

# Astro 282: Problem Set 1

Due April 8

## 1 Problem 1: Natural Units

Cosmologists and particle physicists like to suppress units by setting the fundamental constants  $c$ ,  $\hbar$ ,  $k_B$  to unity so that there is one fundamental unit. Structure formation cosmologists generally prefer Mpc as a unit of length, time, inverse energy, inverse temperature. Early universe people use GeV. Familiarize yourself with the elimination and restoration of units in the cosmological context.

Here are some fundamental constants of nature

Planck's constant	$\hbar = 1.0546 \times 10^{-27} \text{ cm}^2 \text{ g s}^{-1}$
Speed of light	$c = 2.9979 \times 10^{10} \text{ cm s}^{-1}$
Boltzmann's constant	$k_B = 1.3807 \times 10^{-16} \text{ erg K}^{-1}$
Fine structure constant	$\alpha = 1/137.036$
Gravitational constant	$G = 6.6720 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$
Stefan-Boltzmann constant	$\sigma = a/4 = \pi^2/60$ $a = 7.5646 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
Thomson cross section	$\sigma_T = 8\pi\alpha^2/3m_e^2 = 6.6524 \times 10^{-25} \text{ cm}^2$
Electron mass	$m_e = 0.5110 \text{ MeV}$
Neutron mass	$m_n = 939.566 \text{ MeV}$
Proton mass	$m_p = 938.272 \text{ MeV}$
Planck mass	$m_{\text{pl}} = G^{-1/2} = 1.221 \times 10^{19} \text{ GeV}$

and here are some unit conversions:

1 s	$= 9.7157 \times 10^{-15} \text{ Mpc}$
1 yr	$= 3.1558 \times 10^7 \text{ s}$
1 Mpc	$= 3.0856 \times 10^{24} \text{ cm}$
1 AU	$= 1.4960 \times 10^{13} \text{ cm}$
1 K	$= 8.6170 \times 10^{-5} \text{ eV}$
1 $M_\odot$	$= 1.989 \times 10^{33} \text{ g}$
1 GeV	$= 1.6022 \times 10^{-3} \text{ erg}$
	$= 1.7827 \times 10^{-24} \text{ g}$
	$= (1.9733 \times 10^{-14} \text{ cm})^{-1}$
	$= (6.5821 \times 10^{-25} \text{ s})^{-1}$

- Define the Hubble constant as  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  where  $h$  is a dimensionless number observed to be  $h \approx 0.7$ . Convert  $H_0$  into (a) eV, (b)  $\text{Mpc}^{-1}$ , (c)  $\text{Gyr}^{-1}$ . [Corresponds to upper limit on the mass of a dark energy particle, the inverse length scale of the observable universe, inverse approximate age.]
- Define the critical density  $\rho_{\text{crit}} = 3H_0^2/8\pi G$ . A universe at the critical density is spatially flat. Convert it into (a)  $\text{g cm}^{-3}$ , (b)  $\text{GeV}^4$ , (c)  $\text{eV cm}^{-3}$ , (d) protons  $\text{cm}^{-3}$ , (e)  $M_\odot \text{ Mpc}^{-3}$ . Cosmologists tend to define densities in units of the critical density and call it “ $\Omega$ ”. For example, if the density in the cosmological constant is  $\rho_\Lambda = 2\rho_{\text{crit}}/3$ , cosmologists will say  $\Omega_\Lambda = 2/3$ . What is the energy scale (i.e.  $\rho_\Lambda^{1/4}$ ) in (f) GeV of such a value of the cosmological constant? Compare that to the Planck mass. You should note that such a value is far from “natural”!
- Take the temperature of the CMB as  $T_{\text{CMB}} = 2.728 \text{ K}$  to (a) eV. With the Stephan-Boltzmann law  $\rho_\gamma = \pi^2 T_{\text{CMB}}^4/15$  convert this to (b)  $\text{eV cm}^{-3}$  (c)  $\text{g cm}^{-3}$ , and (d)  $\Omega_\gamma = \rho_\gamma/\rho_{\text{crit}}$ . Given a black body, the number density is  $n_\gamma = 2.404 T_{\text{CMB}}^3/\pi^2$  (e) express that in photons  $\text{cm}^{-3}$ . Given that the photons travel at the speed of light, what is the characteristic flux of photons, i.e. (f) photons  $\text{s}^{-1} \text{ cm}^{-2}$ ?