

Astro 282: Problem Set 3  
Due April 26

## 1 Problem 1: Angular Scale of the Horizon

Here you will calculate the scale associated with the first peak in the CMB spectrum and begin to explore its scaling with cosmological parameters. [See solution set from # 2 and or helper handout if you get stuck.]

- (a) In a flat  $\Omega_m = 1$  universe, with no radiation, calculate the horizon scale  $\eta$  at  $z = 1000$ . What is the angular scale subtended on the sky by this scale today? express your result in degrees and angular frequency  $\ell = 2\pi/\theta$ . That the CMB is smooth above this scale is known as the horizon problem; causal physics generates anisotropies below this scale – in particular the CMB acoustic peaks. (b) Calculate the same quantities now including the radiation content from the photons and neutrinos. How big is the difference for  $h = 0.7$ ?  $h = 0.4$ ? From the scaling with  $h$  can you guess what happens in a low  $\Omega_m = 1/3$  universe? (Hint: remember that what is going on is the matter-radiation ratio changes the expansion rate. If you change  $h$  by say 10% up how do you compensate with  $\Omega_m$ ? This is an example of a parameter degeneracy.)

## 2 Problem 2: Baryon Loading

Acoustic oscillations depend on the interplay between pressure and gravitational forces changing the momentum of particles in the plasma. The important quantity that controls the effect of the baryons is the ratio of momentum densities in the plasma. The relativistic generalization of the momentum density  $nmv = \rho v$  is  $(p + \rho)v$ .

- Calculate the momentum density ratio

$$R(a, \Omega_b h^2) = \frac{(\rho_b + p_b)v_b}{(\rho_\gamma + p_\gamma)v_\gamma} \quad (1)$$

and evaluate it at  $z = 1000$ . Recall that rapid scattering forces the baryons and photons to move together so that you may set  $v_b = v_\gamma$  here. Recall from class the equation of state  $p/\rho$  of the two species.