

# *Astro 242*

The Physics of Galaxies and the Universe: Lecture Notes

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# Syllabus

- Text: An Introduction to Modern Astrophysics 2nd Ed., Carroll and Ostlie
- First class Wed Jan 6. Reading period Mar 11-12
- Jan 6: Milky Way Galaxy
- Jan 13: Nature of Galaxies
- Jan 20: Galactic Evolution
- Jan 27: Active Galaxies
- Midterm Feb 10 on material above
- Feb 3: Structure of the Universe
- Feb 10: In class Midterm
- Feb 17, 24: Cosmology
- Mar 3, 10: Early Universe

# Common Themes

- Mapping out the Universe marching out in distance from Earth  
Start with closest system: Galaxy  
End with furthest system: whole Universe
- Limitations imposed by the ability to measure only a handful of quantities, all from our vantage point in the Galaxy  
Common tools: distance measures, number counts
- Inferences on the dynamical nature of the systems by using physical laws to interpret observations  
Common tools: mass inferences from Newtonian dynamics, General Relativity

Set 1:

Milky Way Galaxy

# Astrophysical units

- Length scales
- $1\text{AU} = 1.496 \times 10^{13}\text{cm}$  – Earth-sun distance – used for solar system scales
- $1\text{pc} = 3.09 \times 10^{18}\text{cm} = 2.06 \times 10^5\text{AU}$  – 1AU subtends 1arcsecond on the sky at 1pc – distances between nearby stars

Defined by measuring parallax of nearby stars to infer distance - change in angular position during Earth's orbit: par(allax arc)sec(ond)

$$\frac{1\text{AU}}{1\text{pc}} = \frac{1}{2.06 \times 10^5} = 4.85 \times 10^{-6} = \frac{\pi}{60 * 60 * 180} = 1''$$

- $1\text{kpc} = 10^3\text{pc}$  – distances in the Galaxy
- $1\text{Mpc} = 10^6\text{pc}$  - distances between galaxies
- $1\text{Gpc} = 10^9\text{pc}$  - scale of the observable universe

# Astrophysical units

- Fundamental observables are the **flux**  $F$  (energy per unit time per unit area) or **brightness** (+ per unit solid angle) and **angular position** of objects in a given **frequency** band
- Related to the physical quantities, e.g. the **luminosity** of the object  $L$  if the distance to the object is known

$$F = \frac{L}{4\pi d^2}$$

- Solar luminosity

$$L_{\odot} = 3.839 \times 10^{26} \text{W} = 3.839 \times 10^{33} \text{erg/s}$$

- Frequency band defined by filters - in limit of infinitesimal bands, the whole **frequency spectrum** measured – “spectroscopy”

# Astrophysical units

- **Relative flux** easy to measure - absolute flux requires calibration of filter: (apparent) **magnitudes** (originally defined by eye as filter)

$$m_1 - m_2 = -2.5 \log(F_1/F_2)$$

- **Absolute magnitude**: apparent magnitude of object at  $d = 10\text{pc}$

$$m - M = -2.5 \log(d/10\text{pc})^2 \rightarrow \frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5}$$

- If frequency spectrum has **lines**, **Doppler shift** gives relative or **radial velocity** of object  $V_r$  aka **redshift**  $z$

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 + \frac{V_r}{c}$$

(where  $V_r > 0$  denotes recession and redshift) used to measure velocity for dynamics of systems, including universe as whole

# Astrophysical units

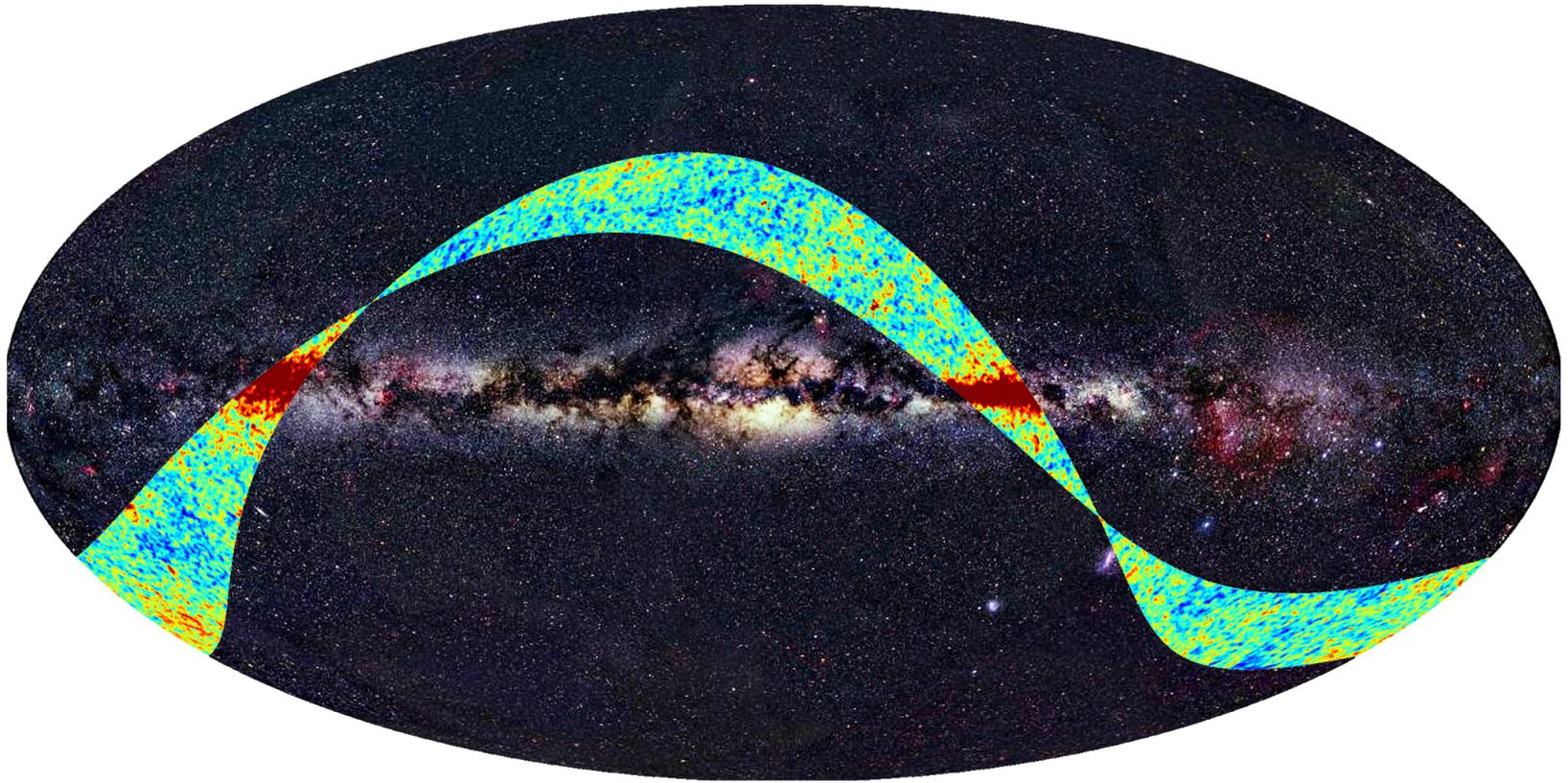
- Masses in units of solar mass  $M_{\odot} = 1.989 \times 10^{33} \text{g}$
- Mass measurement always boils down to inferring gravitational force necessary to keep test object of mass  $m$  with a velocity  $v$  bound
- For circular motion - centripetal force

$$\frac{mv^2}{r} \approx \frac{GmM}{r^2} \rightarrow M \approx \frac{v^2 r}{G}$$

- Requires a measurement of velocity and a measurement or estimate of size
- Various systems will have order unity correction to this circular-motion based relation

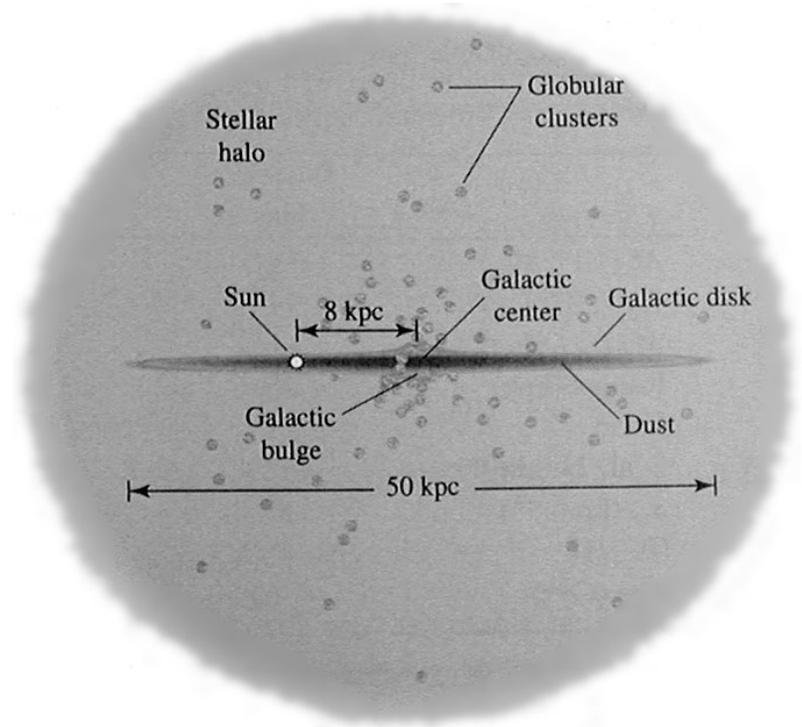
# Starlight: Optical Image

- Color overlay: microwave background



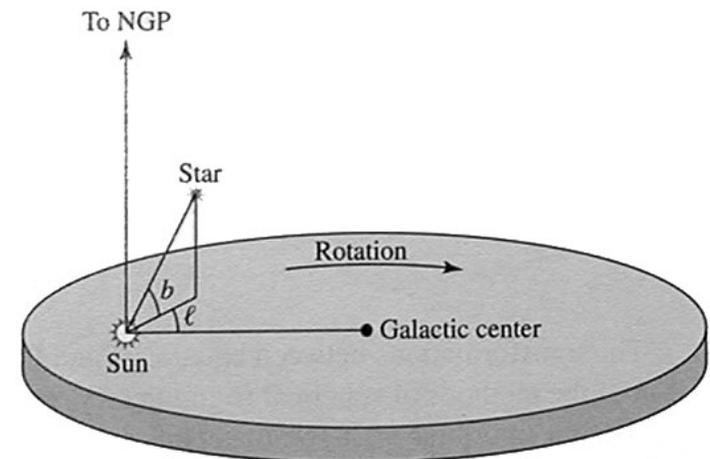
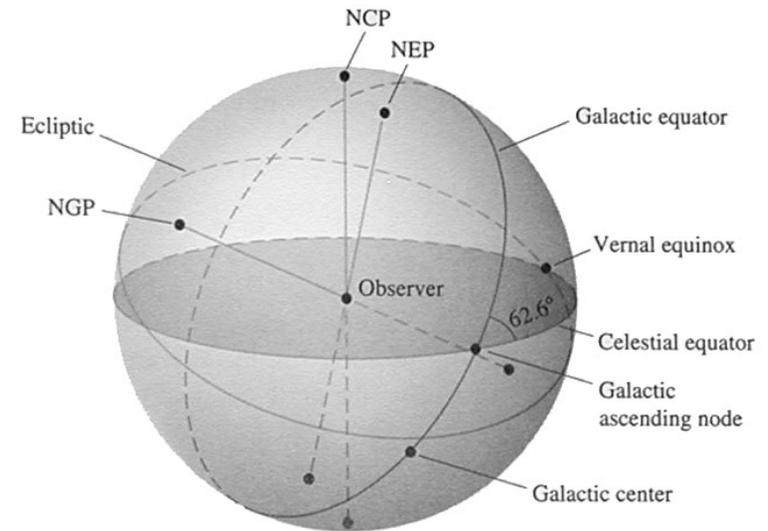
# Galactic Census

- From such data, infer the **structure of the galaxy**
- Sun is embedded in a **stellar disk**  $\sim 8$  kpc from the galactic center
- Extent of disk  $\sim 25$  kpc radius, **spiral structure**
- Thickness of **neutral gas disk**  $< 0.1$  kpc
- Thickness of **thin disk** of young stars  $\sim 0.35$  kpc
- Thickness of **thick disk**  $\sim 1$  kpc



# Galactic Census

- Central **stellar bulge**  
radius  $\sim 4$  kpc, with **central bar**
- **Supermassive black hole**, inferred from large mass within 120AU (solar system scale) of center
- Extended spherical **stellar halo** with **globular clusters**, radius  $> 100$  kpc
- Extended **dark matter halo**, radius  $> 200$  kpc



# Mass and Luminosity

- Neutral gas disk:  $M \sim 0.5 \times 10^{10} M_{\odot}$
- Thin disk:  $M \sim 6 \times 10^{10} M_{\odot}$ ,  $L_B \sim 1.8 \times 10^{10} L_{\odot}$
- Thick disk:  $M \sim 0.2 - 0.4 \times 10^{10} M_{\odot}$ ,  $L_B \sim 0.02 \times 10^{10} L_{\odot}$
- Bulge:  $M \sim 1 \times 10^{10} M_{\odot}$ ,  $L_B \sim 0.3 \times 10^{10} L_{\odot}$
- Supermassive black hole mass  $3.7 \pm 0.2 \times 10^6 M_{\odot}$
- Stellar halo:  $M \sim 0.3 \times 10^{10} M_{\odot}$ ,  $L_B \sim 0.1 \times 10^{10} L_{\odot}$
- Dark matter halo:  $M \sim 2 \times 10^{12} M_{\odot}$
- Total:  $M \sim 2 \times 10^{12} M_{\odot}$ ,  $L_B \sim 3.6 \times 10^{10} L_{\odot}$

# Methods: Star Counts

- One of the oldest methods for inferring the structure of the galaxy from 2D sky maps is from **star counts**
- History: **Kapteyn** (1922), building on early work by Herschel, used star counts to map out the structure of the galaxy
- Fundamental assumptions
  - Stars have a known (distribution in) **absolute magnitude**
  - No obscuration**
- Consider a star with known absolute magnitude  $M$  (magnitude at 10pc). Its **distance** can be inferred from the **inverse square law** from its **observed**  $m$  as

$$\frac{d(m - M)}{10\text{pc}} = 10^{(m-M)/5}$$

# Methods: Star Counts

- Combined with the **angular position** on the sky, the **3d position** of the star can be measured - mapping the galaxy
- Use the star counts to determine **statistical properties**: number density of stars in each patch of sky
- A **fall off** in the number density in radial distance would determine the **edge** of the galaxy
- Suppose there is an **indicator** of absolute magnitude like spectral type that allows stars to be **selected** to within  $dM$  of  $M$
- Describe the underlying quantity to be extracted as the **spatial number density** within  $dM$  of  $M$ :  $n_M(M, \mathbf{r})dM$

# Methods: Star Counts

- The observable is say the **total number** of stars **brighter** than a limiting **apparent magnitude**  $m$  in a **solid angle**  $d\Omega$
- Stars at a given  $M$  can only be observed out to a distance  $d(m - M)$  before their apparent magnitude falls below the limit
- So there is radial distance limit to the volume observed
- **Total number** observed out to in solid angle  $d\Omega$  within  $dM$  of  $M$  is **integral** to that limit

$$N_M dM = \left[ \int_0^{d(m-M)} n_M(M, r) r^2 dr \right] d\Omega dM$$

- **Differentiating** with respect to  $d(m - M)$  provides a measurement of  $n_M(M, r)$

# Methods: Star Counts

- So dependence of counts on the limiting magnitude  $m$  determines the number density and e.g. the edge of the system
- In fact, if there were no edge to system the total flux would diverge as  $m \rightarrow 0$  - volume grows as  $d^3$  flux decreases as  $d^{-2}$ : Olber's paradox
- Generalizations of the basic method:
- Selection criteria is not a perfect indicator of  $M$  and so  $dM$  is not infinitesimal and some stars in the range will be missed -  $S(M)$  and  $M$  is integrated over - total number

$$N = \int_{-\infty}^{\infty} dM S(M) N_M$$

# Methods: Star Counts

- Alternately use **all stars** [weak or no  $S(M)$ ] but assume a **functional form** for  $n_M$  e.g. derived from local estimates and assumed to be the same at larger  $r$

In this case, measurements determine the **normalization** of a distribution with **fixed shape** and determine

$$n(\mathbf{r}) = \int_{-\infty}^{\infty} n_M(M, \mathbf{r}) dM$$

- Similar method applies to **mapping** out the **Universe** with **galaxies**

# Methods: Star Counts

- Kapteyn used all of the stars (assumed to have the same  $n_M$  shape in  $r$ )
- He inferred a flattened spheroidal system of  $< 10\text{kpc}$  extent in plane and  $< 2\text{kpc}$  out of plane: too small
- Missing: interstellar extinction dims stars dropping them out of the sample at a given limiting magnitude

# Methods: Variable Stars

- With a good indicator of absolute magnitude or “standard candle” one can use individual objects to map out the structure of the Galaxy (and Universe)
- History: Shapley (1910-1920) used RR Lyrae and W Virginis variable stars - with a period-luminosity relation  
Radial oscillations with a density dependent sound speed - luminosity and density related on the instability strip  
Calibrated locally by moving cluster and other methods
- Measure the period of oscillation, infer a luminosity and hence an absolute magnitude, infer a distance from the observed apparent magnitude

# Methods: Variable Stars

- Inferred a 100kpc scale for the Galaxy - overestimate due to differences in types of variable stars and interstellar extinction
- Apparent magnitude is dimmed by extinction leading to the variable stars being less distant than they appear
- Both Kapteyn and Shapley off because of dust extinction: discrepancy between two independent methods indicates systematic error
- Caveat emptor: in astronomy always want to see a cross check with two or more independent methods before believing result you read in the NYT!

# Interstellar Extinction

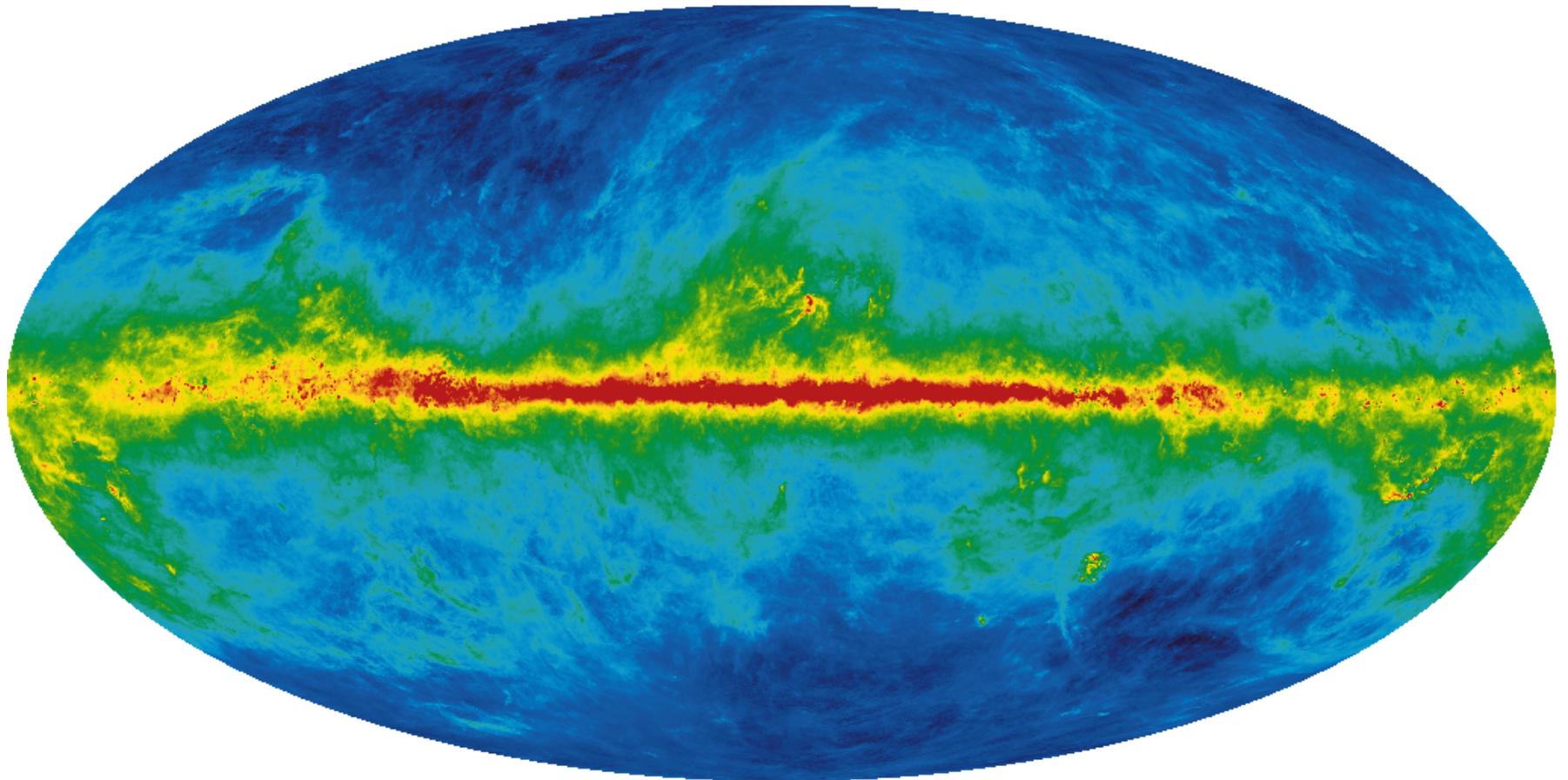
- Dust (silicates, graphite, hydrocarbons) in ISM (Chap 12) dims stars at visible wavelengths making true distance less than apparent
- Distance formula modified to be

$$\frac{d}{10\text{pc}} = 10^{(m_\lambda - M_\lambda - A_\lambda)/5}$$

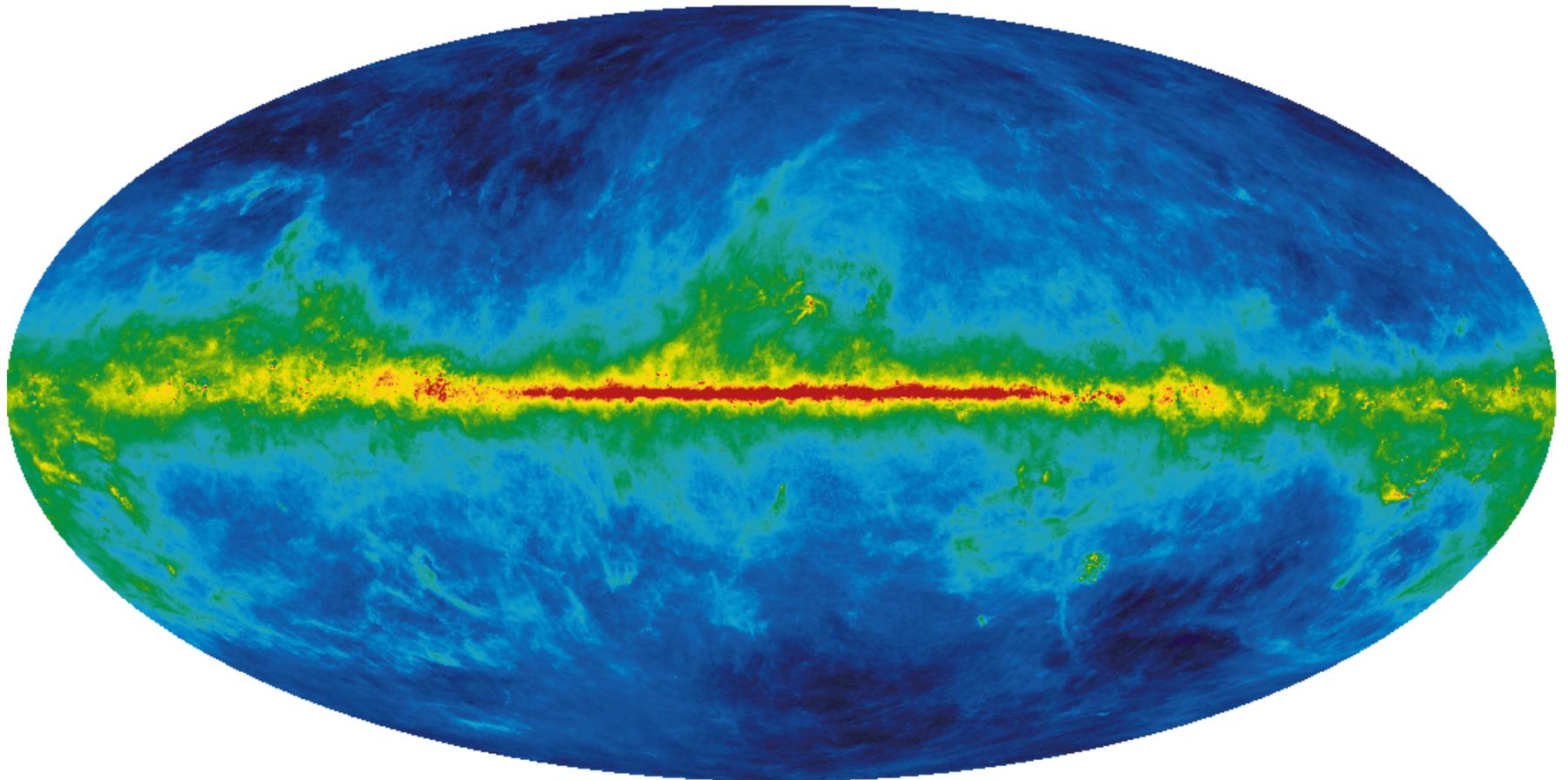
where the extinction coefficient  $A_\lambda \geq 0$  depends on wavelength  $\lambda$

- Extinction also depends on direction, e.g. through the disk, through a giant molecular cloud, etc. Typical value at visible wavelengths and in the disk is 1 mag/kpc
- Dust emits or reradiates starlight in the infrared - maps from these frequencies [IRAS, DIRBE] can be used to calibrate extinction

# Dust Emission



# Extinction Correction



# Kinematic Distances to Stars

- Only nearby stars have their distance measured by parallax - further than a parsec the change in angle is  $< 1$  arcsec:

$$p(\text{arcsec}) = 1\text{pc}/d$$

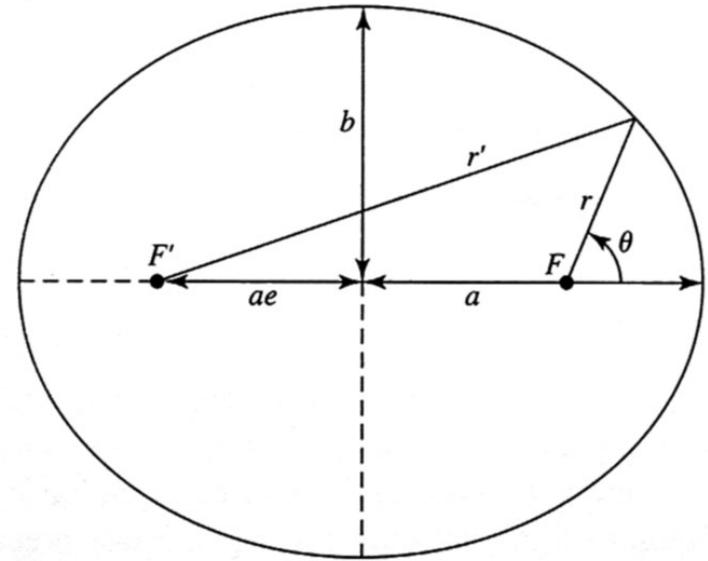
- If proper motion across the sky can be measured from the change in angular position  $\mu$  in rad/s

$$v_t = \mu d$$

- Often  $v_t$  can be inferred from the radial velocity and a comparison with  $\mu$  gives distance  $d$  given assumption of the dynamics
- Example: Keplerian orbits of stars around galactic center  
 $R_0 = 7.6 \pm 0.3\text{kpc}$
- Example: Stars in a moving cluster share a single total velocity whose direction can be inferred from apparent convergent motion (see Fig 24.30)

# Methods: Stellar Kinematics

- Can infer more than just distance: SMBH
- Galactic center: follow orbits of stars close to galactic center
- One star: orbital period 15.2yrs, eccentricity  $e = 0.87$ , perigalacticon distance (closest point on orbit to  $F$ ) 120 AU =  $1.8 \times 10^{13}$  m
- Estimate mass:  $a = ae - r_p$  so semimajor axis



$$a = \frac{r_p}{1 - e} = 1.4 \times 10^{14} \text{ m}$$

# Methods: Stellar Kinematics

- Kepler's 3rd law

$$M = \frac{4\pi^2 a^3}{GP^2} = 7 \times 10^{36} \text{kg} = 3.5 \times 10^6 M_{\odot}$$

- That much mass in that small a radius can plausibly only be a (supermassive) black hole
- Note that this is an example of the general statement that masses are estimated by taking

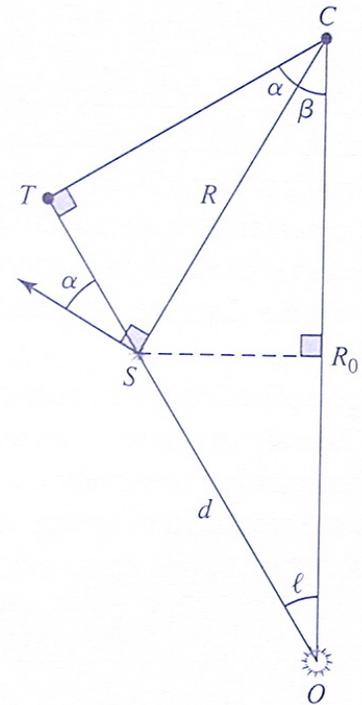
$$M \approx \frac{v^2 r}{G} = \frac{(2\pi a)^2 a}{GP^2} = \frac{4\pi^2 a^3}{GP^2}$$

# Methods: Stellar Kinematics

- Stars around sun - higher velocity, lower metallicity stars: thick disk, lower velocity higher metallicity stars orbiting with sun in thin disk
- Halo stars vs sun or LSR suggests orbital speed of  $\Theta_0 = 220$  km/s
- Differential rotation  $\Theta(R) = R\Omega(R)$  where  $\Omega(R)$  is the angular velocity curve— observables are radial and tangential motion with respect to LSR

$$v_r = R\Omega \cos \alpha - R_0\Omega_0 \sin \ell$$

$$v_t = R\Omega \sin \alpha - R_0\Omega_0 \cos \ell$$



# Methods: Stellar Kinematics

- $d$  (parallax) and  $R_0$  are known observables,  $R$  is not - eliminate with trig relations

$$R \cos \alpha = R_0 \sin \ell \quad R \sin \alpha = R_0 \cos \ell - d$$

- Eliminate  $R$

$$v_r = (\Omega - \Omega_0) R_0 \sin \ell$$

$$v_t = (\Omega - \Omega_0) R_0 \cos \ell - \Omega d$$

solve for  $\Omega(R)$  locally where

$$\begin{aligned} \Omega - \Omega_0 &\approx \frac{d\Omega}{dR} (R - R_0) \\ &\approx \frac{1}{R_0} \left( \frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right) (R - R_0) \quad [\Omega = \Theta/R] \end{aligned}$$

and  $d \ll R_0$ ,  $\cos \beta \approx 1$

# Methods: Stellar Kinematics

- Reduce with trig identities

$$R_0 = d \cos \ell + R \cos \beta \approx d \cos \ell + R$$

$$R - R_0 \approx -d \cos \ell$$

$$\cos \ell \sin \ell = \frac{1}{2} \sin 2\ell$$

$$\cos^2 \ell = \frac{1}{2} (\cos 2\ell + 1)$$

to obtain

$$v_r \approx Ad \sin 2\ell$$

$$v_t \approx Ad \cos 2\ell + Bd$$

# Methods: Stellar Kinematics

- Oort constants

$$A = -\frac{1}{2} \left[ \frac{d\Theta}{dR} - \frac{\Theta_0}{R_0} \right] = -\frac{R_0}{2} \frac{d\Omega}{dR}$$
$$B = -\frac{1}{2} \left[ \frac{d\Theta}{dR} + \frac{\Theta_0}{R_0} \right]$$

- Observables  $v_r$ ,  $v_t$ ,  $\ell$ ,  $d$ : solve for Oort's constants. From Hipparcos

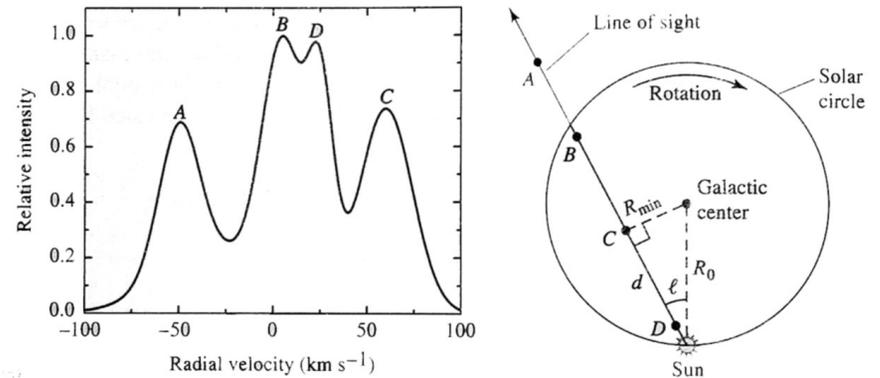
$$A = 14.8 \pm 0.8 \text{ km/s/kpc}$$

$$B = -12.4 \pm 0.6 \text{ km/s/kpc}$$

- Angular velocity  $\Omega = v/r$  decreases with radius: differential rotation. Physical velocity  $\Theta(R)$ :  $d\Theta/dR|_{R_0} = -(A + B) = -2.4$  km/s/kpc decreases slowly compared with 220km/s - flat rotation curve

# Methods: 21 cm

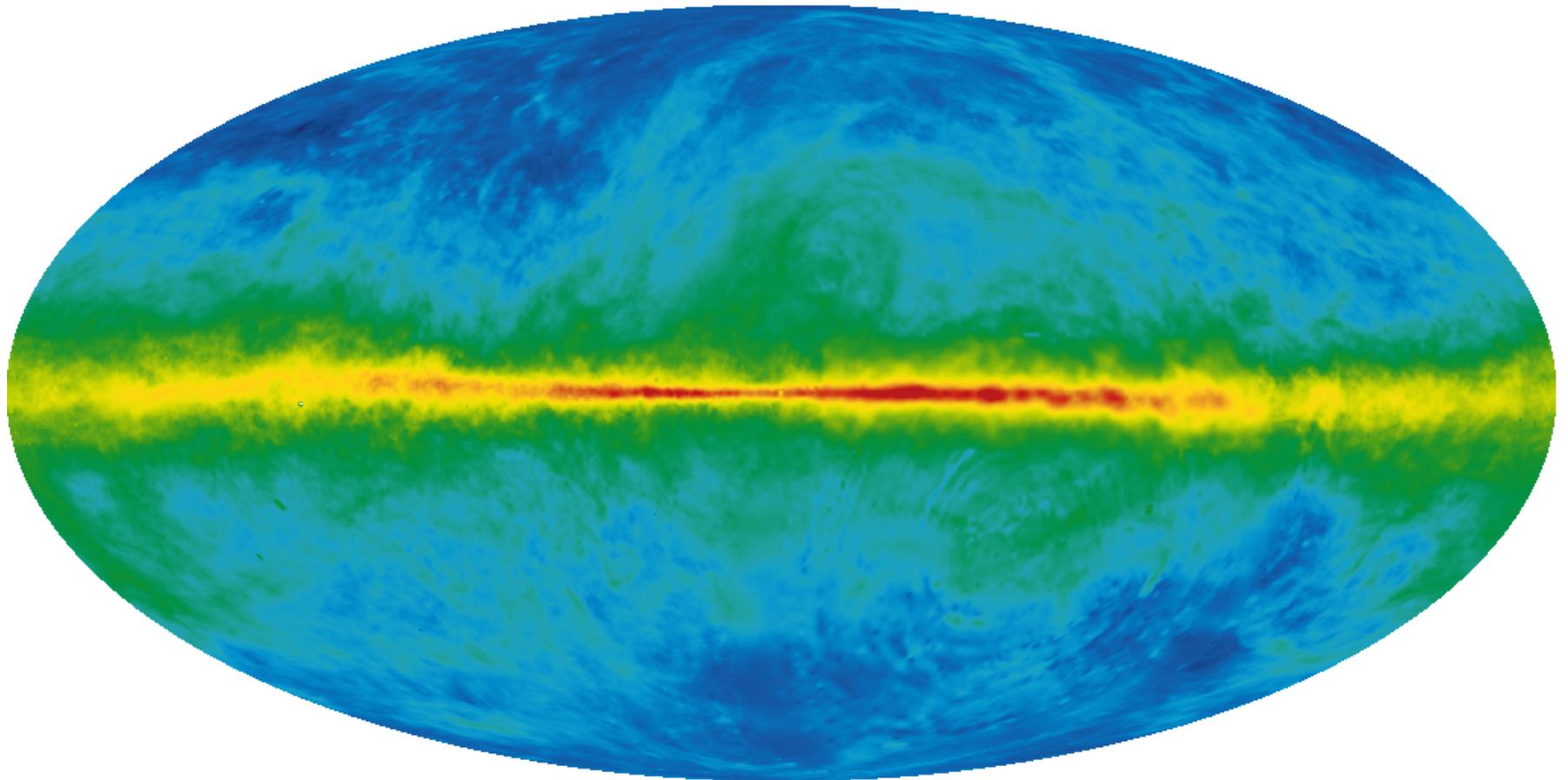
- Spin interaction of the electron and proton leads to a spin flip transition in neutral hydrogen with wavelength 21cm
- Line does not suffer substantial extinction and can be used to probe the neutral gas and its radial velocity from the Doppler shift throughout the galaxy
- No intrinsic distance measure
- Neutral gas is distributed inhomogeneously in clouds leading to distinct peaks in emission along each sight line



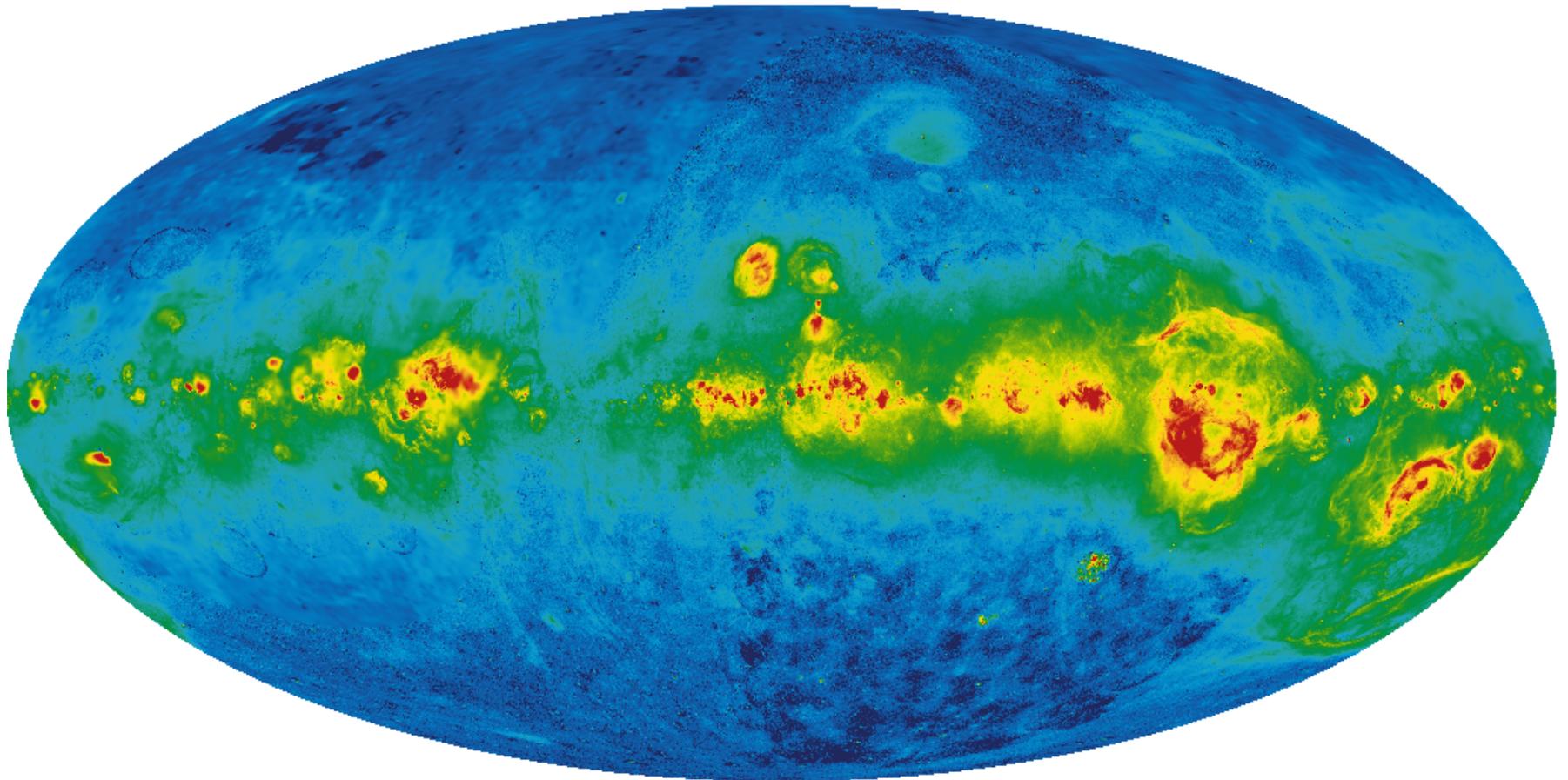
# Methods: 21 cm

- Due to projection of velocities along the line of sight and differential rotation, the highest velocity occurs at the closest approach to the galactic center or tangent point
- Build up a rotation curve interior to the solar circle  $R < R_0$
- Rotation curve steeply rises in the interior  $R < 1\text{kpc}$ , consistent with near rigid body rotation and then remains flat out through the solar circle

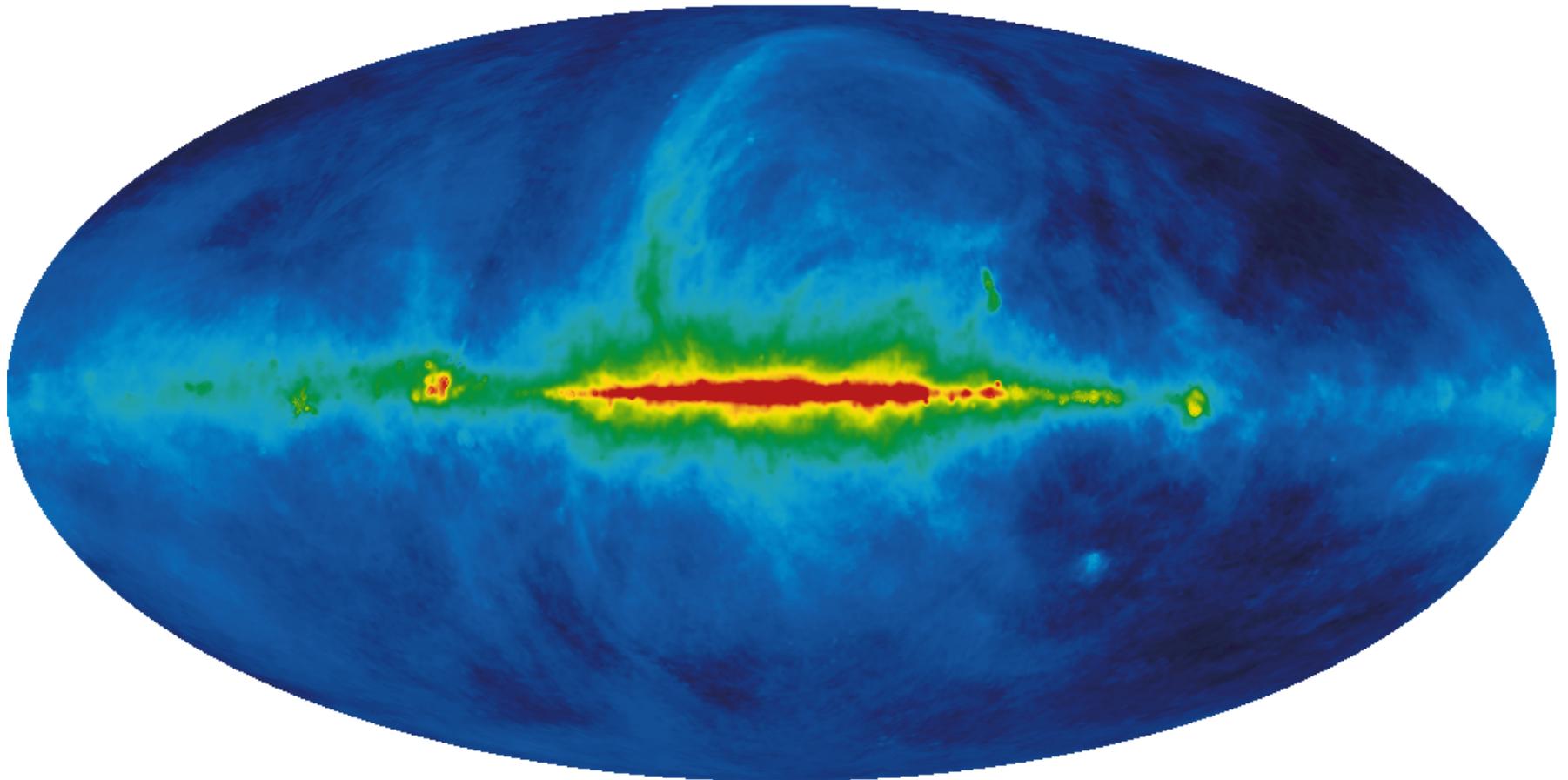
# Neutral Gas: 21cm Emission



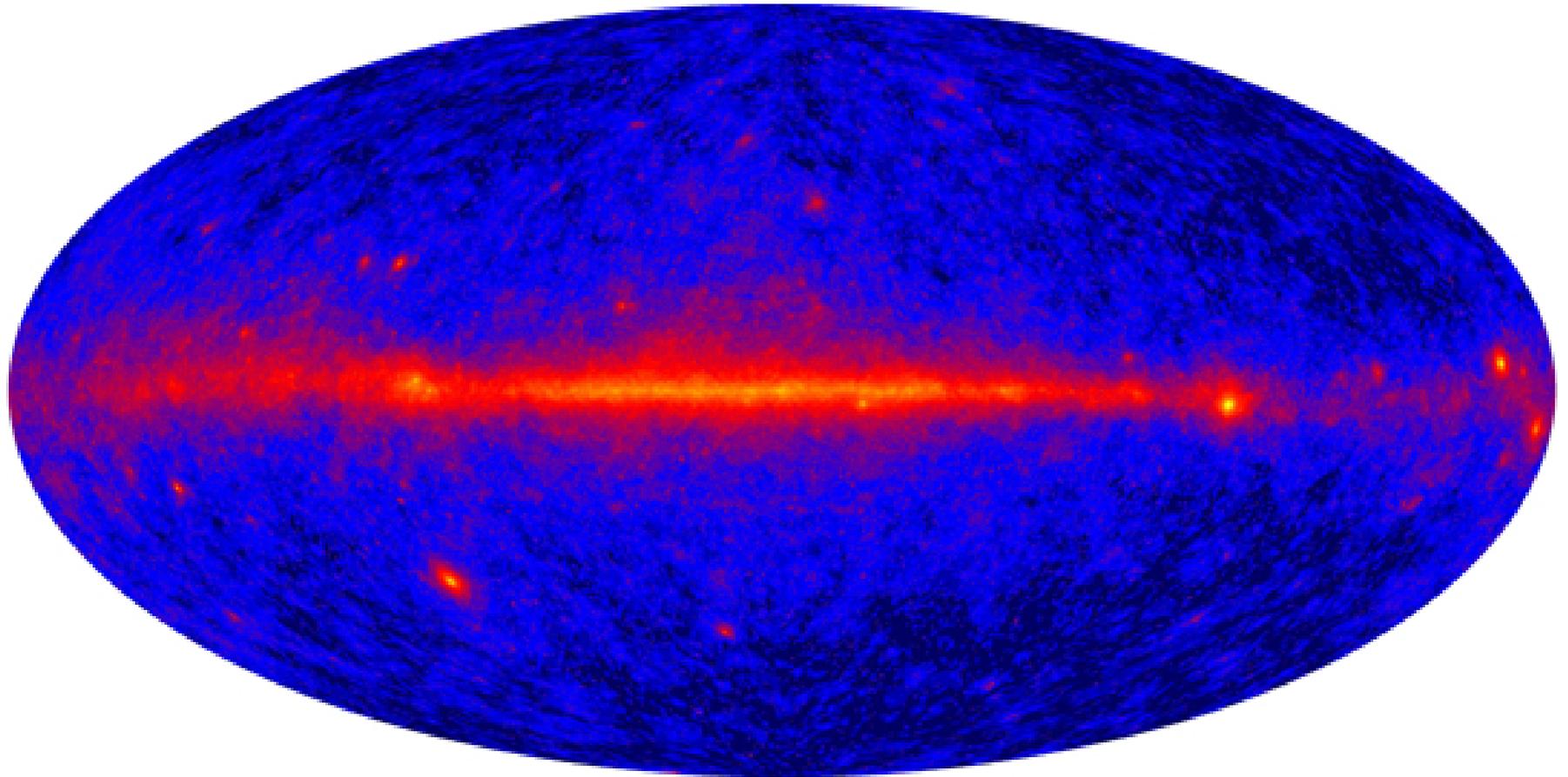
# Ionized Gas: H $\alpha$ Line Emission



# Cosmic Rays in $B$ Field: Synchrotron

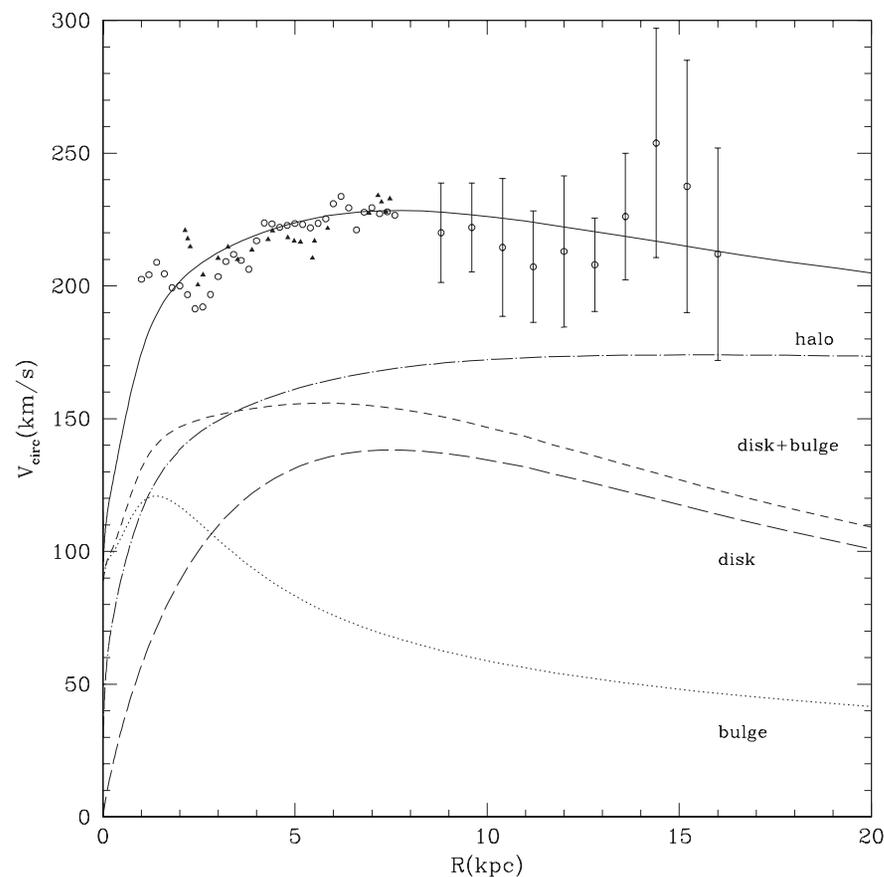


# Gamma Rays



# Methods: Rotation Curves

- Extending the rotation curve beyond the solar circle with objects like Cepheids whose distances are known reveals a flat curve out to  $\sim 20\text{kpc}$
- Mass required to keep rotation curves flat much larger than implied by stars and gas. Consider a test mass  $m$  orbiting at a radius  $r$  around an enclosed mass  $M(r)$



# Methods: Rotation Curves

- Setting the centripetal force to the gravitational force

$$\frac{mv^2(r)}{r} = \frac{GM(r)m}{r^2}$$

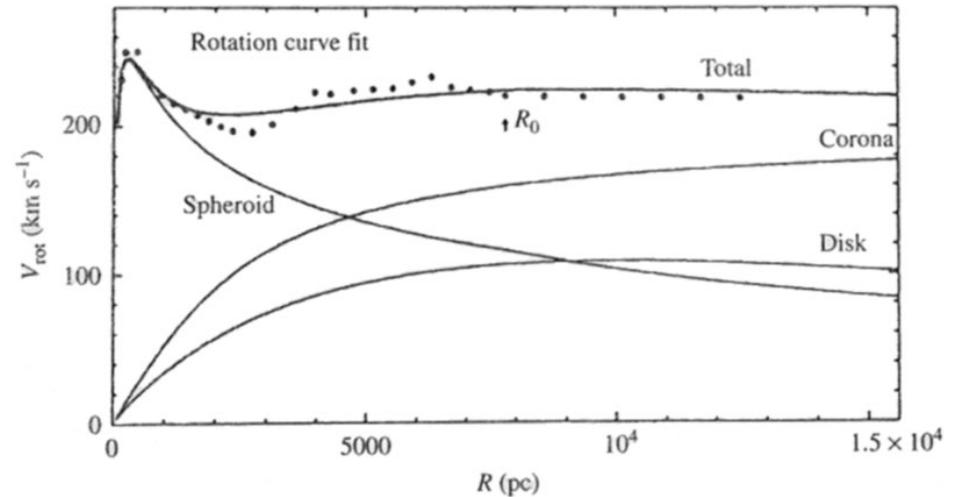
$$v(r) = \left( \frac{GM(r)}{r} \right)^{1/2}$$

Side note: this is the fundamental way masses are measured - balance internal motions of luminous matter with gravitational force - other examples: virial theorem with velocity dispersion, hydrostatic equilibrium with thermal motions

- Measuring the rotation curve  $v(r)$  is equivalent to measuring the mass profile  $M(r)$  or density profile  $\rho(r) \propto M(r)/r^3$

# Methods: Rotation Curves

- Flat rotation curve  $v(r) = \text{const}$  implies  $M \propto r$  - a mass linearly increasing with radius
- Rigid rotation implies  $\Omega = v/r = \text{const}$ .  $v \propto r$  or  $M \propto r^3$  or  $\rho = \text{const}$
- Rotation curves in other galaxies show the same behavior: evidence that “dark matter” is ubiquitous in galaxies



# Methods: Rotation Curves

- Consistent with dark matter density given by

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^2}$$

- Also consistent with the NFW profile predicted by cold dark matter (e.g. weakly interacting massive particles or WIMPs)

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

# Methods: Gravitational Lensing

- Rotation curves leave open the question of what dark matter is
- Alternate hypothesis: dead stars or black holes - massive astrophysical compact halo object “MACHO”
- MACHOs have their mass concentrated into objects with mass comparable to the sun or large planet
- A MACHO at an angular distance  $u = \theta/\theta_E$  from the line of sight to the star will gravitationally lens or magnify the star by a factor of

$$A(u) = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}$$

where  $\theta_E$  is the Einstein ring radius in projection

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

# Methods: Gravitational Lensing

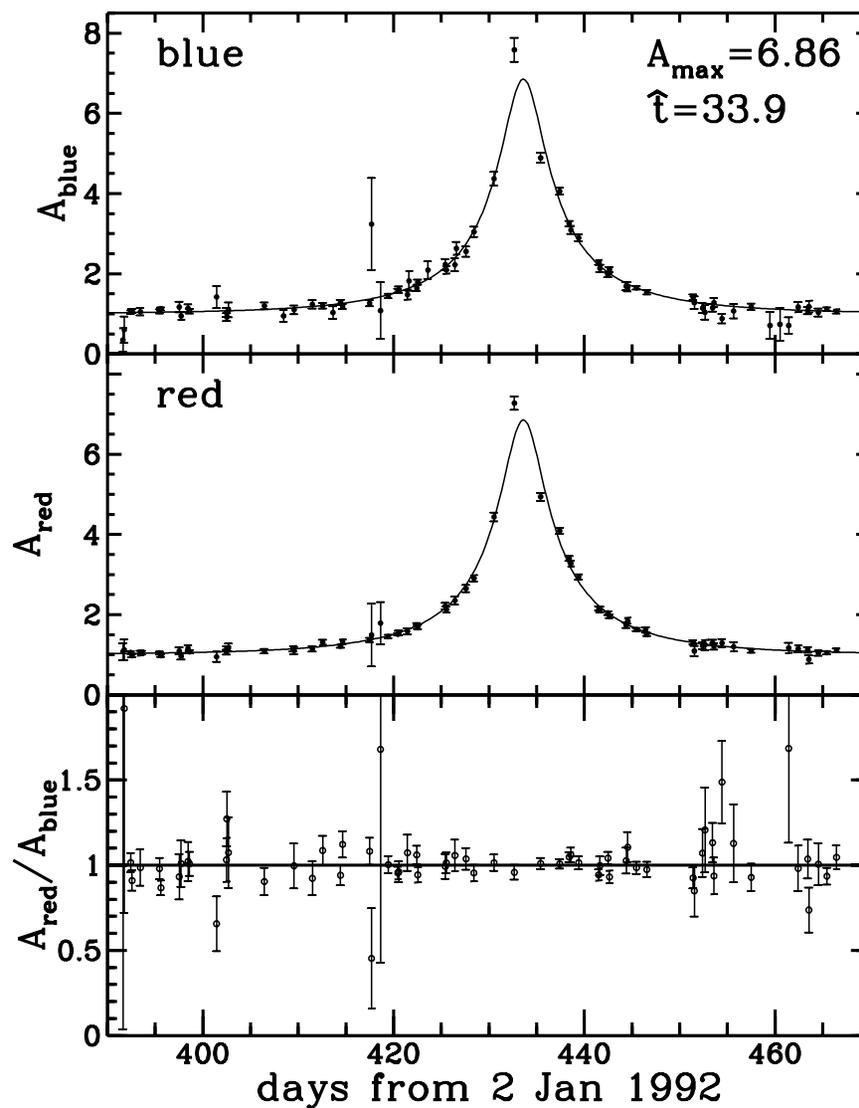
- A MACHO would move at a velocity typical of the disk and halo  $v \sim 200\text{km/s}$  and so the star behind it would brighten as it crossed the line of sight to a background star. With  $u_{\min}$  as the distance of closest approach at  $t = 0$

$$u^2(t) = u_{\min}^2 + \left( \frac{vt}{d_L \theta_E} \right)^2$$

- Monitor a large number of stars for this characteristic brightening. Rate of events says how much of the dark matter could be in MACHOs.

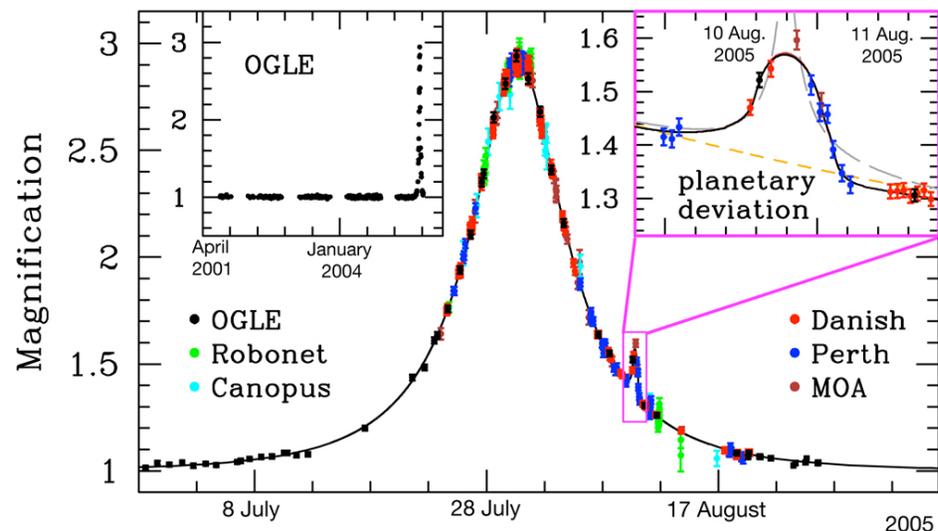
# Methods: Gravitational Lensing

- In the 1990's large searches measured the rate of microlensing in the halo and bulge and determined that only a small fraction of its mass could be in MACHOs



# Methods: Gravitational Lensing

- Current searches (toward the bulge) are used to find planets
- Enhanced microlensing by planet around star leads to a blip in the brightening.



Light Curve of OGLE-2005-BLG-390