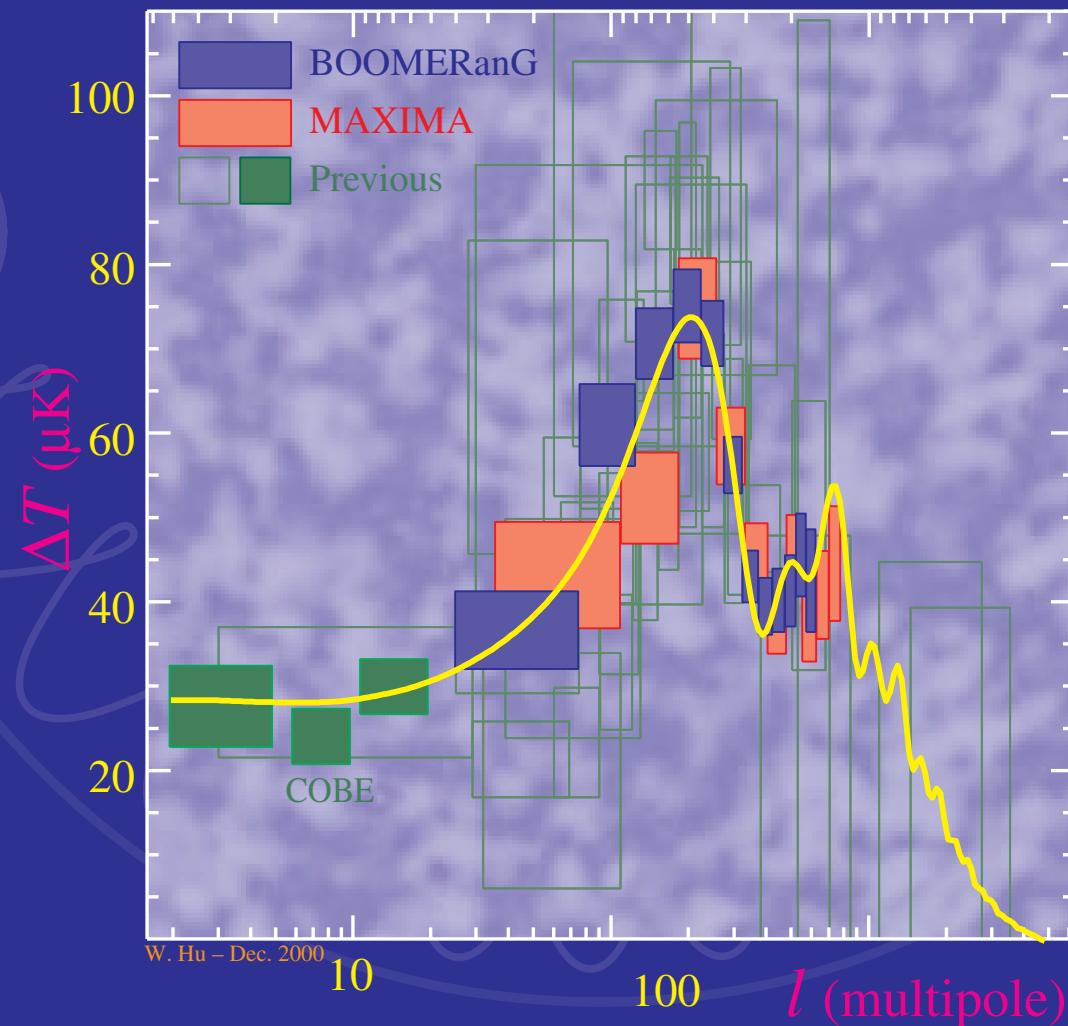


An Acoustic Primer



Wayne Hu
Astro 448

Parameter Estimation

The Ubiquitous Fisher Matrix

- The Fisher matrix is defined in terms of the likelihood (or signal C_l and noise N_l power spectra) as

$$F_{ij} = -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle = \frac{1}{2} \sum_{\ell} \frac{(2\ell + 1) f_{\text{sky}}}{(\mathcal{C}_{\ell} + \mathcal{N}_{\ell})^2} \frac{\partial C_{\ell}}{\partial p_i} \frac{\partial C_{\ell}}{\partial p_j}$$

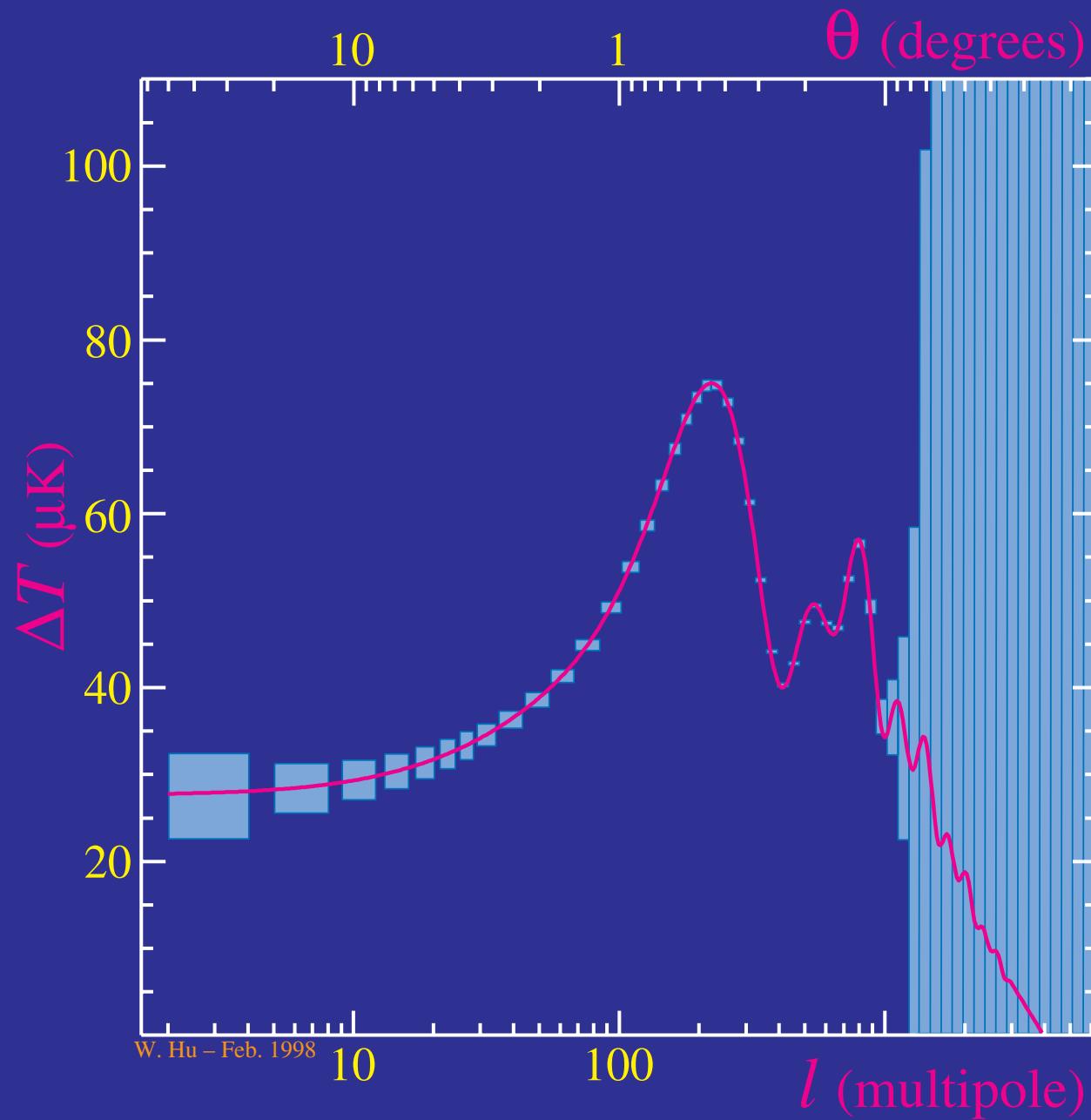
- Its inverse, the curvature matrix, gives the optimal errors on p , $\sigma^2 = (F^{-1})_{ii}$ including sampling and noise variance
- Useful for identifying degeneracies / constrained directions
- Problems: accuracy of derivatives and underlying parameterization can lead to widely diverging estimates:

$\sigma(h)$	$\sim 1\%$	Jungman et al. (1996)
	$\sim 200\%$	Eisenstein, Hu, Tegmark (1999)

MAP:

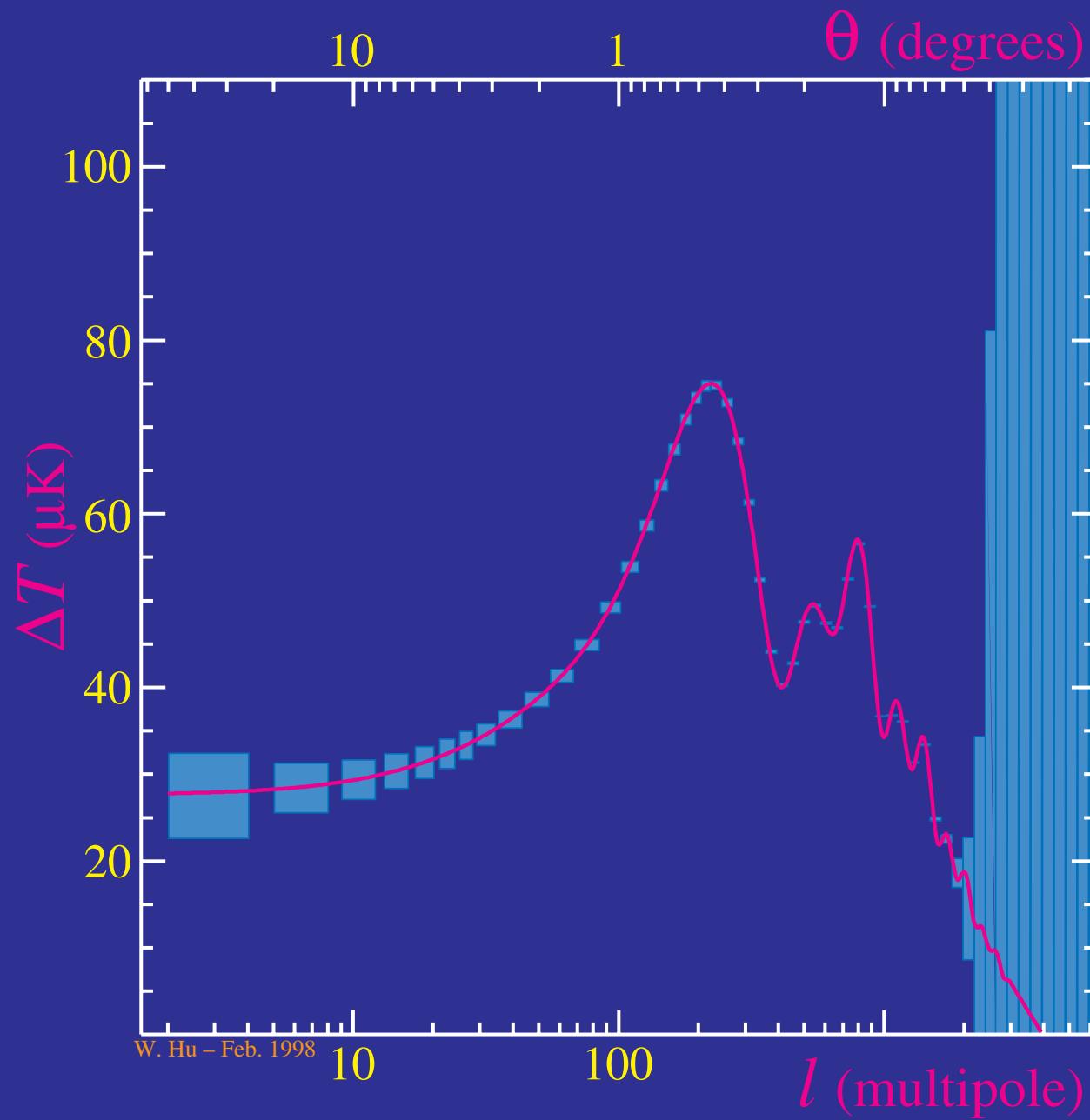
$\Omega_m h^2$	$\Omega_b h^2$	m_v	Ω_Λ	Ω_K	τ	n_S	T/S	A
0.029	0.0026	0.76	1.0	0.29	0.64	0.11	0.45	1.2

Projected MAP Errors



W. Hu – Feb. 1998

Projected Planck Errors



Degeneracies

- Multiple cosmological parameters have (nearly) degenerate effects on the power spectrum
- Example: reionization and gravity waves

Polarization

Why Measure the Polarization?

- Virtues of polarization
 - unlike temperature anisotropies, generated by scattering only
 - tensor field on the sky; carries more info than scalar temperature
- Uses of polarization
 - verify the gravitational instability paradigm: fluctuations present during last scattering: Cole's demon – IC's not acoustic features
 - probe the reionization epoch: remove a leading source of ambiguity (degeneracy) in the temperature power spectrum
 - get higher statistics on the acoustic peaks and their underlying parameters
 - reconstruct the scalar, vector, tensor nature of the perturbations and hence the cosmology even if ab initio models are wrong
 - test inflationary models by measuring the gravity wave amplitude: energy scale and shape of inflaton potential

Polarization

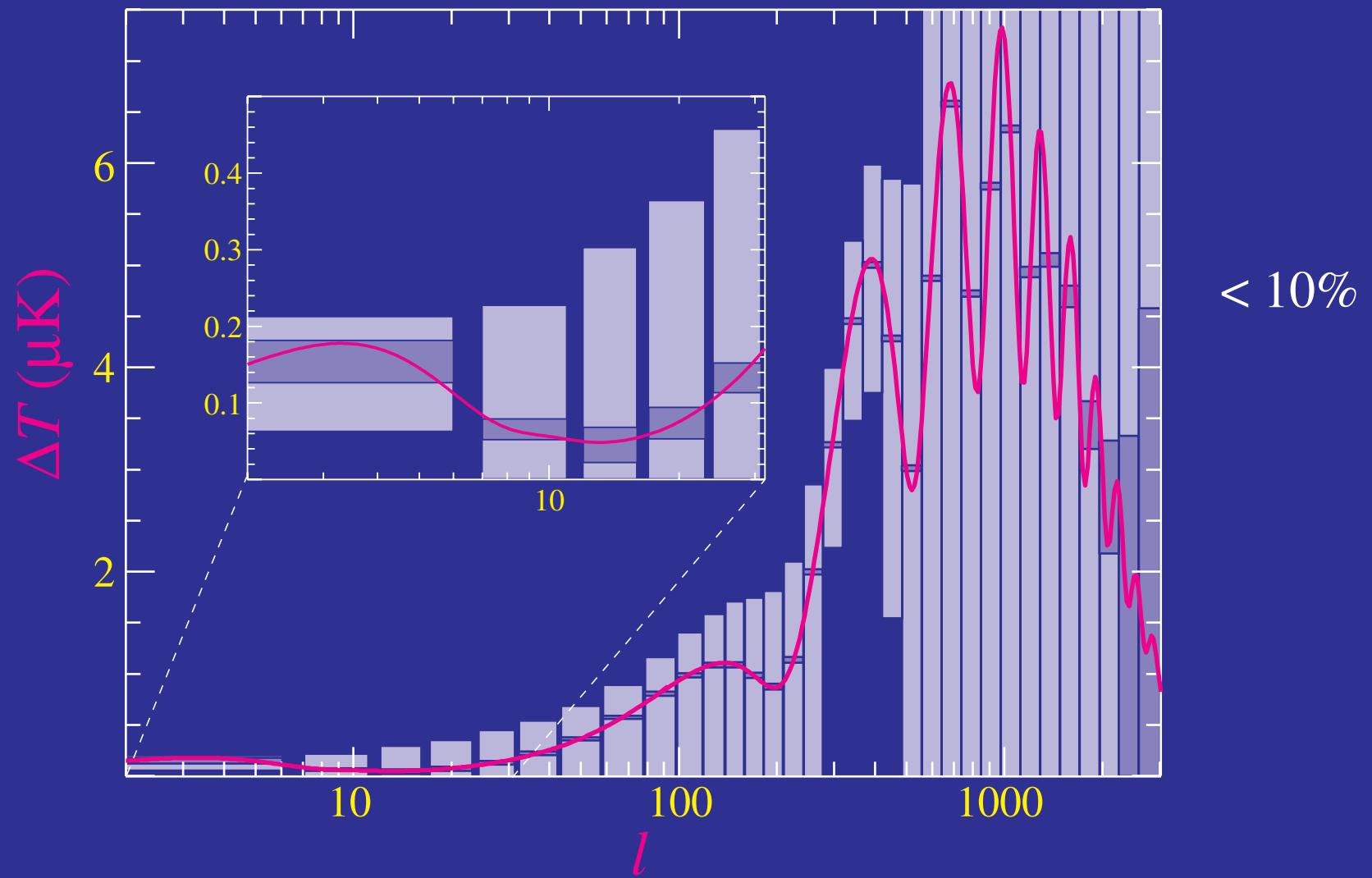
- Thomson of quadrupole temperature anisotropy
- Linear polarization:

Polarization Generation

- Quadrupole anisotropies generated in optically thin regime
- Anisotropies <10% polarized

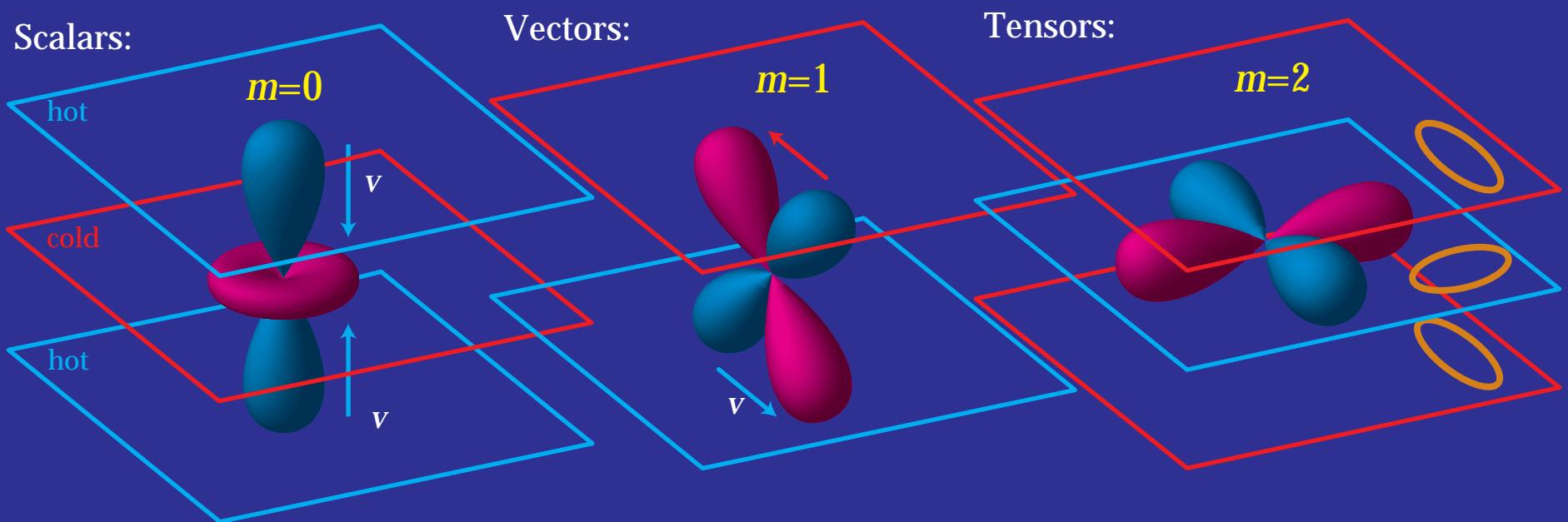
Why Polarization is Difficult

- Source of polarization is the scattering of quadrupole anisotropies
- Rapid scattering destroys quadrupole anisotropies
- Polarization only from the optically thin period before full transparency



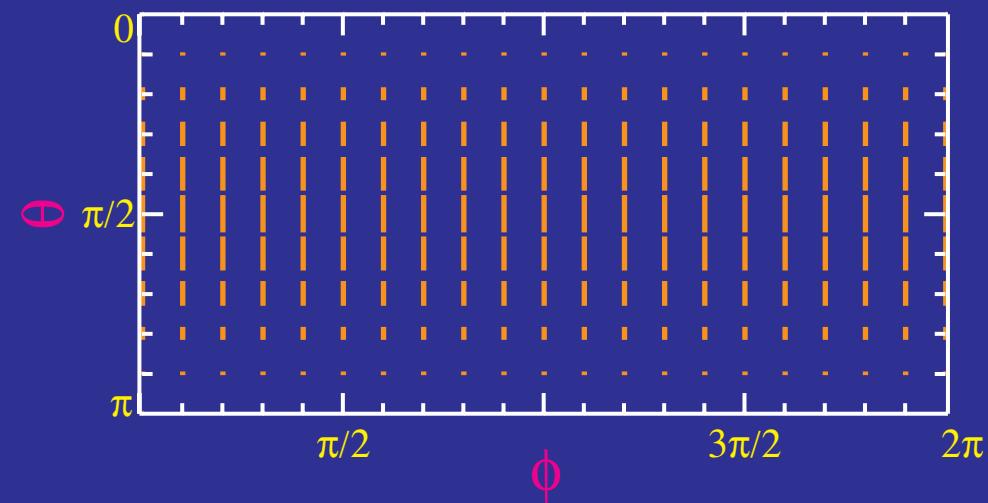
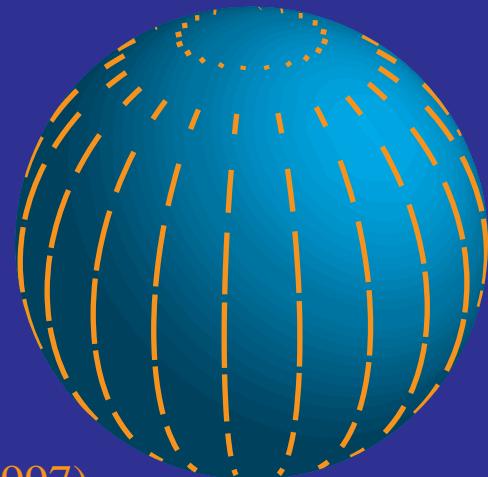
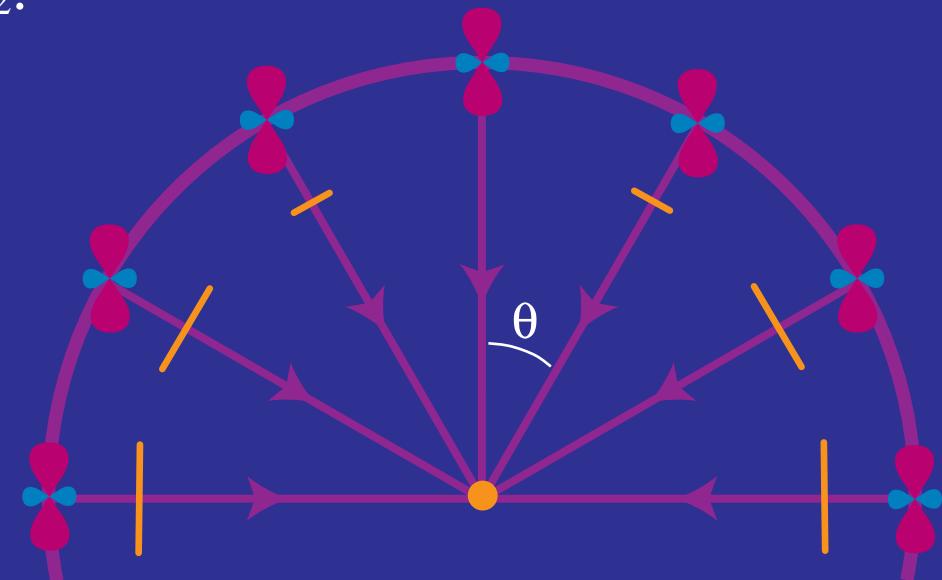
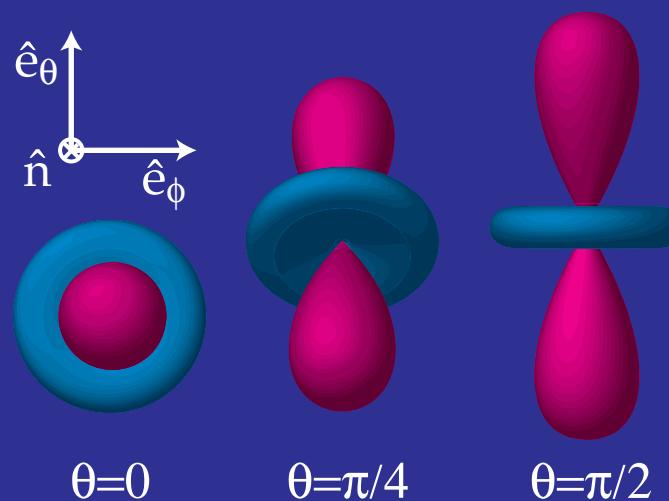
Perturbations & Their Quadrupoles

- Orientation of quadrupole relative to wave (\mathbf{k}) determines pattern
- Scalars (density) $m=0$
- Vectors (vorticity) $m=\pm 1$
- Tensors (gravity waves) $m=\pm 2$



Polarization on the Sphere

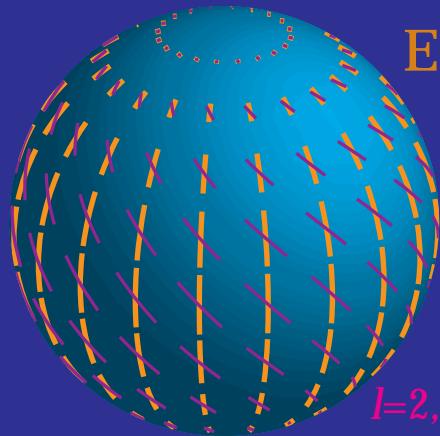
- Polarization direction oriented with the cold lobe of the quadrupole
- A local observer will see a $\sin^2\theta$ pattern of Q -polarization: spin–spherical harmonic: $l=2$, $m=0$, $s=2$: ${}_2Y_2^0$.



Hu & White (1997)

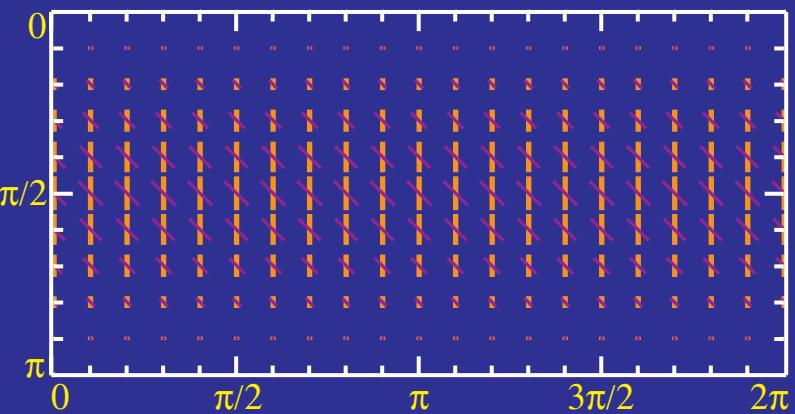
Polarization Patterns

Scalars

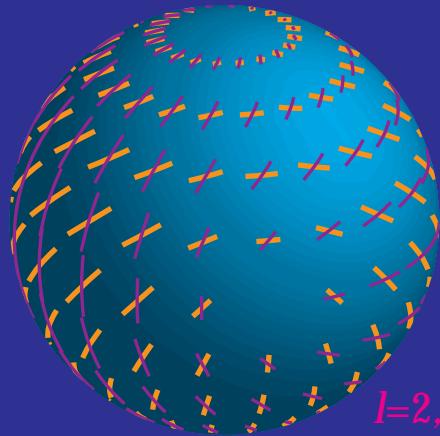


E, B

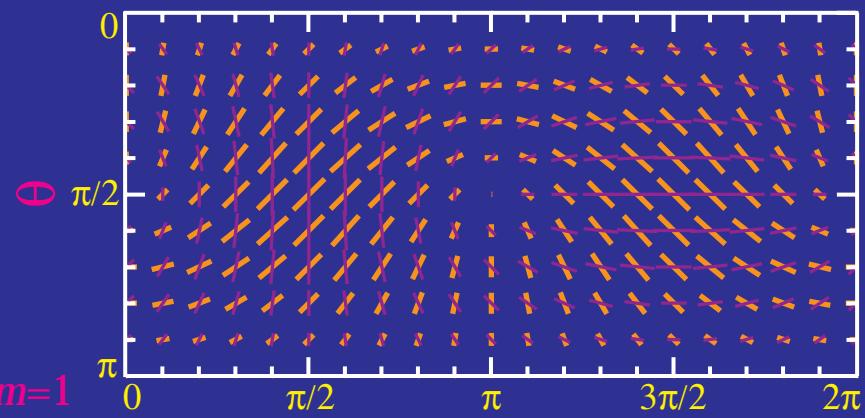
$l=2, m=0$



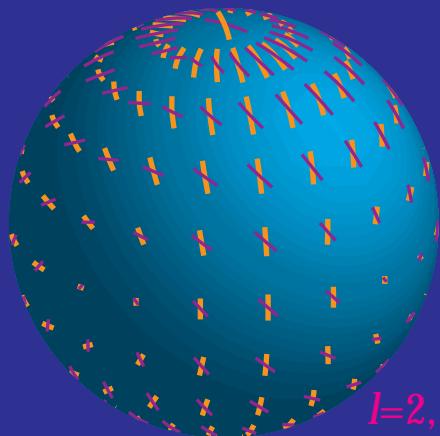
Vectors



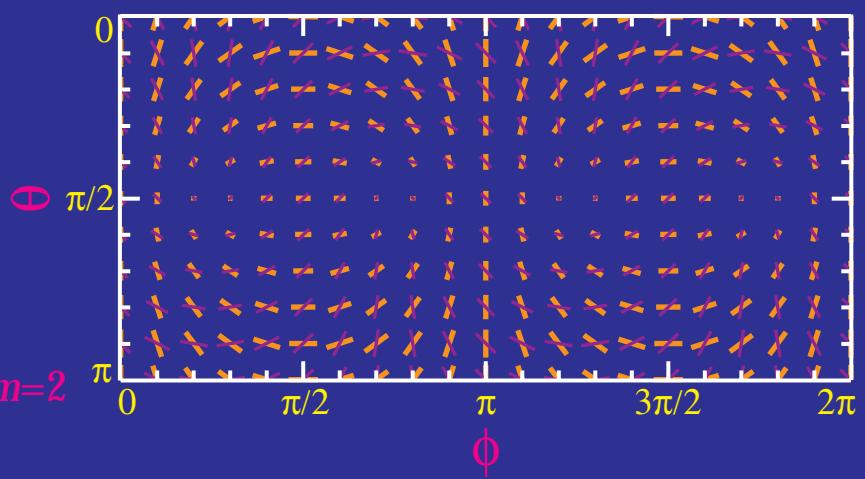
$l=2, m=1$



Tensors

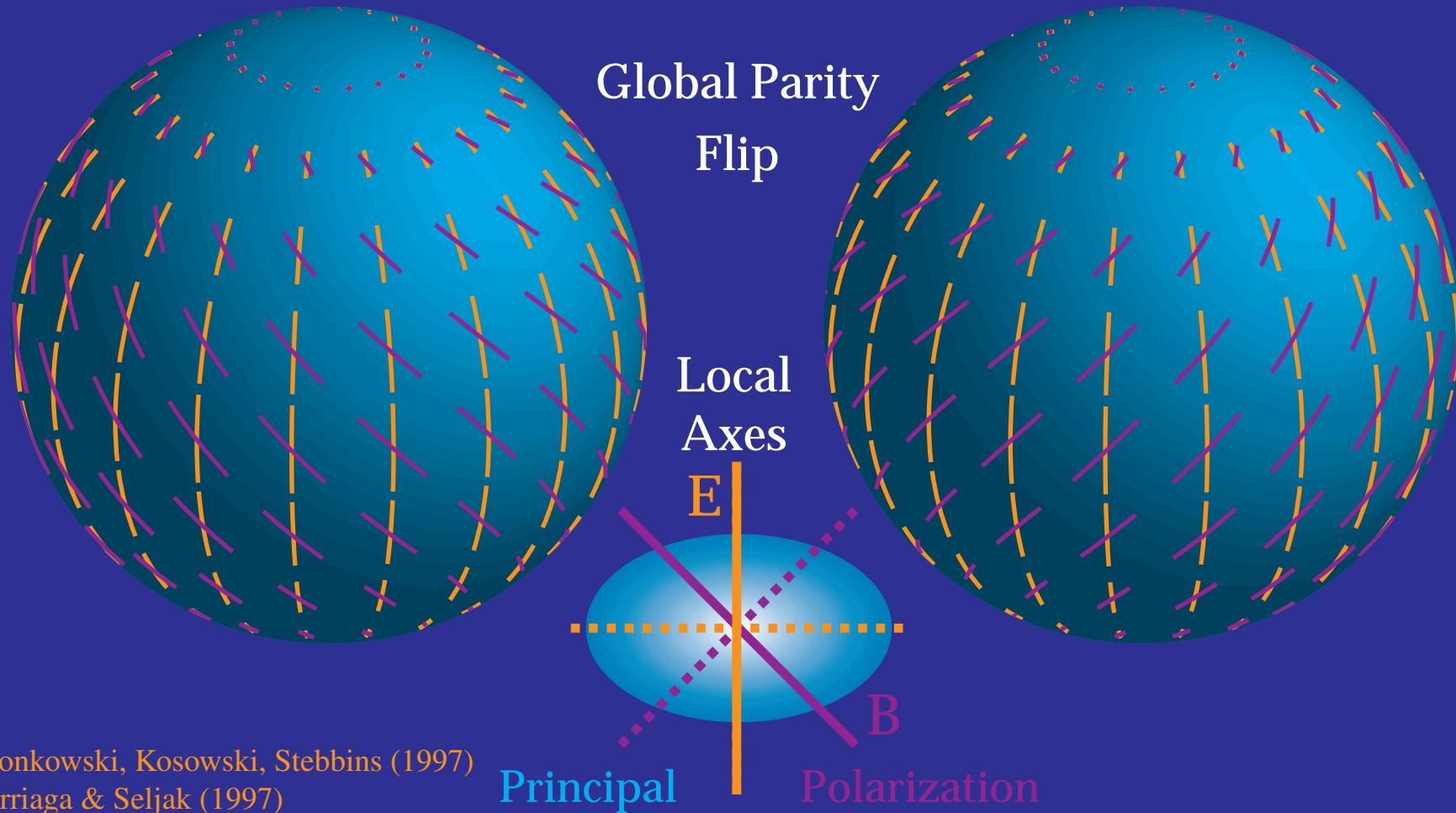


$l=2, m=2$



Electric & Magnetic Patterns

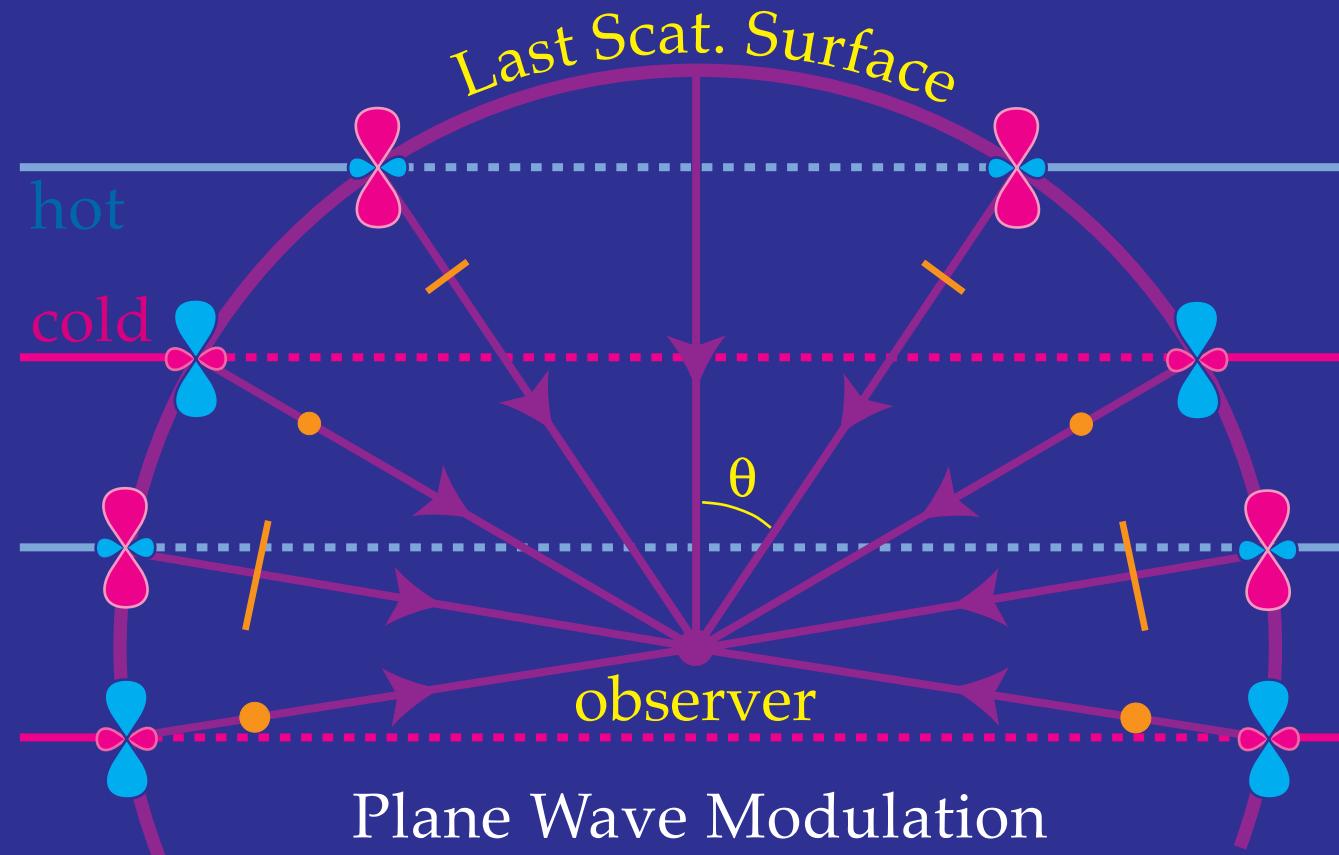
- Global view: behavior under parity
- Local view: alignment of principle vs. polarization axes



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)
Hu & White (1997)

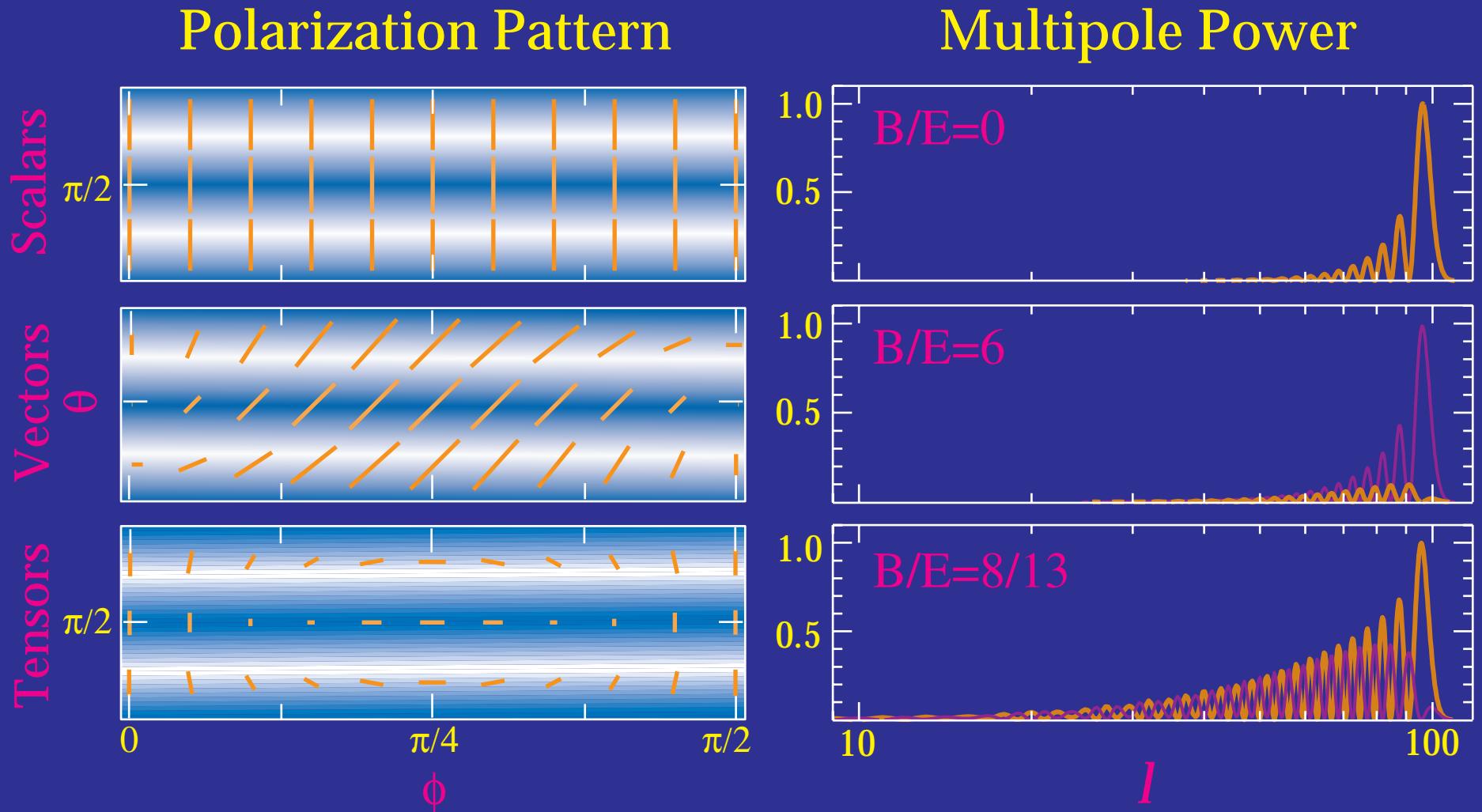
Local vs. Observable Polarization

- Thomson scattering generates a pure E -pattern locally
- Plane wave perturbation modulates the amplitude
- If modulation:
 - in a 0° or 90° direction then E
 - in a 45° direction as polarization then B



Patterns and Perturbation Types

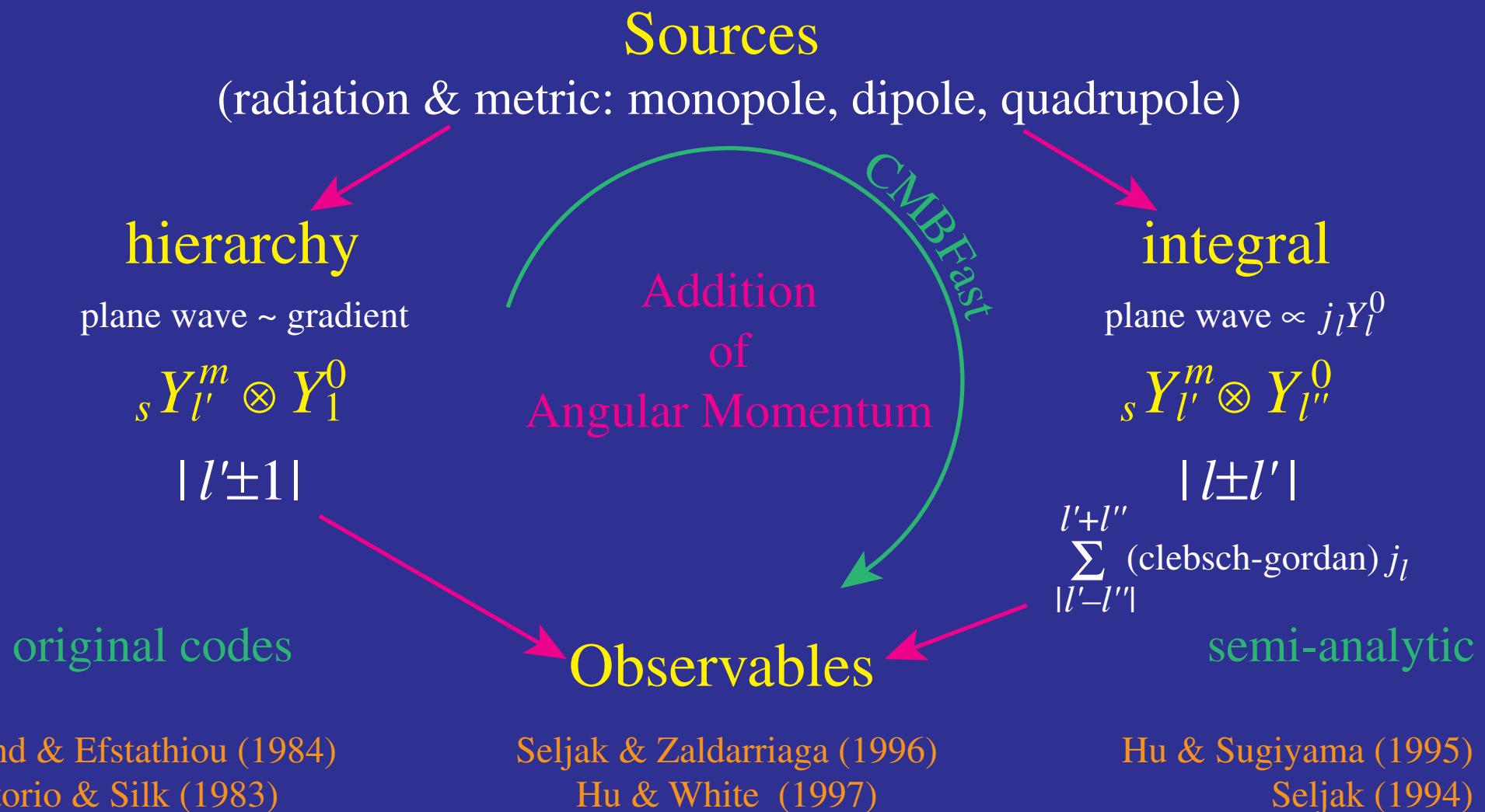
- Amplitude modulated by plane wave → Principle axis
- Direction detemined by perturbation type → Polarization axis



Mechanics of the Calculation

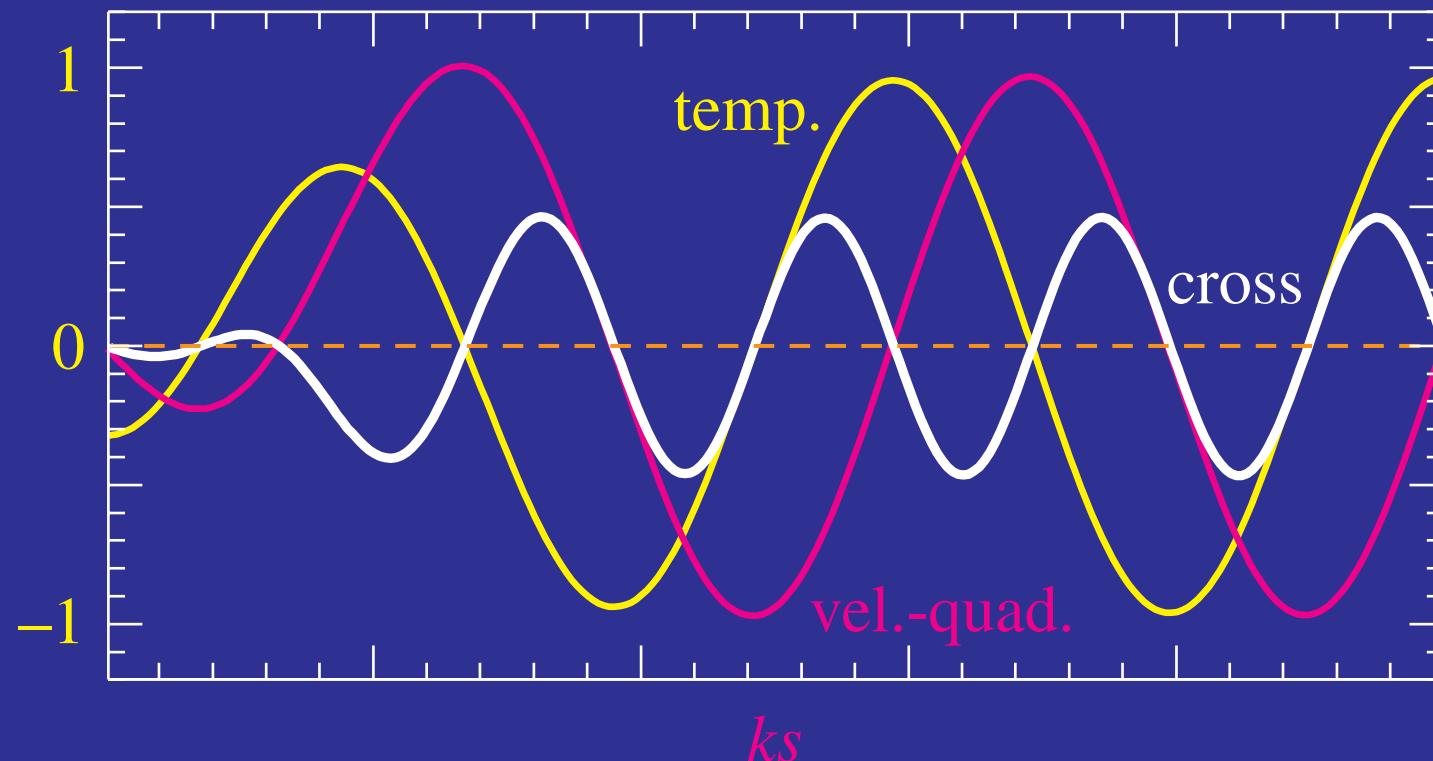
- Radiation distribution: $f(\mathbf{x}=\text{position}, t=\text{time}; \mathbf{n}=\text{direction}, v=\text{frequency})$
- Expand in basis functions: (local angular dependence \otimes spatial dependence)

$${}_s Y_l^m \otimes e^{i\mathbf{k}\cdot\mathbf{x}}$$

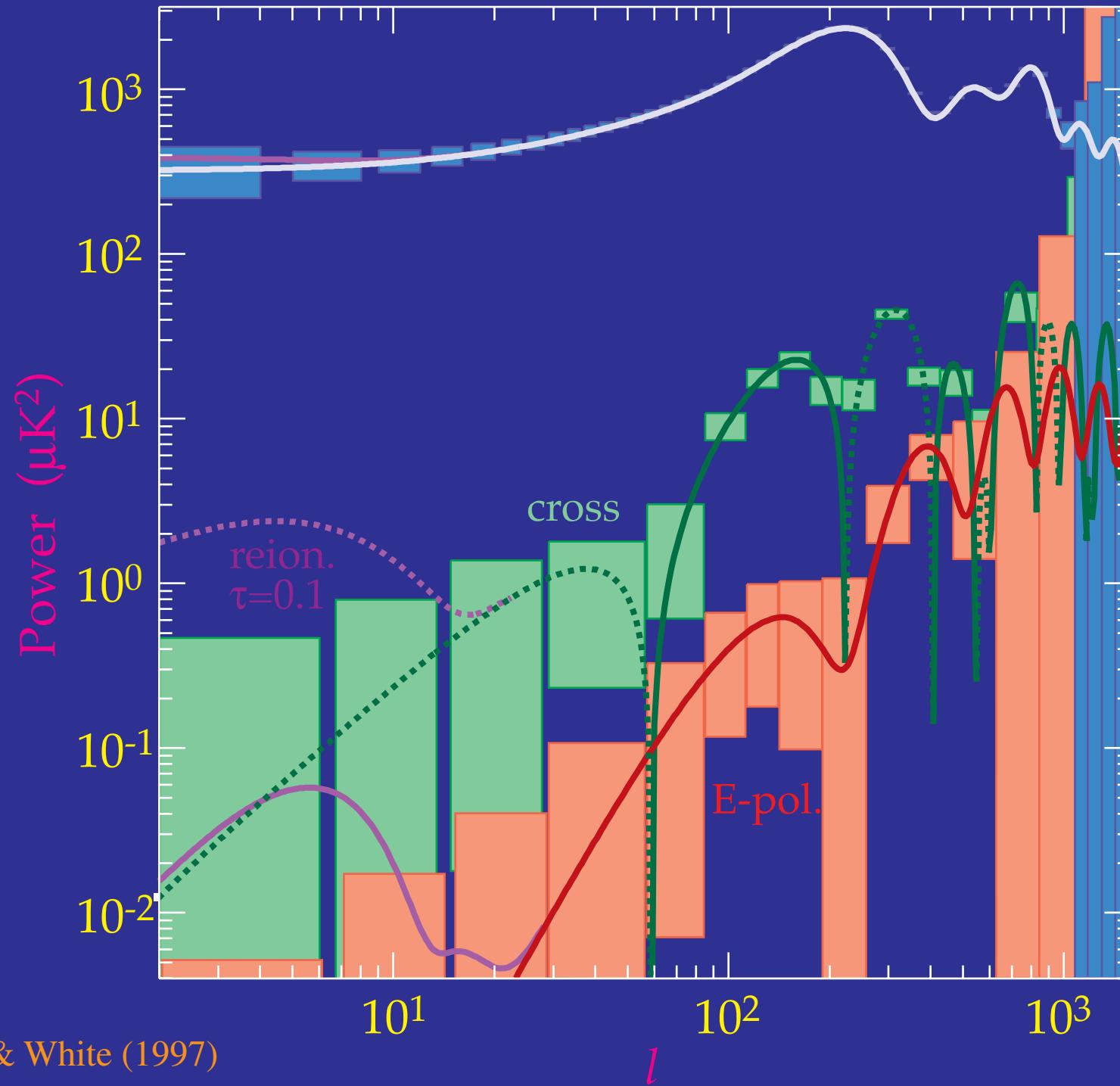


Acoustic Peaks in the Polarization

- Scalar quadrupole follows the velocity perturbation
- Acoustic velocity out of phase with acoustic temperature
- Correlation oscillates at twice the frequency



Scalar Power Spectra



Testing Inflation



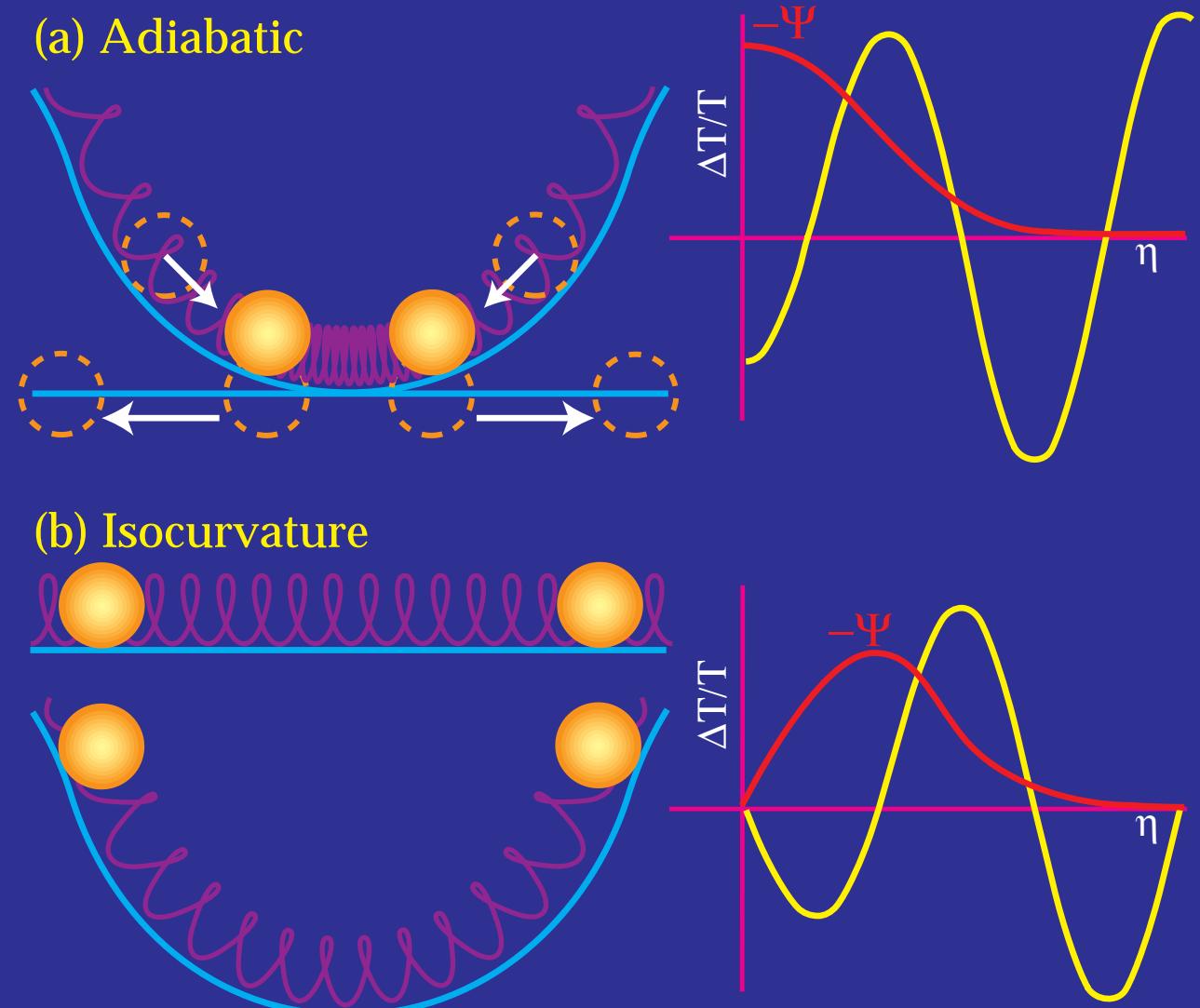
Testing Inflation

- Inflation required to causally carry density fluctuations outside horizon
- Naive test: above 1° , we are looking above the horizon at last scattering hence any power indicates inflation
- Problem: temperature anisotropies can be generated after last scattering
- Solution: find effects confined to last scattering
 - acoustic oscillations: probes potentials just before horizon crossing
 - pros – easy to measure
 - cons – indirect (dynamical assumptions)

polarization: causal scalar fluctuations fall off rapidly (l^6) outside horizon (if first peak right then scalar dominated)
pros – direct
cons – difficult to measure

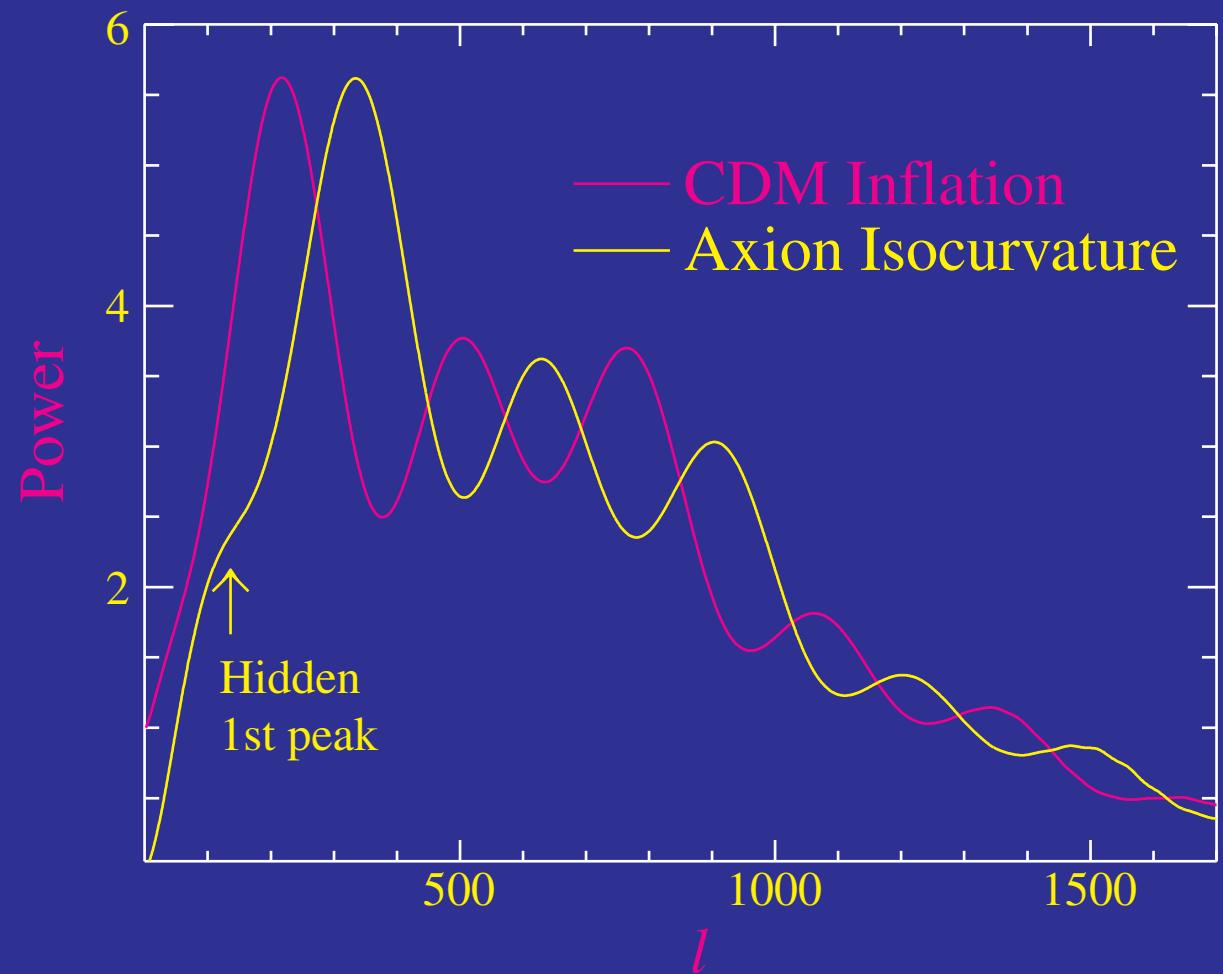
Testing Inflation / Initial Conditions

- Superluminal expansion (inflation) required to generate superhorizon curvature (density) perturbations
- Else perturbations are isocurvature initially with matter moving causally
- Curvature (potential) perturbations drive acoustic oscillations
- Ratio of peak locations
- Harmonic series:
 - curvature 1:2:3...
 - isocurvature 1:3:5...

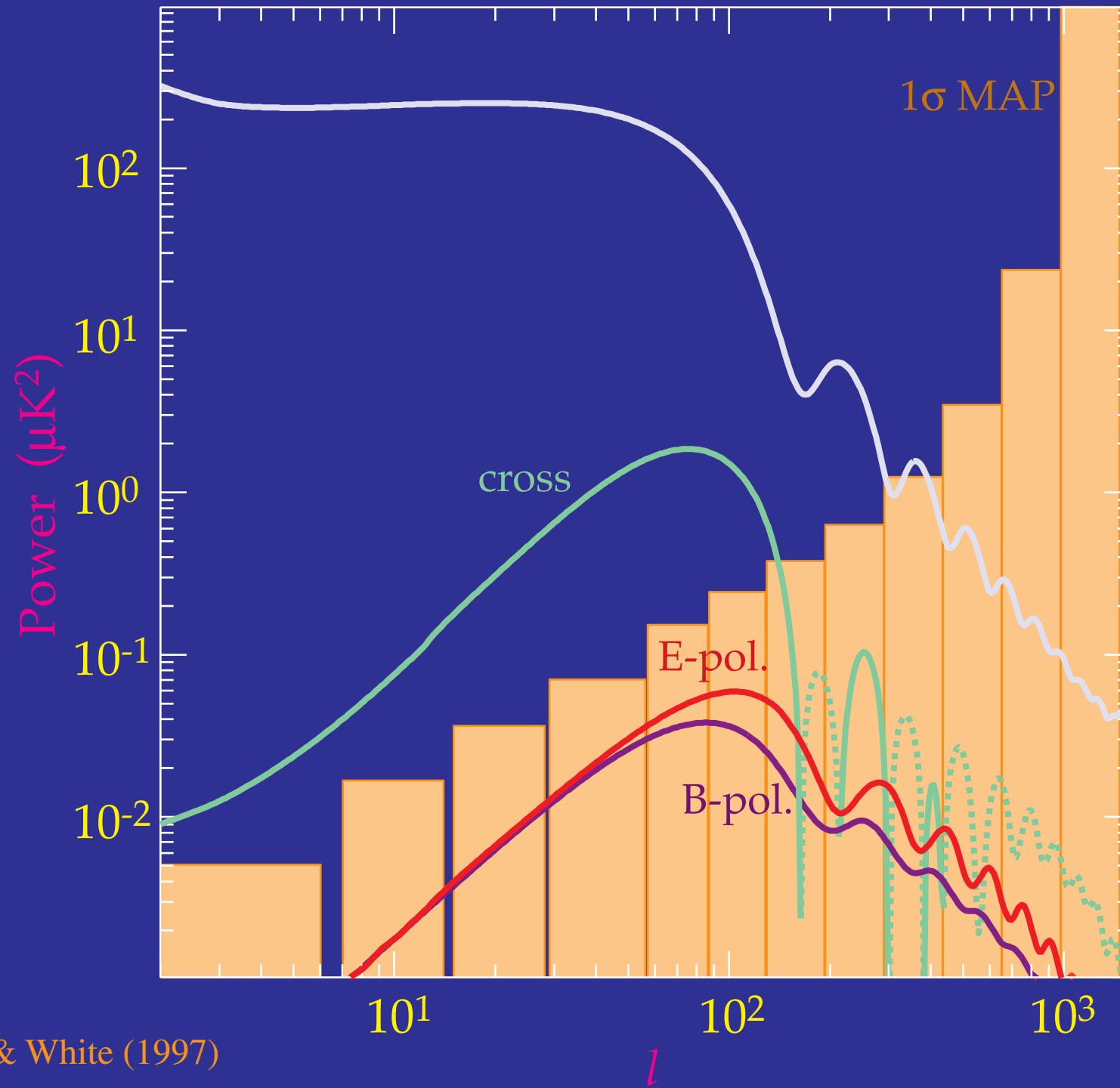


Testing Inflation / Initial Conditions

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curvature 1:2:3...
isocurvature 1:3:5...



Tensor Power Spectra

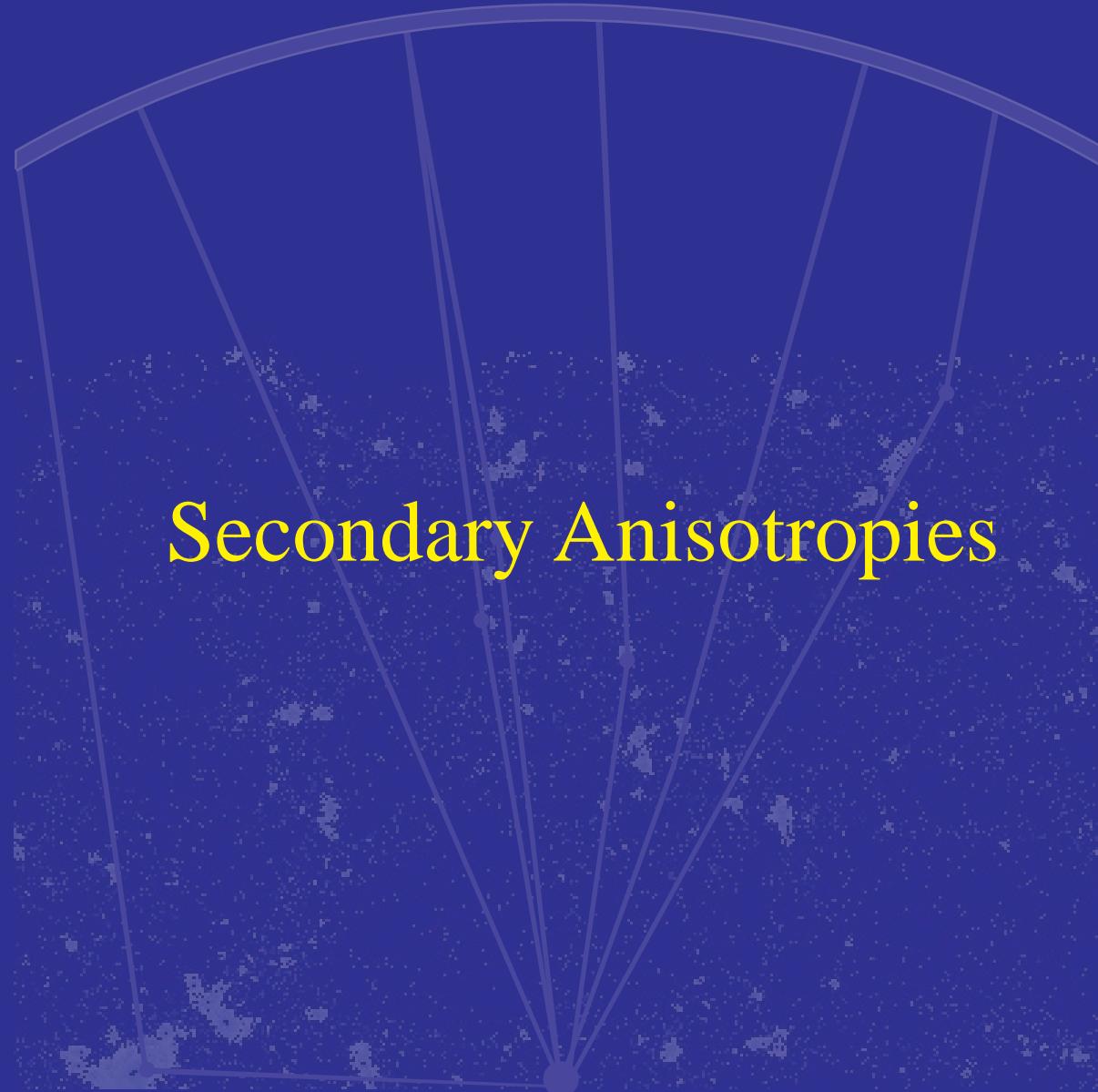


Hu & White (1997)

Inflationary Dynamics

- Tensor Amplitude $\propto V$ (inflaton potential)
- Current upper limits: inflationary energy scale $< 2 \times 10^{16}$ GeV
- Tensor / Scalar amplitude $\propto (V'/V)^2$
Scalar slope function of $(V'/V)^2, (V''/V)$
- Constrain shape, test models of inflation
Planck Errors: $T/S \quad \pm 0.35$ (temp) ± 0.012 (+pol.)
 $n_s \quad \pm 0.04$ (temp) ± 0.008 (+pol.)
- Consistency Relation – test of slow-roll inflation
Tensor slope $\propto (V'/V)^2$
- Meaningful test by Planck only possible if T/S close to current limits
- Next generation detectors (Paul Richards talk)?

Secondary Anisotropies

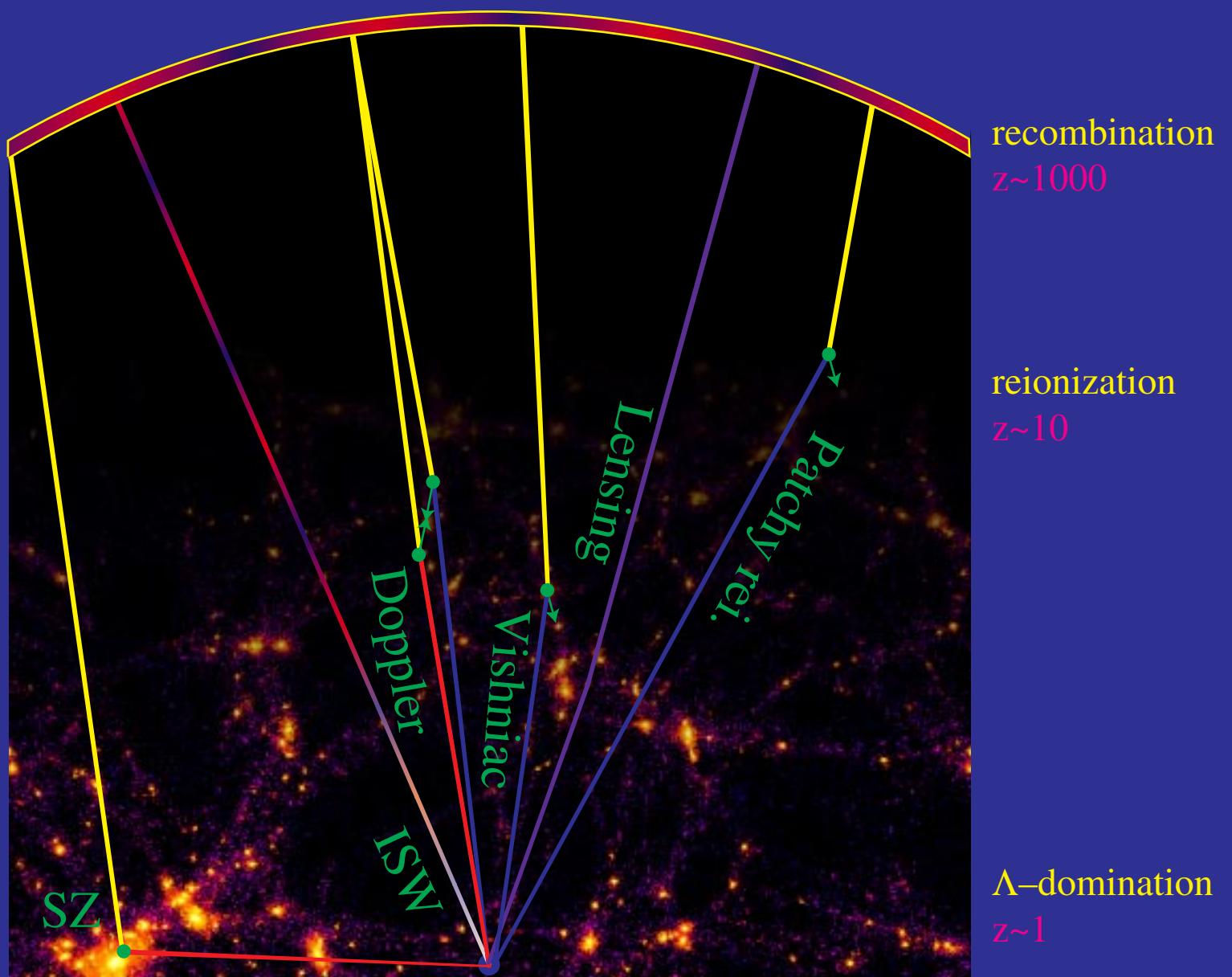


Secondary Anisotropies

- CMB photons **traverse** the large-scale structure of the universe
- Scattering (\sim few%), gravitational **redshift**, lensing

Physics of Secondary Anisotropies

Primary Anisotropies



Secondary Anisotropies: Power Spectra

- Gravitational Effects

 - ISW Effect

 - (redshift from decaying potentials)

 - Weak Lensing

 - (smooths peaks and generates power $<1'$)

- Scattering Effects

 - Doppler Effect

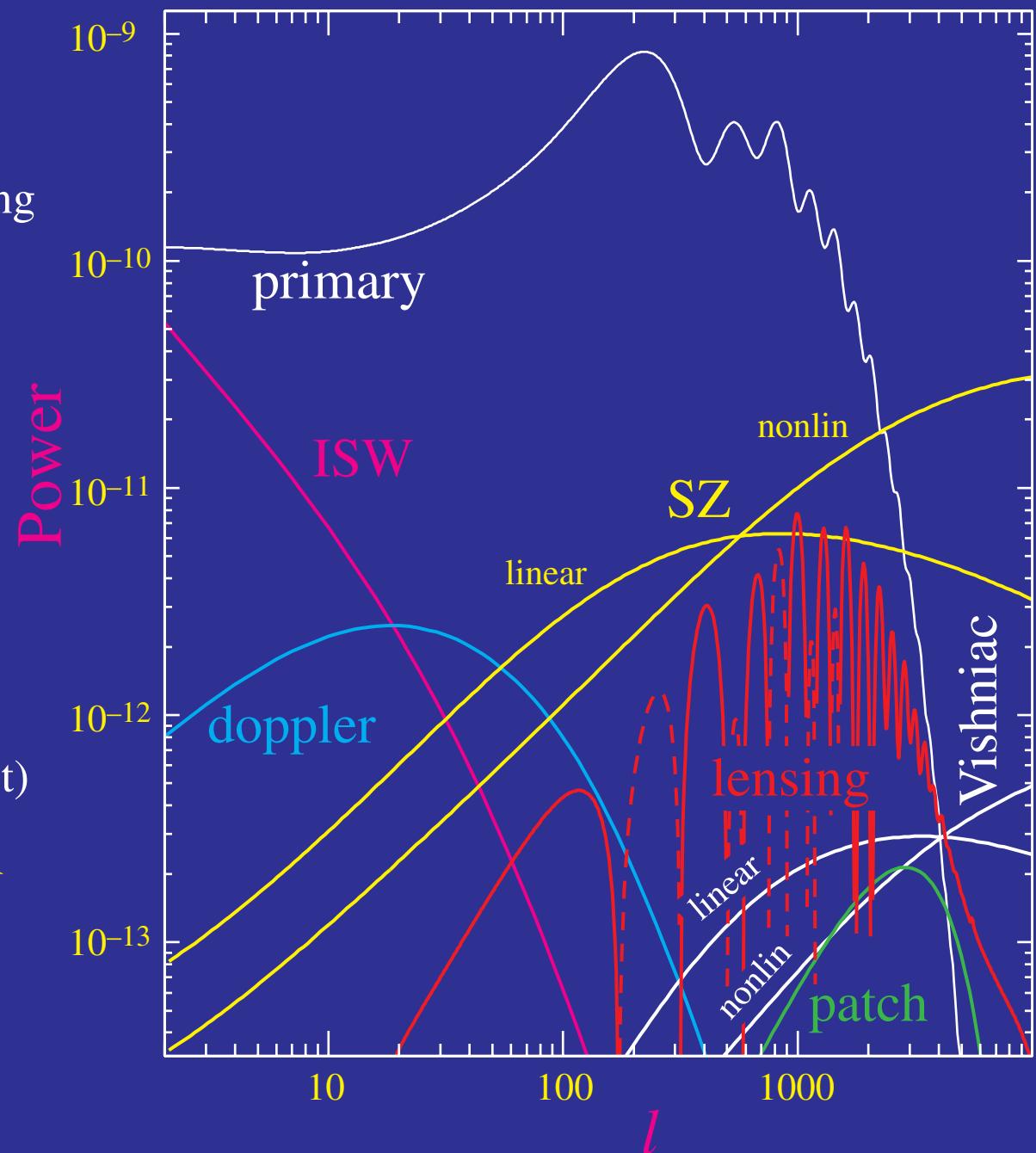
 - Vishniac Effect

 - (LSS kinetic SZ effect)

 - Patchy Reionization

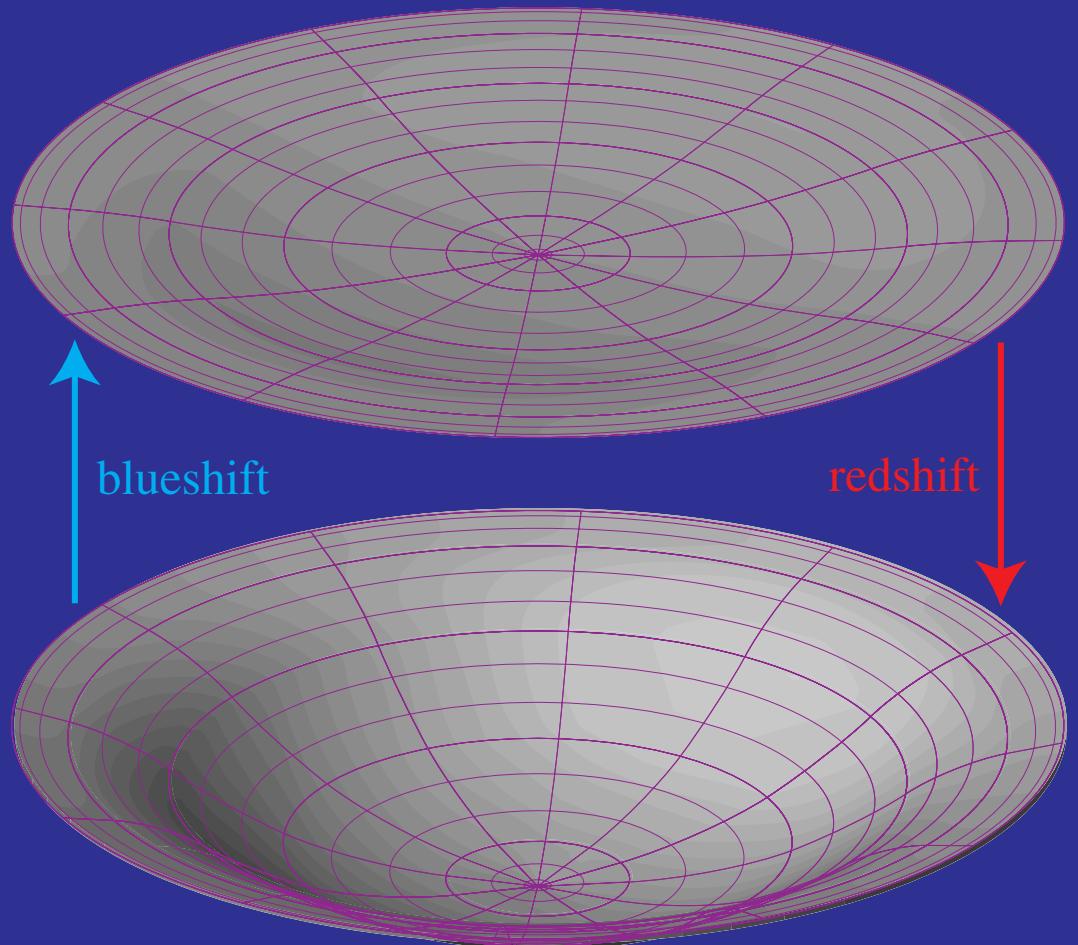
 - SZ effect

 - (LSS thermal)



Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00}=-(1+\Psi)^2 \delta_{ij}$



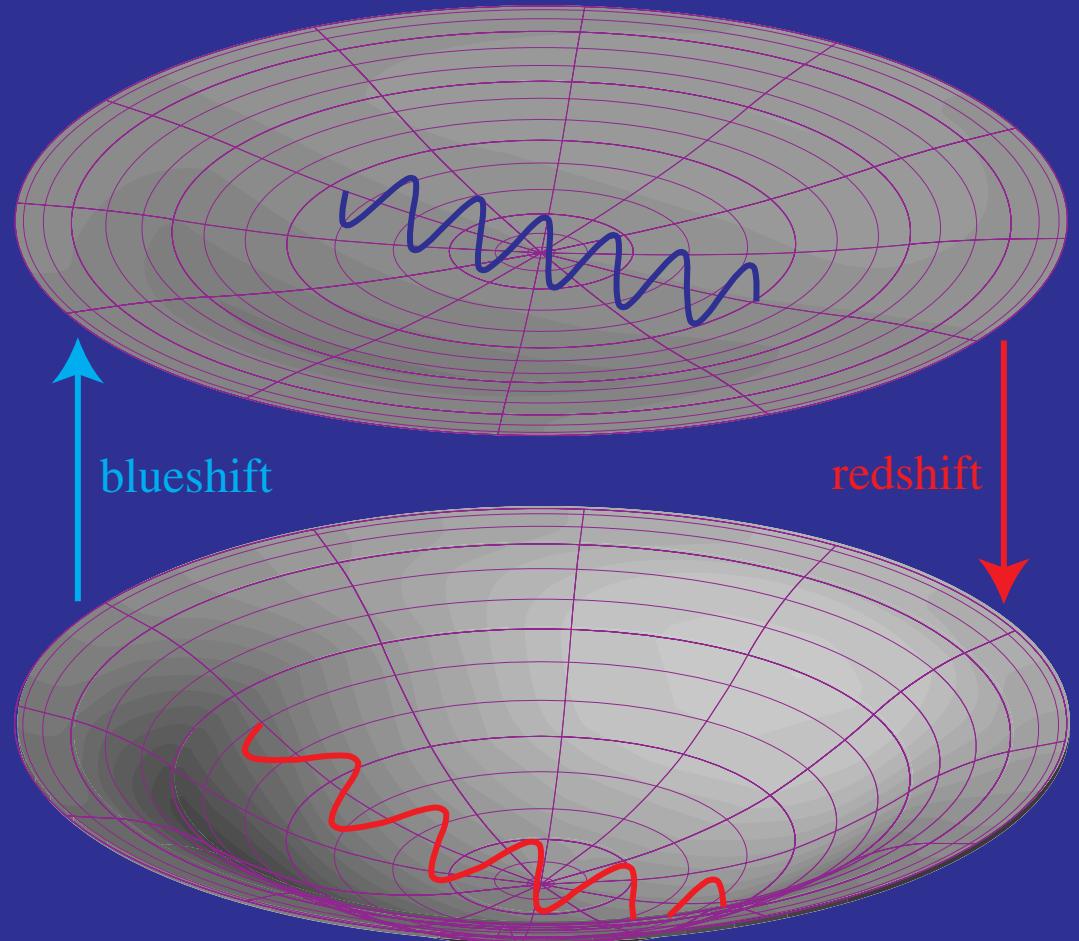
Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00} = -(1+\Psi)^2 \delta_{ij}$

- Perturbed cosmological redshift

$$g_{ij} = a^2(1+\Psi)^2 \delta_{ij}$$

$$\delta T/T = -\delta a/a = \Psi$$



Integrated Sachs–Wolfe Effect

- Potential redshift: $g_{00} = -(1+\Psi)^2 \delta_{ij}$

- Perturbed cosmological redshift

$$g_{ij} = a^2(1+\Psi)^2 \delta_{ij}$$

$$\delta T/T = -\dot{\delta a}/a = \Psi$$

- Time-varying potential

Rapid compared with λ/c

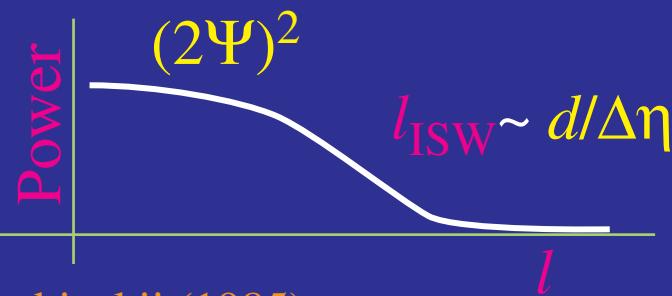
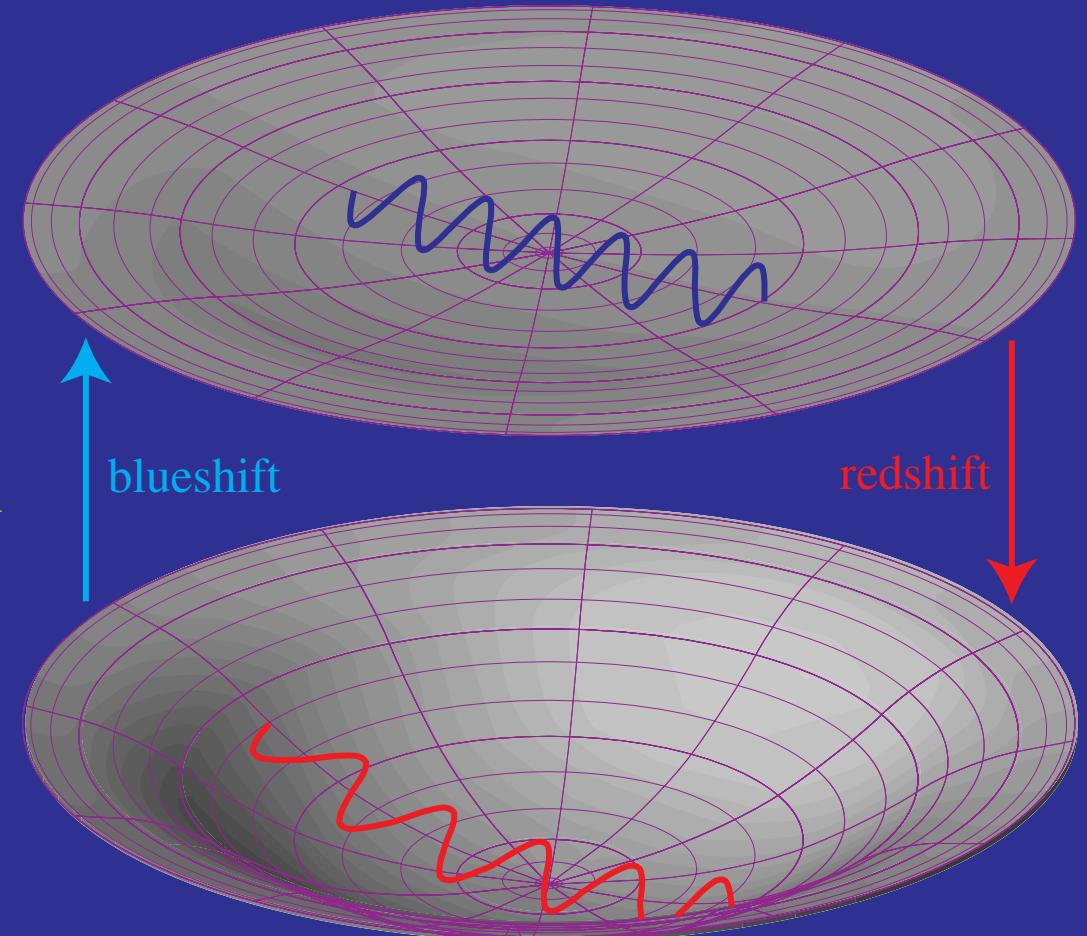
$$\delta T/T = -2\Delta\Psi$$

Slow compared with λ/c

redshift–blueshift cancel

- Imprint characteristic time

scale of decay in angular spectrum



Kofman & Starobinskii (1985)

Hu & Sugiyama (1994)

Calculation of Secondary Anisotropies

- Addition of angular momentum gives

$$\text{multipole moment} = \int \left(\begin{array}{c} \text{clebsch} \\ \text{gordan} \end{array} \right) \left(\begin{array}{c} \text{bessel} \\ \text{function} \end{array} \right) \text{Source} d\left(\begin{array}{c} \text{line of} \\ \text{sight} \end{array} \right)$$

- Primary anisotropies: source sharply peaked at last scattering
Tight Coupling Approximation:

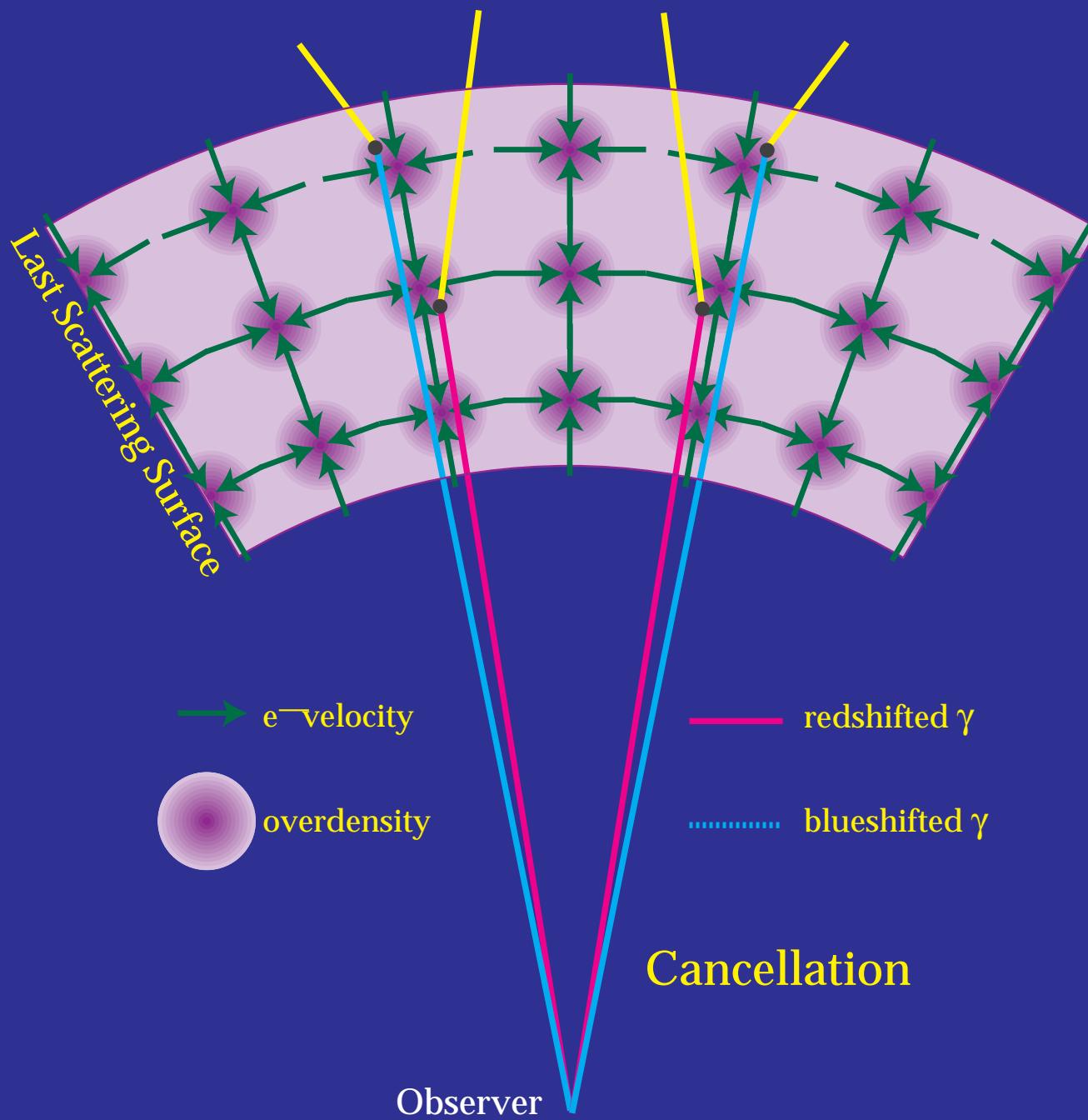
$$\text{multipole moment} \sim \left(\begin{array}{c} \text{clebsch} \\ \text{gordan} \end{array} \right) \left(\begin{array}{c} \text{bessel} \\ \text{function} \end{array} \right) \int \text{Source} d\left(\begin{array}{c} \text{line of} \\ \text{sight} \end{array} \right)$$

- Secondary anisotropies: source slowly-varying in time
Weak Coupling Approximation:

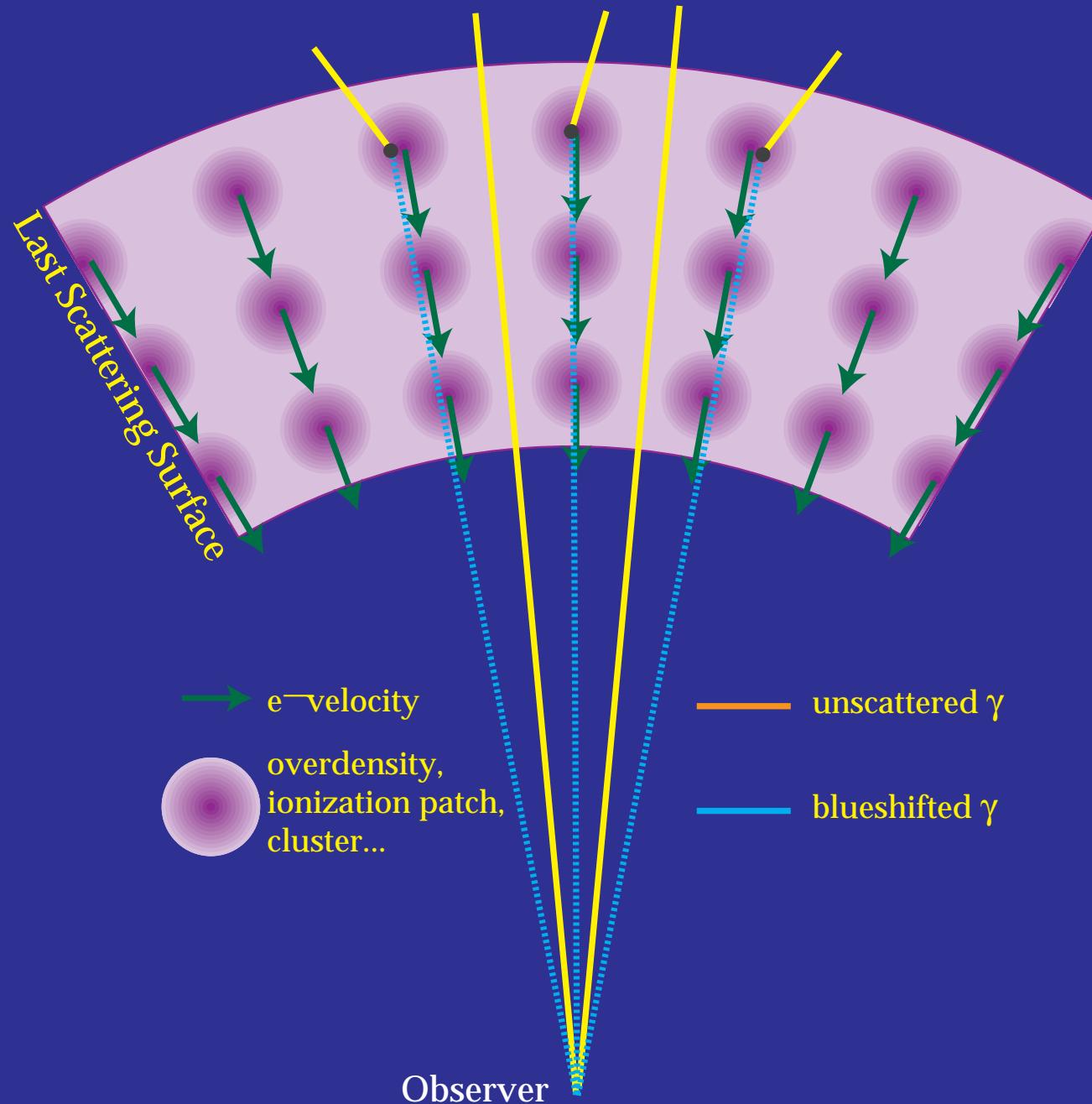
$$\text{multipole moment} \sim \text{Source} \left(\begin{array}{c} \text{clebsch} \\ \text{gordan} \end{array} \right) \int \left(\begin{array}{c} \text{bessel} \\ \text{function} \end{array} \right) d\left(\begin{array}{c} \text{line of} \\ \text{sight} \end{array} \right)$$

- Log power spectrum of CMB $\sim (cg)^*$ Log power spectrum of source / l
- Scalar source and scalar field on sky: weak coupling = limber approx.

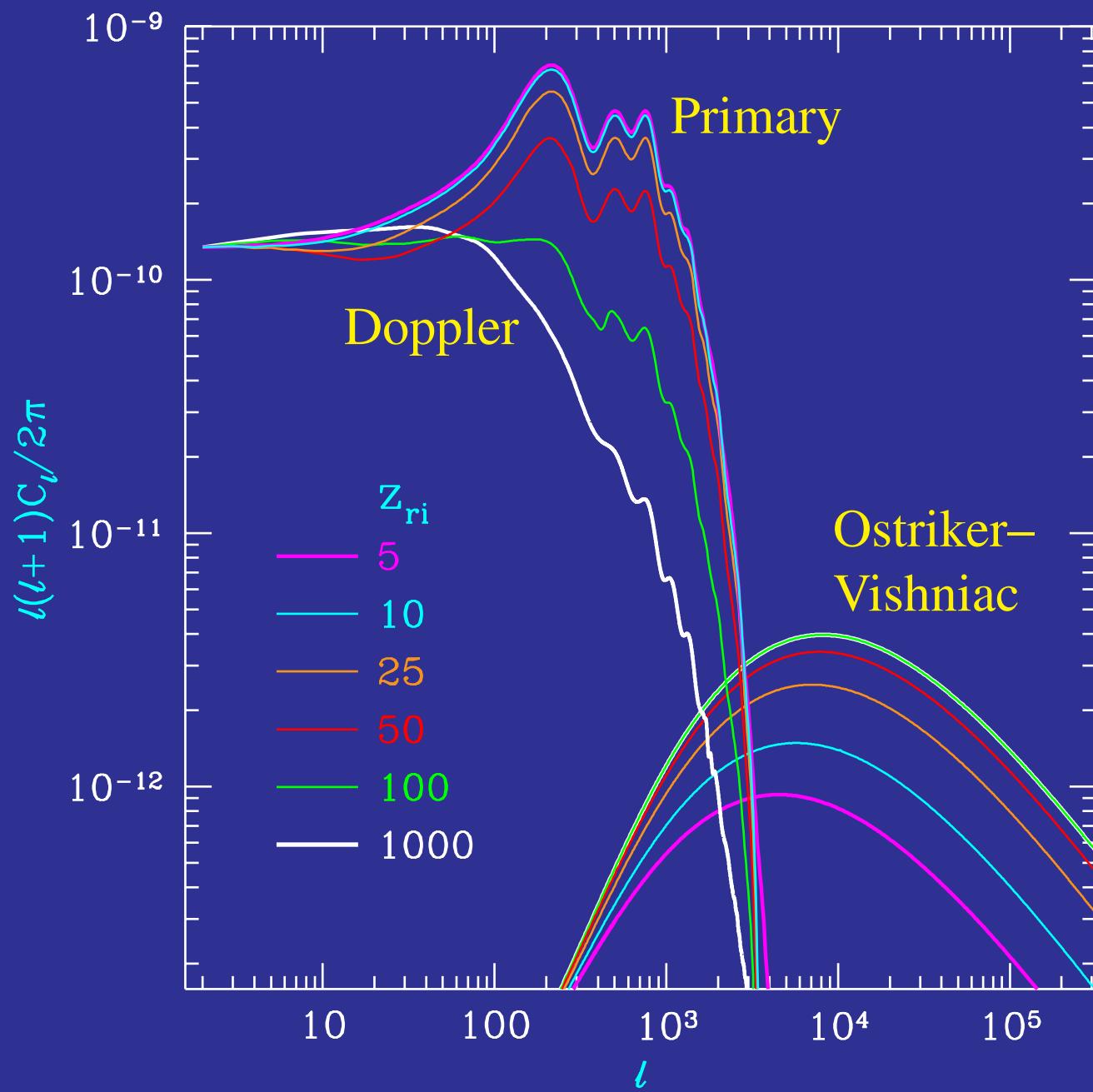
Cancellation of the Linear Effect



Modulated Doppler Effect



Ostriker–Vishniac Effect



<http://background.uchicago.edu>



talk available as a pdf file

Future Directions

- Precision measurements will allow sharp consistency tests of the inflationary CDM paradigm and determination of its underlying parameters
- Polarization can in principle enable probes of the early universe (inflationary dynamics)
 - [challenge to extract from foregrounds and systematic effects]
- Secondary anisotropies probe the evolution of large scale structure in the universe
 - [challenge to separate from primary anisotropies, foregrounds and each other]
spectral signatures, non-gaussian signatures and cross correlation

Index