Astro 321

Lecture Notes Set 6

Wayne Hu

CMB Normalization

- Normalization of potential, hence inflationary power spectrum, set by CMB observations, aka COBE or WMAP normalization
- Angular power spectrum:

$$C_\ell = \frac{4\pi\Delta_T^2(\ell/D_*)}{2\ell(\ell+1)} = \frac{2\pi}{\ell(\ell+1)}\Delta_T^2(\ell/D_*)$$

- $\ell(\ell+1)C_\ell/2\pi = \Delta_T^2$ is commonly used log power
- Sachs-Wolfe effect says $\Delta_T^2 = \Delta_\Phi^2/9$, $\Phi = \frac{3}{5}\zeta$ initial
- Observed number at recombination

$$\Delta_T^2 = \left(\frac{28\mu\mathrm{K}}{2.725\times 10^6\mu\mathrm{K}}\right)^2$$

$$\Delta_\Phi^2 \approx (3\times 10^{-5})^2$$

$$\Delta_\zeta^2 \approx (5\times 10^{-5})^2$$

COBE vs WMAP Normalization

- Given that the temperature response to an inflationary initial perturbation is known for all k through the Boltzmann solution of the acoustic physics, one can translate Δ_T^2 to Δ_ζ^2 at the best measured $k \approx \ell/D_*$.
- The CMB normalization was first extracted from COBE at $\ell \sim 10$ or $k \sim H_0$. A low ℓ normalization point suffers from cosmic variance: only $2\ell + 1$ samples of a given ℓ mode.
- WMAP measures very precisely the first acoustic peak at $\ell \approx 200$. This is the current best place to normalize the spectrum ($k \sim 0.02$ Mpc^{-1}).
- To account for future improvements, WMAP chose $k = 0.05$ Mpc^{-1} as the normalization point. Taking out the CMB transfer function $\Delta_\zeta^2(k = 0.05) = (5.07 \times 10^{-5})^2$ consistent with a scale invariant spectrum from $0.0002 - 0.05$ Mpc^{-1}

Transfer Function

- Transfer function transfers the initial Newtonian curvature to its value today (linear response theory)

$$T(k) = \frac{\Phi(k, a=1)}{\Phi(k, a_{\rm init})} \frac{\Phi(k_{\rm norm}, a_{\rm init})}{\Phi(k_{\rm norm}, a=1)}$$

- Conservation of Bardeen curvature: Newtonian curvature is a constant when stress perturbations are negligible: above the horizon during radiation and dark energy domination, on all scales during matter domination
- When stress fluctuations dominate, perturbations are stabilized by the Jeans mechanism
- Hybrid Poisson equation: Newtonian curvature, comoving density perturbation $\Delta \equiv (\delta\rho/\rho)_{\rm com}$ implies Φ decays

$$(k^2 - 3K)\Phi = 4\pi G \rho \Delta \sim \eta^{-2} \Delta$$

Transfer Function

- Freezing of Δ stops at $\eta_{\rm eq}$

$$\Phi \sim (k\eta_{\rm eq})^{-2}\Delta_H \sim (k\eta_{\rm eq})^{-2}\Phi_{\rm init}$$

- Transfer function has a k^{-2} fall-off beyond $k_{\rm eq} \sim \eta_{\rm eq}^{-1}$

$$\eta_{\rm eq} = 15.7(\Omega_m h^2)^{-1}\left(\frac{T}{2.7K}\right)^2 \rm Mpc$$

- Small correction since growth with a smooth radiation component is logarithmic not frozen
- Transfer function is a direct output of an Einstein-Boltzmann code

Fitting Function

- Alternately accurate fitting formula exist, e.g. pure CDM form:

$$T(k(q)) = \frac{L(q)}{L(q) + C(q)q^2}$$

$$L(q) = \ln(e + 1.84q)$$

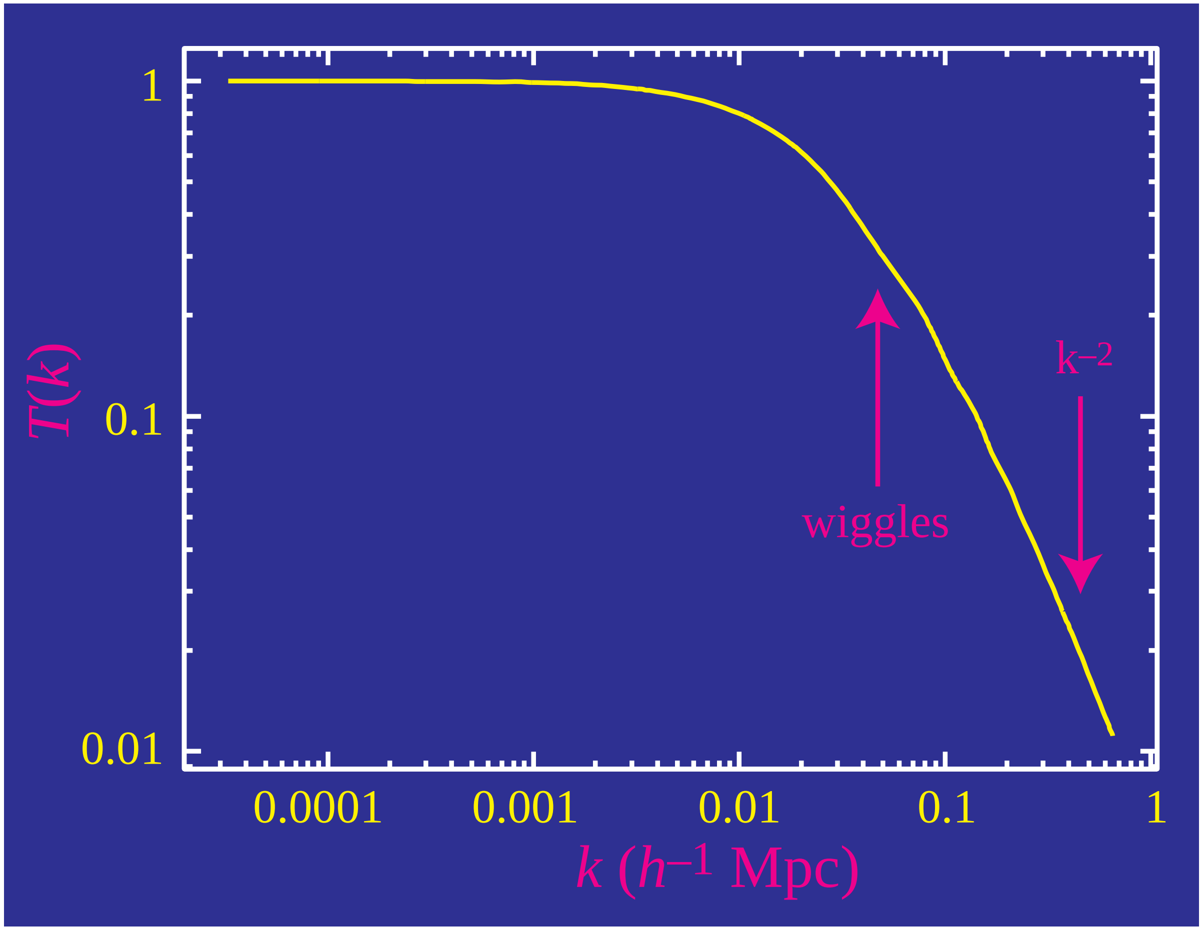
$$C(q) = 14.4 + \frac{325}{1 + 60.5q^{1.11}}$$

$$q = k/\Omega_m h^2 \mathrm{Mpc}^{-1} (T_{\rm CMB}/2.7K)^2$$

- In h Mpc^{-1}, the critical scale depends on $\Gamma \equiv \Omega_m h$ also known as the shape parameter

Transfer Function

- Numerical calculation



Baryon Wiggles

- Baryons caught up in the acoustic oscillations of the CMB and impart acoustic wiggles to the transfer function. Density enhancements are produced kinematically through the continuity equation $\delta_b \sim (k\eta)v_b$ and hence are out of phase with CMB temperature peaks
- Dissipation of the acoustic oscillations eliminates both the CMB and baryon perturbations – known as Silk damping for the baryons. This suppression and the general fact that baryons are caught up with photons was one of the main arguments for CDM
- Detected first (so far only) in the SDSS LRG survey.
- An excellent standard ruler for angular diameter distance $D_A(z)$ since it does not evolve in redshift in linear theory
- Radial extent of wiggles gives $H(z)$ (not yet seen in data)

Massive Neutrinos

- Neutrino dark matter suffers similar effects and hence cannot be the main component of dark matter in the universe
- Relativistic stresses of a light neutrino slow the growth of structure
- Neutrino species with cosmological abundance contribute to matter as $\Omega_\nu h^2 = \sum m_\nu / 94\mathrm{eV}$, suppressing power as $\Delta P/P \approx -8\Omega_\nu/\Omega_m$
- Current data from SDSS galaxy survey and CMB indicate $\sum m_\nu < 1.7\mathrm{eV}$ (95% CL) and with Lyα forest < 0.42 eV.

Growth Function

- Same physics applies to the dark energy dominated universe
- Under the dark energy sound horizon or Jeans scale, dark energy density frozen. Potential decays at the same rate for all scales

$$G(a) = \frac{\Phi(k_{\rm norm}, a)}{\Phi(k_{\rm norm}, a_{\rm init})} \qquad {}' \equiv \frac{d}{d\ln a}$$

- Continuity + Euler + Poisson

$$G'' + \left(1 - \frac{\rho''}{\rho'} + \frac{1}{2}\frac{\rho_c'}{\rho_c}\right) G' + \left(\frac{1}{2}\frac{\rho_c' + \rho'}{\rho_c} - \frac{\rho''}{\rho'}\right) G = 0$$

where ρ is the Jeans unstable matter and ρ_c is the critical density

Dark Energy Growth Suppression

- Pressure growth suppression: $\delta \equiv \delta\rho_m/\rho_m \propto aG$

$$\frac{d^2G}{d\ln a^2} + \left[\frac{5}{2} - \frac{3}{2}w(z)\Omega_{DE}(z)\right]\frac{dG}{d\ln a} + \frac{3}{2}[1 - w(z)]\Omega_{DE}(z)G = 0\,,$$

  where $w \equiv p_{DE}/\rho_{DE}$ and $\Omega_{DE} \equiv \rho_{DE}/(\rho_m + \rho_{DE})$ with initial conditions $G = 1$, $dG/d\ln a = 0$

- As $\Omega_{DE} \to 0$ $g =$const. is a solution. The other solution is the decaying mode, elimated by initial conditions

- As $\Omega_{DE} \to 1$ $g \propto a^{-1}$ is a solution. Corresponds to a frozen density field.

Power Spectrum Normalization

- Present (or matter dominated) vs inflationary initial conditions (normalized by CMB):

$$\Delta^2_\Phi(k,a) \approx \frac{9}{25}\Delta^2_{\zeta_i}(k_{\rm norm})G^2(a)T^2(k)\left(\frac{k}{k_{\rm norm}}\right)^{n-1}$$

- Density field

$$\begin{aligned}
k^2\Phi &= 4\pi G a^2 \Delta\rho_m \\
&= \frac{3}{2}H_0^2\Omega_m\frac{\Delta\rho_m}{\rho_m}\frac{1}{a} \\
\Delta^2_\Phi &= \frac{9}{4}\left(\frac{H_0}{k}\right)^4\Omega_m^2 a^{-2}\Delta^2_m \\
\Delta^2_m &= \frac{4}{25}\Delta^2_{\zeta_i}(k_{\rm norm})\Omega_m^{-2}a^2G^2(a)T^2(k)\left(\frac{k}{k_{\rm norm}}\right)^{n-1}\left(\frac{k}{H_0}\right)^4
\end{aligned}$$

Antiquated Normalization Conventions

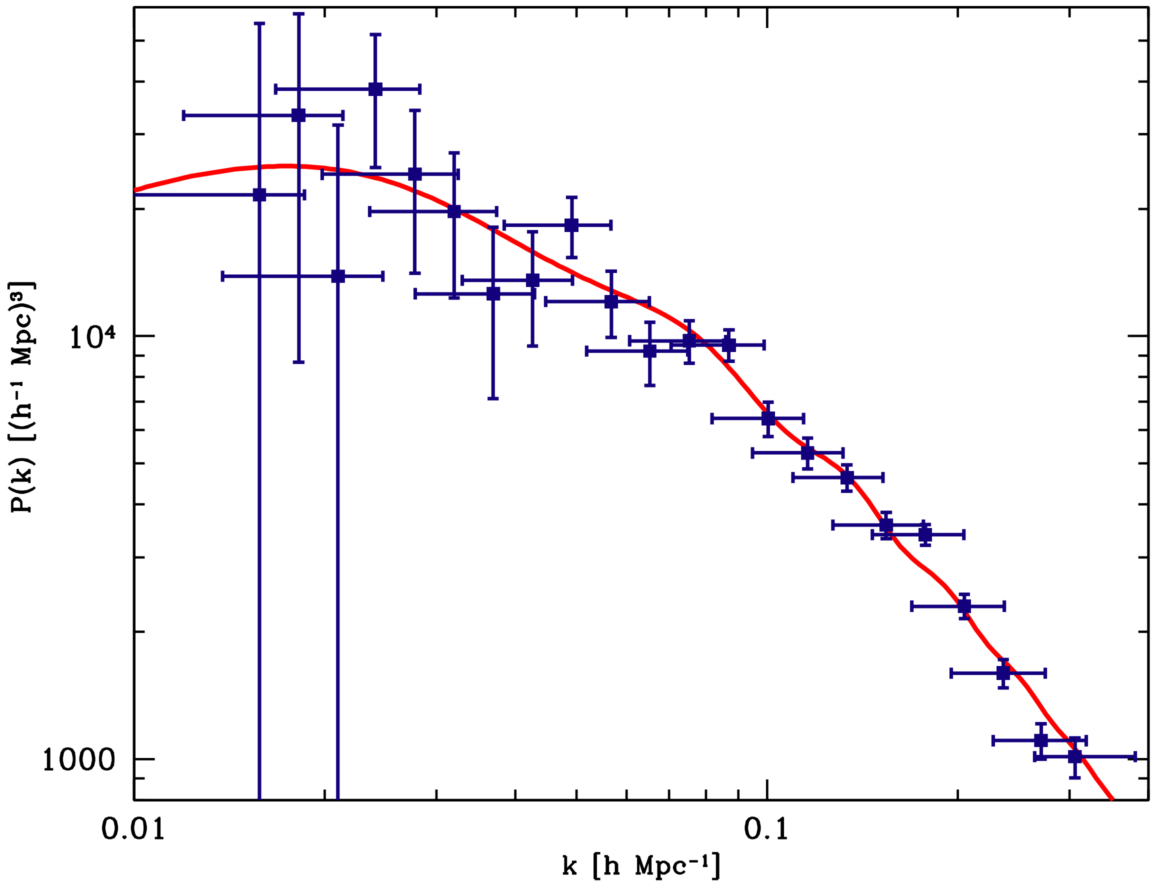
- Current density field on the horizon scale $k = H_0$

$$\delta_H^2 = \frac{4}{25}\Delta^2_{\zeta_i}(k_{\rm norm})\Omega_m^{-2}a^2G^2(a) = (2G(1)/\Omega_m \times 10^{-5})$$

- σ_8, RMS of density field filtered by tophat of $8h^{-1}$Mpc

Power Spectrum

- SDSS data



- Power spectrum defines large scale structure observables: galaxy clustering, velocity field, Lyα forest clustering, cosmic shear

Velocity field

- Continuity gives the velocity from the density field as

$$
\begin{aligned}
v = -\dot{\Delta}/k &= -\frac{aH}{k}\frac{d\Delta}{d\ln a} \\
&= -\frac{aH}{k}\Delta\frac{d\ln(aG)}{d\ln a}
\end{aligned}
$$

- In a ΛCDM model or open model $d\ln(aG)/d\ln a \approx \Omega_m^{0.6}$
- Measuring both the density field and the velocity field (through distance determination and redshift) allows a measurement of Ω_m
- Practically one measures $\beta = \Omega_m^{0.6}/b$ where b is a bias factor for the tracer of the density field, i.e. with galaxy numbers $\delta n/n = b\Delta$
- Can also measure this factor from the redshift space power spectrum - the Kaiser effect where clustering in the radial direction is apparently enhanced by gravitational infall

Redshift Space Power Spectrum

- Kaiser effect is separable from the real space clustering if one measures modes parallel and transverse to the line of sight. Redshift space distortions only modify the former
- 2D power spectrum in “s” or redshift space

$$P_s(k_\perp, k_\parallel) = \left[1 + \beta \left(\frac{k_\parallel}{k}\right)^2\right]^2 b^2 P(k)$$

where $k^2 = k_\parallel^2 + k_\perp^2$ and $k_\perp$ is a 2D vector transverse to the line of sight

Power Spectrum Errors

- The precision with which the power spectrum can be measured is ultimately limited by sample variance from having a finite survey volume $V = L^3$. This is basically a mode counting argument. The errors on the power spectrum are given by

$$\left(\frac{\Delta P_s}{P_s}\right)^2 = \frac{2}{N_k}$$

where N_k is the number of modes in a range of $\Delta k_\perp$, $\Delta k_\parallel$. This is determined by the k-space volume and the fundamental mode of the box $k_0 = 2\pi/L$ which sets the cell size in the volume

$$\left(\frac{\Delta P_s}{P_s}\right)^2 = \frac{2}{\frac{V}{(2\pi)^3} 2\pi k_\perp \Delta k_\perp \Delta k_\parallel}$$

Lyman-α Forest

- QSO spectra absorbed by neutral hydrogen through the Lyα transition.
- The optical depth to absorption is (with ds in physical scale)

$$\tau(\nu) = \int ds x_{\rm HI} n_b \sigma_\alpha \sim \int ds x_{\rm HI} n_b \Gamma \phi(\nu) \lambda^2$$

  where $x_{\rm HI}$ is the neutral fraction, $\Gamma = 6.25 \times 10^8 s^{-1}$ is the transition rate and $\lambda = 1216$A is the Lyα wavelength and $\phi(\nu)$ is the Lorentz profile. For radiation at a given emitted frequency ν_0 above the transition, it will redshift through the transition
- Resonant transition: lack of complete absorption, known as the lack of a Gunn-Peterson trough indicates that the universe is nearly fully ionized $x_{\rm HI} \ll 1$ out to the highest redshift quasar $z \sim 6$; indications that this is near the end of the reionization epoch

Lyman-α Forest

- In ionization equilibrium, the Gunn-Peterson optical depth is a tracer of the underlying baryon density which itself is a tracer of the dark matter $\tau_{GP} \propto \rho_b^2 T^{-0.7}$ with $T(\rho_b)$.

$$\frac{d(1 - x_{\rm HI})}{dt} = -x_{\rm HI} \int d\nu \frac{4\pi J_\nu}{h\nu} \sigma_\nu + (1 - x_{\rm HI})^2 n_b R$$

  where σ_ν is the photoionization cross section (sharp edge at threshold and falling in frequency means $J_\nu \approx J_{21}$) and $R \propto T^{-0.7}$ is the recombination coefficient.

- Given an equation of state from simulations of $p \propto \rho^\gamma$

$$x_{\rm HI} \propto \frac{\rho_b R}{J_{21}} \propto \frac{\rho_b T^{-0.7}}{J_{21}}, \quad \tau_{GP} \propto \frac{\rho_b^{2-0.7(\gamma-1)}}{J_{\rm HI}}$$

- Clustering in the Lyα forest reflects the underlying power spectrum modulo an overall ionization intensity J_{21}

Gravitational Lensing

- Gravitational potentials along the line of sight $\hat{\mathbf{n}}$ to some source at comoving distance D_s lens the images according to (flat universe)

$$\phi(\hat{\mathbf{n}}) = 2 \int dD \frac{D_s - D}{D D_s} \Phi(D\hat{\mathbf{n}}, \eta(D))$$

remapping image positions as

$$\hat{\mathbf{n}}^I = \hat{\mathbf{n}}^S + \nabla_{\hat{\mathbf{n}}} \phi(\hat{\mathbf{n}})$$

- Since absolute source position is unknown, use image distortion defined by the Jacobian matrix

$$\frac{\partial n_i^I}{\partial n_j^S} = \delta_{ij} + \psi_{ij}$$

Weak Lensing

- Small image distortions described by the convergence κ and shear components (γ_1, γ_2)

$$\psi_{ij} = \begin{pmatrix} \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & \kappa + \gamma_1 \end{pmatrix}$$

where $\nabla_{\hat{\mathbf{n}}} = D\nabla$ and

$$\psi_{ij} = 2\int dD \frac{D(D_s - D)}{D_s} \nabla_i \nabla_j \Phi(D\hat{\mathbf{n}}, \eta(D))$$

- In particular, through the Poisson equation the convergence (measured from shear) is simply the projected mass

$$\kappa = \frac{3}{2}\Omega_m H_0^2 \int dD \frac{D(D_s - D)}{D_s} \frac{\Delta(D\hat{\mathbf{n}}, \eta(D))}{a}$$