Astro 321: Problem Set 3
Due Jan. 29

1 Problem 1: Predict the CMB Temperature

- Assume that deuterium forms when the background temperature is $T = 10^9$K. Require that neutron capture be efficient enough to form light elements but not so efficient as to leave no deuterium so that $\langle \sigma v \rangle n_b t \sim 1$. (a) With $\langle \sigma v \rangle = 4.6 \times 10^{-20}$ cm^3 s^{-1}, and the age of the universe at $T = 10^9$K calculated in the first problem set, estimate the baryon density n_b under this condition. (b) Assuming a current baryon number density corresponding to $\Omega_b h^2 = 0.02$, what is the scale factor at $T = 10^9$? (c) What is the temperature of the background today?

2 Problem 2: Compton-y distortions

Recall that the Kompaneets equation is given by

$$\frac{\partial f}{\partial t} = \frac{d\tau}{dt} \frac{k_B T_e}{m_e c^2} \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial f}{\partial x} + f + f^2 \right) \right] , \tag{1}$$

where $x = h\nu / kT_e$.

- Show that evolution under the Kompaneets equation preserves the number (density) of photons.

Consider small deviations of the spectrum from the blackbody form

$$f = \frac{1}{e^{h\nu/k_B T} - 1} . \tag{2}$$

- Show
$$f + f^2 \approx -\frac{T}{T_e} \frac{\partial f}{\partial x} , \tag{3}$$

- Transform variables from time t to the Compton-y parameter
$$y = \int dt \frac{d\tau}{dt} \frac{k_B (T_e - T)}{m_e c^2} , \tag{4}$$
and write the Kompaneets equation in the form of a diffusion equation $\frac{\partial f}{\partial y} = \ldots$ The Kompaneets equation describes an upwards diffusion in energy of the photons via scattering off hotter electrons.

- Again assuming small deviations, insert the blackbody form eqn. into the right hand side of the Kompaneets/diffusion equation. Trivially integrate the equation to show that the change in the distribution function is given by
$$\frac{\Delta f}{f} = -y x_\nu e^{x_\nu} f \left(4 - x_\nu \coth \frac{x_\nu}{2} \right) , \tag{5}$$
where $x_\nu = h\nu / k_B T$.

- Define the effective thermodynamic temperature as the temperature of a blackbody that has the same f at a given frequency as the perturbed spectrum. Convert $\Delta f / f$ to $\Delta T / T$. What happens as $x_\nu \to 0$? What happens at $x_\nu \to \infty$. Argue that there must be a frequency (independent of y) at which $\Delta T / T = 0$. Numerically find this value of x_ν. Convert your answer to frequency (in GHz) and wavelength (cm) assuming $T = 2.726$K. This is known as the null in the thermal Sunyaev Zeldovich effect.