Astro 321: Problem Set 2

Due Jan. 22

1 Problem 1: Conformal Time

- Assume the universe today is flat with both matter (Ω_m) and a cosmological constant (Ω_Λ). (a) Compute the conformal age or horizon of the universe and plot your result for $H_0\eta_0$ as a function of Ω_m. (b) What is the current horizon size for a universe with $\Omega_m = 1/3$ and $h = 1/\sqrt{2}$? (c) What is the mass contained within the current horizon in solar masses. If all objects were $10^{13}h^{-1}$ $M_{\rm sun}$ in mass, how many are in the observable universe.

- Evaluate the conformal age as a function of the scale factor in the above cosmology. What happens when $a \to \infty$. Comment on the implications for establishing causal contact between observers currently separated by much more than a Hubble length.

2 Problem 2: Comoving Distance

- Write down the expression for the conformal time elapsed between some initial epoch $a_i = (1 + z_i)^{-1}$ and a final epoch $a_f = (1 + z_f)^{-1}$. This is also the distance a particle going at the speed of light travels in this interval in comoving coordinates. At what redshift has light travelled halfway across the current horizon (in the cosmology calculated above).

- Calculate the above distance for $z_i = 0.5$ and $z_f = 0.4$. Use this answer to calculate the comoving volume of a spherical shell defined by these two redshifts in the cosmology above. Repeat this for a flat $\Omega_m = 1$ model. Which has the bigger volume in a fixed redshift shell?

3 Problem 3: Angular Scale of the Horizon

- In a flat $\Omega_m = 1$ universe, with no radiation, calculate the horizon scale at $z = 1000$. What is the angular scale subtended by this scale today? express your result in degrees and angular frequency $\ell = 2\pi/\theta$. That the CMB is smooth above this scale is known as the horizon problem; causal physics generates anisotropies below this scale – in particular the CMB acoustic peaks.