Astro 321: Problem Set 3

Due Jan. 30

This problem set will begin a series which builds your standard toolkit for structure formation calculations. The motivation of some pieces may seem mysterious for now, but save the codes you build here because you will need them again later in the course.

1 Problem 1: Power Spectra

- Define the power spectrum today as

$$\begin{aligned} \Delta^2(k) &\equiv \frac{k^3}{2\pi^2}P(k) \\ &= \delta_H^2 \left(\frac{k}{H_0}\right)^{3+n} T^2(k) \end{aligned} \tag{1}$$

For reference, δ_H is the normalization of density perturbations on the horizon scale today $k \approx H_0$:

$$\delta_H = 1.94 \times 10^{-5} \Omega_m^{-0.785-0.05\ln\Omega_m} \exp[-0.95(n-1) - 0.169(n-1)^2] \tag{2}$$

is the famous COBE normalization courtesy of Bunn & White, n is the spectral index of the *initial* density fluctuations and $T(k)$ is the *transfer function* which defines the linear response (through gravitational perturbation theory) to the initial perturbations. We will see later where that comes from; for now take it to be defined as

$$\begin{aligned} T(k(q)) &= \frac{L(q)}{L(q) + C(q)q^2} \\ L(q) &= \ln(e + 1.84q) \\ C(q) &= 14.4 + \frac{325}{1 + 60.5q^{1.11}} \end{aligned} \tag{3}$$

q scales k to the horizon size at matter radiation equality and is given for historical reasons by

$$q = \frac{k}{\Omega_m h^2 \mathrm{Mpc}^{-1}} (T_{\rm CMB}/2.7K)^2 \qquad T_{\rm CMB} = 2.728K \tag{4}$$

- Calculate $\eta(a_{\rm eq})$ and show that $k\eta(a_{\rm eq})$ has the above scaling, $q \propto k\eta(a_{\rm eq})$ in its scalings with Ω_m, h, $T_{\rm CMB}$. For what value of q is $k\eta(a_{\rm eq}) = 1$? At this value of q, what is the value of the transfer function T?
- Write a code in the language of your choice to generate $\Delta^2(k)$ with k in units of h Mpc^{-1}. Leave Ω_m, n, h as adjustable parameters. Notice that value of q with k in h Mpc^{-1} depends on $\Omega_m h$, which is usually called the shape parameter "Γ" in the literature.
- For an $\Omega_m = 1/3$, $h = 1/\sqrt{2}$, $n = 1$ cosmology, plot your result of $\Delta^2(k)$. For what k in h Mpc^{-1} is $\Delta^2 = 1$.