $B = -8$ to -15 and surface brightness can only be detected locally

Blue compact dwarf: small with vigorous star formation $B = -14$ to -17 and $M \sim 10^9$.

Galaxy Zoology: Spiral NGC4414



Galaxy Zoology: Spirals

- Spirals are subdivided a , ab , b , bc , c in order of bulge prominence, tightly wound spiral arms, smoothest distribution of stars
- The presence of a central bar is indicated with B
- Milky Way is a $SBbc$, M31 is an Sb
- $S(B)a - c$ smaller range of physical properties compared with ellipticals (table)

TABLE 25.1 Characteristics of Early Spiral Galaxies.

	Sa	Sb	Sc
M_B	-17 to -23	-17 to -23	-16 to -22
M (M_\odot)	10^9 - 10^{12}	10^9 - 10^{12}	10^9 - 10^{12}
$\langle L_{\text{bulge}}/L_{\text{total}} \rangle_B$	0.3	0.13	0.05
Diameter (D_{25} , kpc)	5-100	5-100	5-100
$\langle M/L_B \rangle$ (M_\odot/L_\odot)	6.2 ± 0.6	4.5 ± 0.4	2.6 ± 0.2
$\langle V_{\text{max}} \rangle$ (km s^{-1})	299	222	175
V_{max} range (km s^{-1})	163-367	144-330	99-304
pitch angle	$\sim 6^\circ$	$\sim 12^\circ$	$\sim 18^\circ$
$\langle B - V \rangle$	0.75	0.64	0.52
$\langle M_{\text{gas}}/M_{\text{total}} \rangle$	0.04	0.08	0.16
$\langle M_{\text{H}_2}/M_{\text{H I}} \rangle$	2.2 ± 0.6 (Sab)	1.8 ± 0.3	0.73 ± 0.13
$\langle S_N \rangle$	1.2 ± 0.2	1.2 ± 0.2	0.5 ± 0.2

TABLE 25.2 Characteristics of Late Spiral and Irregular Galaxies.

	Sd/Sm	Im/Ir
M_B	-15 to -20	-13 to -18
M (M_\odot)	10^8 - 10^{10}	10^8 - 10^{10}
Diameter (D_{25} , kpc)	0.5-50	0.5-50
$\langle M/L_B \rangle$ (M_\odot/L_\odot)	~ 1	~ 1
V_{max} range (km s^{-1})	80-120	50-70
$\langle B - V \rangle$	0.47	0.37
$\langle M_{\text{gas}}/M_{\text{total}} \rangle$	0.25 (Scd)	0.5-0.9
$\langle M_{\text{H}_2}/M_{\text{H I}} \rangle$	0.03-0.3	~ 0
$\langle S_N \rangle$	0.5 ± 0.2	0.5 ± 0.2

Galaxy Zoology: Irregulars

- Irregulars classed as *IrrI* if there is any organized structure such as spiral arms
- Otherwise *IrrII* otherwise
- Examples: Large Magellanic Clouds (LMC) is *IrrI* and Small Magellanic Clouds (SMC) is *IrrII*
- Physical properties: tend to be small and faint
- Absolute B magnitude from -13 to -20
- Masses from $10^8 M_{\odot}$ to $10^{10} M_{\odot}$

Galaxy Properties: Luminosity Function

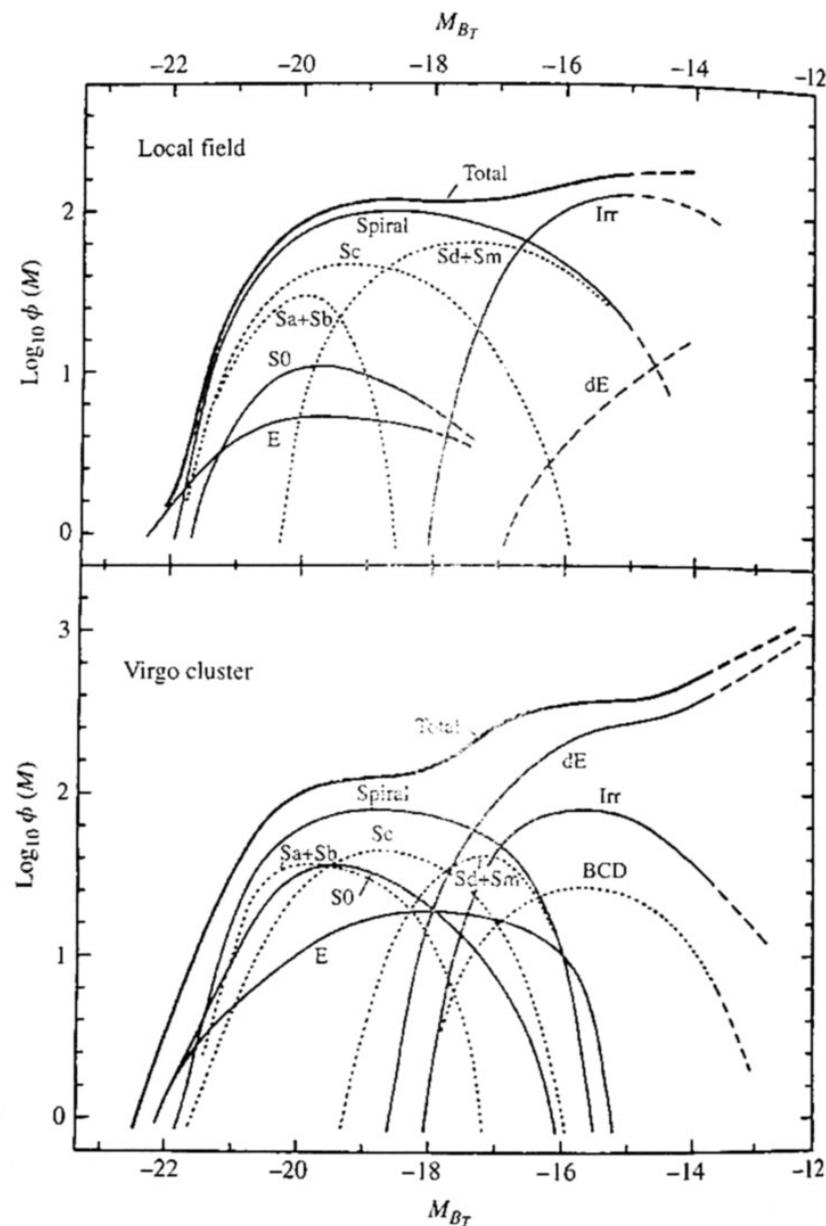
- Abundance as a function of luminosity is called the “luminosity function”. Number of galaxies in dL around L and has a rough shape of a “Schechter function”

$$\phi_L dL \propto L^\alpha e^{-L/L_*} dL$$

$$\phi_M dM \propto 10^{-0.4(\alpha+1)M}$$

$$\times e^{-10^{0.4(M_* - M)}} dM$$

with $\alpha \approx -1$, $M_* = -21$ in B



Galaxy Properties: Luminosity Function

- Luminosity function is to galaxies what the distribution in magnitudes of stars is to star counts
- Galaxy counts probe the galaxy number density as a function of angular position (and redshift) to a limiting magnitude (a “redshift” survey)
- Luminosity function (determined locally) tells you how to interpret the observed counts in terms of a 3D distribution of galaxies

Galaxy Properties: Surface Brightness

- Surface brightness profile defines the effective scale of the bulge and disk components
- Surface brightness μ measured in B -mag arcsec $^{-2}$
- Define r_e as the radius within which 1/2 the light emitted.
- Bulges of spirals and ellipticals follow a Sersic profile where the surface brightness in mag scales as a power law at $r \gg r_e$

$$\mu(r) = \mu_e + 8.3268 \left[\left(\frac{r}{r_e} \right)^{1/n} - 1 \right]$$

where $n = 4$ is the de Vaucouleurs profile and μ_e is the surface brightness at r_e

Galaxy Properties: Surface Brightness

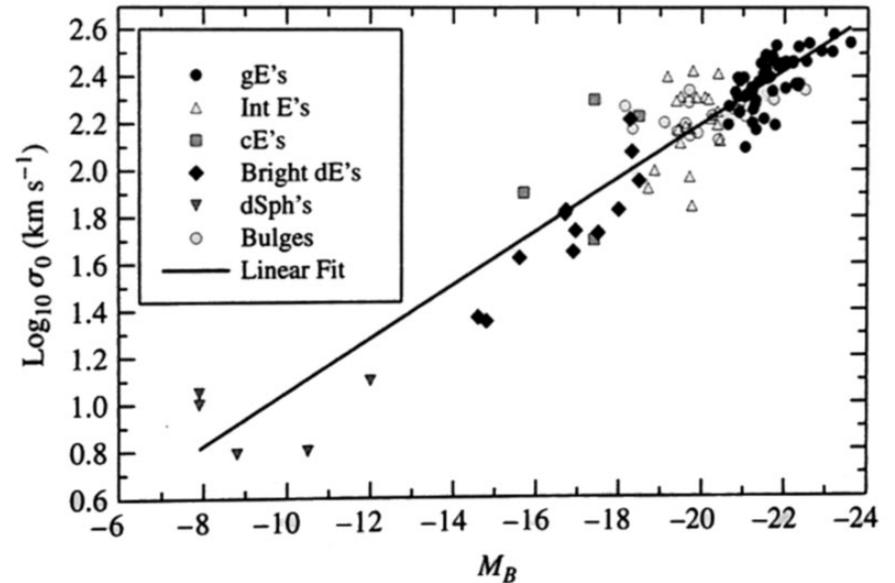
- Disks follow an exponential which in mag scales as

$$\mu(r) = \mu_0 + 1.09 \left(\frac{r}{h_r} \right)$$

where h_r is the characteristic scale length

Galaxy Properties: Fundamental Plane

- Faber Jackson
correlation between luminosity and velocity dispersion of stars (measured from the width of lines from aggregate unresolved stars) $L \propto \sigma_0^4$
- Expected if mass to light and surface brightness a constant.
Consider virial theorem



$$-2\langle K \rangle = \langle U \rangle, \quad -2 \sum_i^N \frac{1}{2} m_i v_i^2 = U$$

Galaxy Properties: Fundamental Plane

- Simplify as equal mass objects composing M

$$-\frac{m}{N} \sum_i^N v_i^2 = \frac{U}{N}$$

- Sum is the average v^2 and is an observable assuming that radial velocities reflect total $\langle v^2 \rangle = 3\langle v_r^2 \rangle \equiv 3\sigma_r^2$

$$-3m\sigma_r^2 = \frac{U}{N}$$

- Potential energy for a constant density spherical distribution of mass $M = Nm$ and radius R

$$\frac{U}{N} = -\frac{3}{5} \frac{GM^2}{NR}, \quad M_{\text{vir}} = \frac{5R\sigma_r^2}{G}$$

Galaxy Properties: Fundamental Plane

- Eliminate R by assuming constant surface brightness

$$L/R^2 = C_{SB} \text{ eliminate } R = (L/C_{SB})^{1/2}$$

$$M_{\text{vir}} = \frac{5\sigma_r^2}{G} (L/C_{SB})^{1/2}$$

- Eliminate M_{vir} by assuming constant mass to light $M/L = 1/C_{ML}$

$$L = C_{ML} \frac{5\sigma_r^2}{G} (L/C_{SB})^{1/2}$$

$$L \propto \sigma_r^4$$

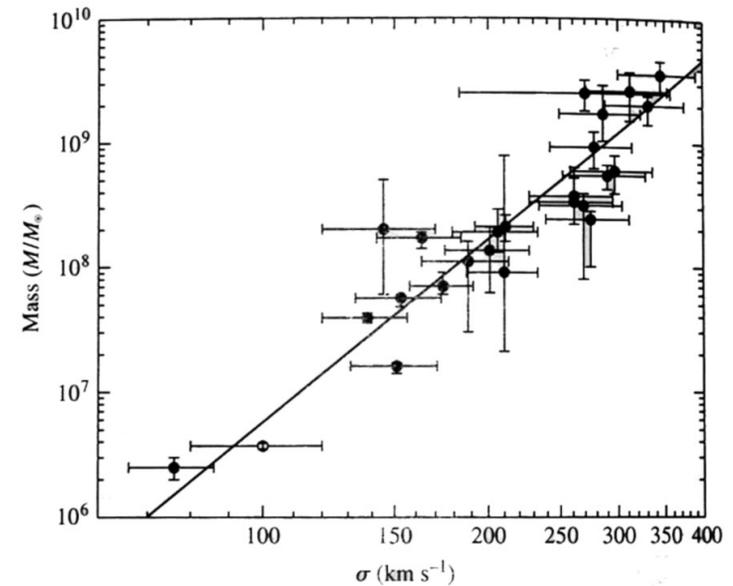
- A tighter relation is obtained by introducing a second observable - either the effective radius

$$L \propto \sigma_r^{2.65} r_e^{0.65}$$

which defines the fundamental plane of ellipticals

Galaxy Properties: SMBH

- A similar argument is used to measure the mass of the central black hole from the velocity dispersion of stars around it in both spirals and ellipticals
- The inferred mass is also correlated with the velocity dispersion much further out in the bulge

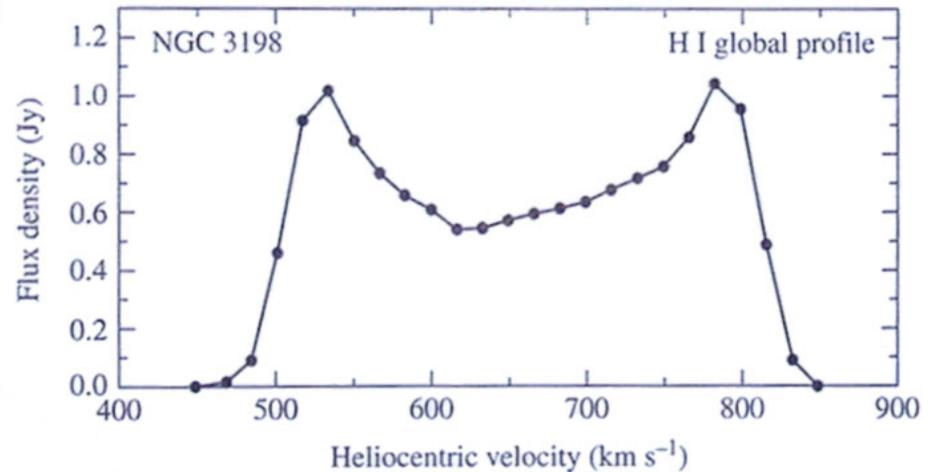


$$M_{bh} \propto \sigma^{\beta} \quad (\beta = 4.86 \pm 0.43)$$

- Assembly of the bulge must be linked to the SMBH formation

Galaxy Properties: v_{\max}

- The maximum velocity in a rotation curve is a robust observable
- The 21 cm line of the disk as a whole reflects the Doppler shifts of the HI participating in the rotation
- Line has a double peaked profile with the peaks near v_{\max} since much of the gas is in the flat part of the rotation curve near the peak

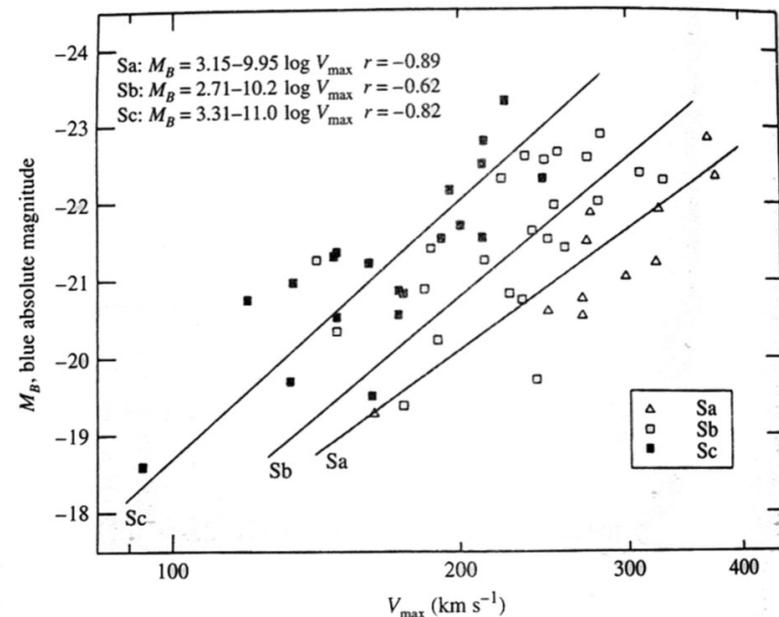


Galaxy Properties: Tully Fisher relation

- Correcting for the inclination from the observed radial velocity v_r

$$\frac{\Delta\lambda}{\lambda_{\text{rest}}} = \frac{v_r}{c} = \frac{v_{\text{max}}}{c} \sin i$$

- Tully and Fisher established that v_{max} is correlated with B band luminosity as approximately $L_B \propto v_{\text{max}}^4$



Galaxy Properties: Tully Fisher relation

- Tully-Fisher relationship is expected if galaxies have a constant mass to light ratio and constant surface brightness
- Enclosed mass

$$M = \frac{v_{\max}^2 R}{G}$$

- Mass to light ratio $M/L = 1/C_{ML}$

$$L = C_{ML} \frac{v_{\max}^2 R}{G}$$

- Surface brightness $L/R^2 = C_{SB}$ eliminate $R = (L/C_{SB})^{1/2}$

$$L = C_{ML} \frac{v_{\max}^2}{G} \left(\frac{L}{C_{SB}} \right)^{1/2}$$

$$L \propto v_{\max}^4$$

Galaxy Properties: Tully Fisher relation

- In absolute magnitude

$$M_B = -2.5 \log_{10} L_B + \text{const}$$

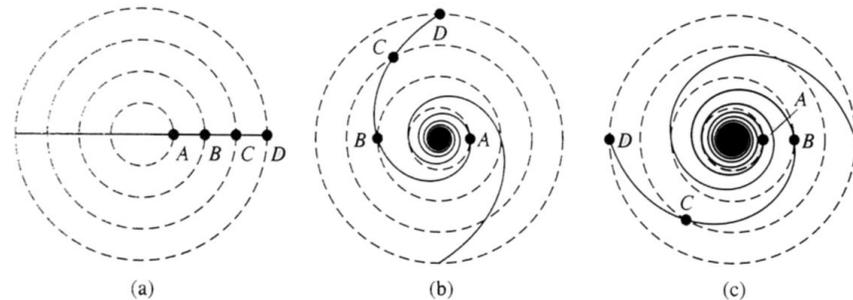
$$M_B = -2.5 \log_{10} v_{\text{max}}^4 + \text{const}$$

$$M_B = -10 \log_{10} v_{\text{max}} + \text{const}$$

- Tully Fisher relation is even tighter in IR bands such as *H* band - less extinction and late type giant stars are better tracers of overall luminosity
- Tully Fisher relation can be used to measure distances: measure v_{max} , infer absolute magnitude and compare to apparent magnitude

Galaxy Properties: Spiral Structure

- Winding problem:
if spiral structure were physical structures, a flat rotation curve would cause the arms to wind up tightly
- Lin-Shu density wave
theory: spiral arms are quasistatic density waves - bunching is like cars in a traffic jam
- Stars pass through the wave/jam and do not cause a winding problem



Galaxy Properties: Spiral Structure

- Consider the orbital motion of a star in cylindrical coordinates (R, ϕ, z) where z is the coordinate out of the disk

$$\frac{d^2 \mathbf{r}}{dt^2} = -\nabla \Phi$$

where Φ is the gravitational potential.

- Assuming axial symmetry for the potential this yields 3 equations for the three directions

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}$$

$$\frac{1}{R} \frac{\partial (R^2 \dot{\phi})}{\partial t} = 0$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}$$

Galaxy Properties: Spiral Structure

- Second equation says that there is no force in the azimuthal direction or torque $\tau = \mathbf{r} \times \mathbf{F}$

$$L_z = MRv_\phi = MR^2\dot{\phi} = \text{const}$$

where M is the mass of the star. Defining $J_z = L_z/M = R^2\dot{\phi}$ the angular momentum per unit mass

$$R\dot{\phi}^2 = \frac{J_z^2}{R^3}$$

- Radial equation becomes

$$\ddot{R} = -\frac{\partial\Phi}{\partial R} + \frac{J_z^2}{R^3}$$

Galaxy Properties: Spiral Structure

- The second term is an angular momentum barrier against radial infall or equivalently the centripetal acceleration required to keep R constant v_ϕ^2 . It can be absorbed into an effective potential

$$\Phi_{\text{eff}} = \Phi + \frac{J_z^2}{2R^2}$$

so that the equations of motion becomes (J_z is a constant in z)

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$$

- Structure of $\Phi_{\text{eff}}(R, z)$ determines motion. In z minimum is at the midplane. In R , minimum forms from the competition of gravity and angular momentum

Galaxy Properties: Spiral Structure

- Minimum found by seeing where slope vanishes (or equivalently where the gravitational and centripetal acceleration match)

$$\frac{\partial \Phi_{\text{eff}}}{\partial R} = \frac{\partial \Phi}{\partial R} - \frac{J_z^2}{R^3} = 0$$

- Orbits near this stable minimum m oscillate around it:

$$\rho \equiv R - R_m$$

$$\Phi_{\text{eff}} \approx \Phi_{\text{eff},m} + \frac{1}{2} \kappa^2 \rho^2 + \frac{1}{2} \nu^2 z^2$$

where $\kappa^2 = \partial^2 \Phi_{\text{eff}} / \partial R^2|_m$ and $\nu^2 = \partial^2 \Phi_{\text{eff}} / \partial z^2|_m$

- Equations of motion

$$\ddot{\rho} = -\kappa^2 \rho$$

$$\ddot{z} = -\nu^2 z$$

Galaxy Properties: Spiral Structure

- Star executes simple harmonic motion around minimum:

$$\rho(t) = A_R \sin \kappa t$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

where ζ is a phase factor and we have defined $t = 0$ to eliminate the other phase factor

- Azimuthal coordinate given in terms of radial motion

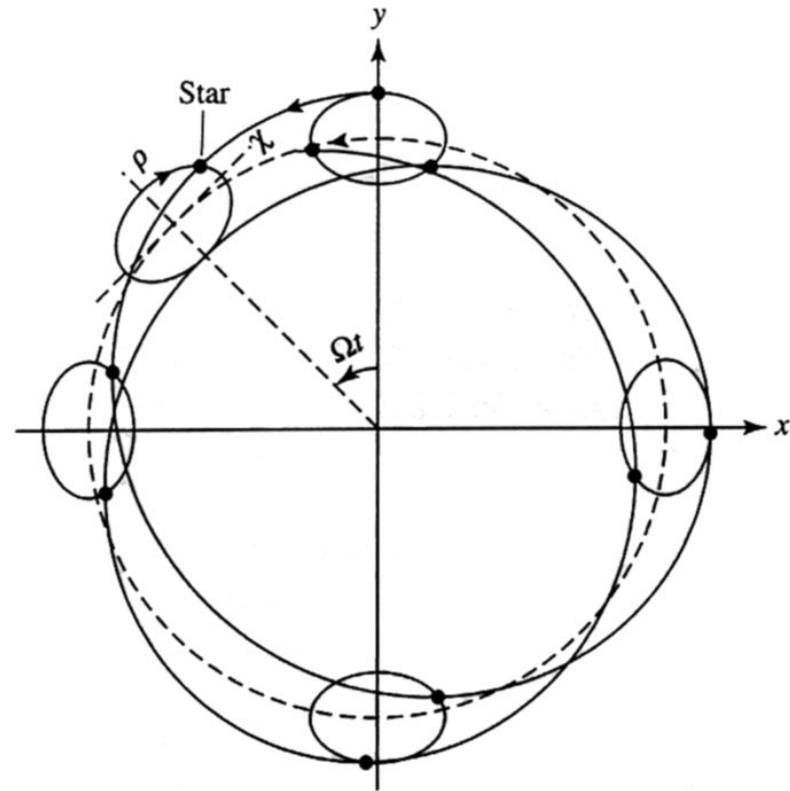
$$\dot{\phi} = \frac{J_z}{R^2} \approx \frac{J_z}{R_m^2} \left(1 - 2 \frac{\rho(t)}{R_m} \right)$$

$$\phi(t) = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t$$

where the unperturbed angular frequency $\Omega = J_z / R_m^2$

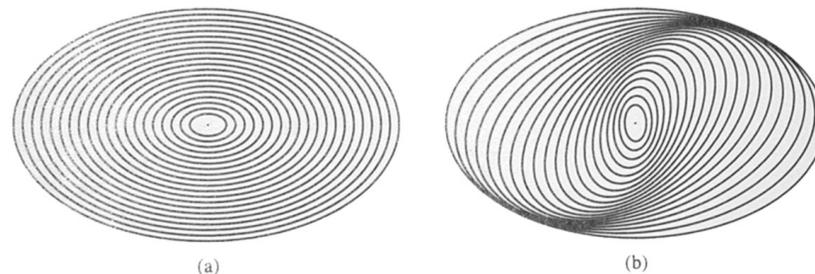
Galaxy Properties: Spiral Structure

- Star executes epicyclic motion or rosette
- κ also known as epicyclic frequency
- Relative to the unperturbed orbit (corrotating with the local angular speed Ω , star executes a simple retrograde closed orbit around R_m



Galaxy Properties: Spiral Structure

- If the epicyclic frequency $\kappa/\Omega = m/n$ integer ratio then the orbit is closed in the fixed frame: star executes m epicycles during n orbits
- More generally, can always go into a rotating frame “local pattern speed” Ω_{lp} where this condition is true and orbits are closed



$$m(\Omega - \Omega_{lp}) = n\kappa$$

- An $(n = 1, m = 2)$ is shown for a case where Ω_{lp} is independent of R : if axis of orbit ovals are aligned then bar structure, if rotated then a two armed bar.

Galaxy Properties: Spiral Structure

- Only pattern is stationary - stars are continuously orbiting and piling up in the arms
- Non-constancy of the Ω_{lp} will still cause winding but of the pattern and typically at a slower rate for $(1, 2)$.
- Where the local pattern speed matches the global pattern speed Lindblad resonances occur where the epicyclic amplitude increases due to forcing from the local density enhancement - can destroy spiral pattern.
- N -body simulations show formation of transient $m = 2$ arm patterns and long lived bar instability.