Astro 280: Problem Set 5
Due May 3

1 Problem 1: Oscillator Equation

Recall from class that a photon-dominated fluid without gravitational forces obeys the simplified continuity and Euler equations

$$\begin{aligned}\dot{\Theta} &= -\frac{k}{3}v_\gamma \\ \dot{v}_\gamma &= k\Theta\,. \end{aligned} \tag{1}$$

Here $\Theta \equiv \Delta T/T$ is the temperature perturbation and v_γ is the bulk velocity of the photons. Overdots are conformal time derivatives and k is the (comoving) wavenumber.

- (a) Combine the two equations into a single second order differential equation for Θ.
- (b) Write down the general solution to this equation $\Theta(\eta)$ in terms of the two initial conditions $\Theta(0)$ and $\dot{\Theta}(0)$

2 Problem 2: Adiabatic (Inflationary) Initial Conditions

Assume that the initial conditions are $\Theta(0) = \Psi/3$ and $\dot{\Theta}(0) = 0$. You will find it helpful to recall the general expression for conformal time in a matter-radiation universe from PS # 2.

- (a) Write down the expression for the wavenumbers k_n that reach their nth extrema by $z = 1000$. You may leave this expression in terms of $\eta_* = \eta(z = 1000)$.
- (b) Generalize the calculation from PS # 3 and determine the peak multipoles ℓ_n in terms of η_* and $\eta_0 = \eta(z = 0)$ [Hint: convert the wavenumbers to wavelengths, determine the angular scale and convert back to angular wavenumber – or do this all at once]. Evaluate these locations of the first 3 peaks for a flat $\Omega_m \approx 1$ and $h = 0.4$ model (recall that this is close to the desired $\Omega_m = 1/3$ and $h = 0.7$ universe). What is the spacing of the peaks $\Delta\ell = \ell_{n+1} - \ell_n$.

3 Problem 3: Isocurvature Initial Conditions

Isocurvature models start with no initial density or temperature perturbations $\Theta(0) = 0$ but with a velocity perturbation corresponding to $\dot{\Theta}(0) = kS$.

- (a) Write down the solution in (1) for this set of initial conditions.
- (b,c) Repeat (2a) and (2b) for this set of initial conditions. If we did not know that the initial conditions were adiabatic and we also did not know that the universe were flat, can we determine both facts from the locations of the peaks themselves? How many peaks would we need?

The qualitative picture you've worked out here applies to the case with gravity but with a few quantitative modifications as discussed in class. Note in particular that the first peak in reality is at a substantially lower multipole than predicted here.