Astro 321: CMB Final Helper Problem Set 4

Evolution of a k-mode

Take the initial conditions for a given k-mode from CMB PS# 3 and evolve the equations of motion in linear perturbation theory for a fully ionized universe.

In the notation of the previous problem sets and class notes, your fundamental variables are for the density perturbations

$$\Delta_\gamma, \Delta_b, \Delta_c \tag{1}$$

for the fluid velocities

$$v_\gamma, v_b, v_c\,, \tag{2}$$

[if you care, these are all in comoving gauge; although v_b is not technically needed for the approximation you are asked to solve, keep it anyway so that you can generalize your code later]. Your auxiliary variables are: the Newtonian curvature Φ, the Newtonian potential Ψ, the conformal time derivative of the Bardeen curvature $\dot\zeta$, the Newtonian temperature perturbation Θ, the anisotropic stress of the photons π_γ and the entropy perturbation in the photon-baryon system σ.

Explicitly, the set of coupled linear differential equations are:

(1) Continuity

$$\begin{aligned}
\dot\Delta_\gamma &= -\frac{4}{3}(kv_\gamma + 3\dot\zeta) \\
\dot\Delta_b &= -(kv_b + 3\dot\zeta) \\
\dot\Delta_c &= -(kv_c + 3\dot\zeta)
\end{aligned} \tag{3}$$

(2) Euler

$$\begin{aligned}
\dot v_\gamma &= -\frac{R}{1+R}\frac{\dot a}{a}v_\gamma + \frac{1}{1+R}k\Theta + k\Psi - \frac{1}{3}\frac{R}{(1+R)^2}k\sigma - \frac{1}{6}\frac{1}{(1+R)}k\pi_\gamma \\
\dot v_b &= \dot v_\gamma \\
\dot v_c &= -\frac{\dot a}{a}v_c + k\Psi
\end{aligned} \tag{4}$$

For which you will need the definitions of the auxiliary parameters:

$$k^2\Phi = 4\pi G a^2 \sum_i \Delta_i \rho_i \tag{5}$$

$$k^2(\Psi+\Phi) = -\frac{8}{3}\pi G a^2 \rho_\gamma \pi_\gamma \tag{6}$$

$$\Theta = \frac{1}{4}\Delta_\gamma - \frac{\dot a}{a} v/k \tag{7}$$

$$v = \frac{\sum_i (\rho_i + p_i) v_i}{\sum_i(\rho_i + p_i)} \tag{8}$$

$$\dot\zeta\left(\frac{\dot a}{a}\right)^{-1} = -\frac{w}{(1+w)}\left(\Delta_\gamma - \frac{2}{3}\pi_\gamma\right) \tag{9}$$

$$w = \frac{1}{3}\frac{\rho_\gamma}{\rho} \tag{10}$$

$$\sigma = (k\dot\tau^{-1})Rv_\gamma \tag{11}$$

$$\pi_\gamma = \frac{32}{15}(k\dot\tau^{-1})v_\gamma\,. \tag{12}$$

where the sums are over the three particle species. You can and should verify all these relationships yourselves from your PS's and notes since I am prone to make sign errors, etc!

Test your code.

Artificially set $\sigma = \pi_\gamma = 0$ to make the system dissipationless.

- Choose a $k \gg \eta_{\rm eq}^{-1}$ and plot the evolution of the fundamental and auxiliary parameters. In particular what is the amplitude of the acoustic oscillation in Θ in terms of $\zeta(0)$? check the answer with the solution given in class. What is the behavior of the Newtonian potential Φ? again check this against what we learned in class.

- Choose a $k \ll \eta_{\rm eq}^{-1}$ and follow the evolution well past horizon crossing (ignoring recombination). What is the value of Φ and Ψ compared with $\zeta(0)$, check your answer. How does the amplitude of the oscillation in $\Theta + \Psi$ behave as you cross $R = 1$, verify the qualitative behavior discussed in class.

Turn dissipation back on.

- For the $k \gg \eta_{\rm eq}^{-1}$ case, when does the acoustic oscillation dissipate, compare that with the random walk (or full dissipation) calculation in class.