

Astro 321: Problem Set 1

Due Jan. 16

1 Problem 1: Units

Convert the following quantities by inserting the appropriate factors of c , \hbar , k_B and unit conversions. You may find Peacock chapter 9 helpful.

- $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ into (a) eV, (b) Mpc^{-1} , (c) Gyr^{-1} . [Corresponds to upper limit on the mass of a dark energy particle, the inverse Hubble length, inverse approximate age.]
- $\rho_{\text{crit}} = 3H_0^2/8\pi G$ into (a) g cm^{-3} , (b) GeV^4 , (c) eV cm^{-3} , (d) protons cm^{-3} , (e) $M_\odot \text{ Mpc}^{-3}$. If the cosmological constant, has $\rho_\Lambda = 2\rho_{\text{crit}}/3$, what is its energy scale in eV (i.e. $\rho_\Lambda^{1/4}$). Compare that to the Planck mass.
- $T_{\text{CMB}} = 2.728\text{K}$ to (a) eV. Assuming a black body distribution, convert this to number density n_γ in photons cm^{-3} and energy density ρ_γ in (a) eV cm^{-3} (b) g cm^{-3} , and $\Omega_\gamma = \rho_\gamma/\rho_{\text{crit}}$.
- $T_\nu = (4/11)^{1/3}T_{\text{CMB}}$. Use this to express n_ν , ρ_ν and Ω_ν in the above units assuming that the neutrinos are relativistic (and fermions and have three species!).
- With the above relic number density, now consider the case where one out of three neutrino species has a mass of 1 eV and the rest are massless. What is the density of relic neutrinos in units of the critical density $\Omega_{\nu, \text{massive}}$. For what mass is the density at the critical value.

2 Problem 2: Age

- Assume the universe today is flat with both matter (Ω_m) and a cosmological constant (Ω_Λ). Compute the age of the universe and plot your result for t_0 in h^{-1} Gyr as a function of Ω_m .
- Assume instead that the universe is open with matter (Ω_m) but no cosmological constant. Compute the age of the universe and plot your result for t_0 in h^{-1} Gyr as a function of Ω_m .
- Assume that there is only matter and radiation in the universe (no cosmological constant) and that the universe is flat. Integrate the age equation to determine the time at which the cosmic temperature was 10^9K and $1/3$ eV.