

# Challenges for Kinetic Unified Dark Matter

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Given that the dark matter and dark energy in the universe affect cosmological observables only gravitationally, their phenomenology may be described by a single stress energy tensor. True unification however requires a theory that reproduces the successful phenomenology of  $\Lambda$ CDM and that requirement places specific constraints on the stress structure of the matter. We show that a recently proposed unification through an offset quadratic kinetic term for a scalar field is exactly equivalent to a fluid with a closed-form barotropic equation of state plus cosmological constant. The finite pressure at high densities introduces a cutoff in the linear power spectrum, which may alleviate the dark matter substructure problem; we provide a convenient fitting function for such studies. Given that sufficient power must remain to reionize the universe, the equation of state today is nonrelativistic with  $p \propto \rho^2$  and a Jeans scale in the parsec regime for all relevant densities. Structure may then be evolved into the nonlinear regime with standard hydrodynamic techniques. In fact, the model is equivalent to the well-studied collisional dark matter with negligible mean free path. If recent observations of the triaxiality of dark matter halos and ram pressure stripping in galaxy clusters are confirmed, this model will be ruled out.

## I. INTRODUCTION

The standard model for cosmology contains dark energy in the form of a cosmological constant and cold dark matter ( $\Lambda$ CDM). While this model explains most observations to date, the dark matter and the dark energy are only observed gravitationally. It thus remains possible that they are a single phenomenon, sometimes called in the literature unified dark matter (UDM).

A candidate for UDM that has been extensively discussed in the literature is the generalized Chaplygin gas. Here the UDM is taken to be a perfect fluid with an equation of state  $p = -A/\rho^\alpha$ , with  $A > 0$  and  $0 < \alpha \leq 1$ ; models with  $\alpha = 1$  [1, 2] are particularly attractive since they represent a Born-Infeld scalar field [3, 4] that admits a string theory interpretation (e.g. [5, 6]). This class of equations of state leads to a *background* evolution that transitions from a matter dominated phase at early times to a de Sitter phase in the asymptotic future in agreement with distance measures [7, 8, 9].

Unfortunately, in linear perturbation theory Chaplygin UDM produces oscillations or exponential growth in the matter power spectrum unless  $\alpha \rightarrow 0$  [10], even after the effect of baryons [11] has been included. The exact phenomenology of these models once nonlinear structures have formed is complicated due to the fact that the pressure decreases with increasing energy density (see e.g. [12, 13]). It is clear, however, that these models differ substantially from the successful  $\Lambda$ CDM model and are unlikely to be viable in their simplest form.

These problems with Chaplygin gas UDM stem from very general considerations and serve as a guide to building more successful UDM models. Fundamentally, a UDM model that mimics the joint stress energy tensor of  $\Lambda$ CDM will reproduce all of its observational successes [14]. Since the UDM stress-energy tensor is covariantly conserved, mimicry in the spatial stresses is all that need

be explicitly enforced [15].

Deviations from perfect mimicry must be small in a very specific sense. The basic requirement for mimicry is that spatial fluctuations in the UDM stresses be negligible in comparison to those in its energy density  $\delta\rho$ , even though the homogeneous or background stress must be of order  $-\rho$  to accelerate the expansion [10, 14]. More specifically, the ratio between the fluctuating stress and the energy density must be of order the *square* of that between the momentum and energy densities, or a non-relativistic velocity squared. This disparity implies that the total pressure is effectively decoupled from the total density, i.e. that the pressure is effectively non-adiabatic [10, 16, 17]. For a generalized Chaplygin gas this requirement is only satisfied as  $\alpha \rightarrow 0$ .

In the linear regime, where the momentum density itself is a perturbation, fluctuations in the spatial stresses must then be a second order effect quantified by a vanishingly small sound speed and anisotropic stress. The internal dynamics of dark matter halos place even more stringent requirements on mimicry. Here the UDM stress energy tensor must embed the information from multiple momentum streams in the dark matter, i.e. the full phase space structure of cold dark matter. A successful UDM model will be at least as complex as  $\Lambda$ CDM in order to reproduce its rich phenomenology detracting somewhat from its appeal.

The prototype for a successful UDM model is a scalar field coherently oscillating in an offset quadratic potential [14]; it satisfies mimicry requirements all the way down to the de-Broglie wavelength of the field [18]. It does not, however, explain the relationship between the  $10^{-33}$  eV flatness of the offset scale and the  $> 10^{-22}$  eV mass scale.

Recently, Scherrer [19] introduced a  $k$ -essence [20] variant of scalar field UDM that also succeeds in linear theory. Here the quadratic potential is replaced by a quadratic kinetic term (see [21] for a possible physical

motivation). An important feature of this model is that its stress-energy tensor is purely kinetic (see [22] for a generalization). We shall show that its equation of state is consequently both barotropic and expressible in closed form. We are consequently able to reduce this kinetic form of UDM to collisional dark matter with a negligible mean free path. Non-linear structure formation for this case has been well-studied in the literature [23, 24].

The outline of the paper is as follows. We begin in §II with a brief review of the basic properties of scalar field UDM. We derive an exact, fully non-linear, equation of state for the purely kinetic UDM model [19] in §III. We then calculate the evolution of linear density fluctuations in §IV. We place constraints on the model from the suppression of the low mass dark matter halo abundance in §V and discuss opportunities and challenges for the model from halo substructure in §VI. We conclude in §VII.

Throughout we will compare the UDM model to a  $\Lambda$ CDM model with a total non-relativistic matter density  $\Omega_m h^2 = \Omega_c h^2 + \Omega_b h^2 = 0.14$ , baryon density  $\Omega_b h^2 = 0.024$ , in a flat cosmology with  $\Omega_\Lambda = 0.73$  and scale invariant adiabatic initial conditions.

## II. SCALAR FIELD UDM

In this section, we briefly review the relationship between scalar fields and perfect fluids and show why scalar fields whose potential and kinetic terms possess a non-vanishing minimum value satisfy the basic requirements of a successful UDM model. The minimum acts as a decoupled source of negative pressure with an equation of state  $p/\rho = -1$  for the background that does not contribute to stress fluctuations.

The stress energy tensor of a classical scalar field  $Q$  with a Lagrangian  $\mathcal{L}$  and kinetic term

$$X = -\frac{1}{2}\nabla_\mu Q \nabla^\mu Q \quad (1)$$

is given by

$$T^\mu_\nu = \mathcal{L}_{,X} \nabla^\mu Q \nabla_\nu Q + \mathcal{L} \delta^\mu_\nu. \quad (2)$$

It takes the form of a perfect fluid

$$T^\mu_\nu = (\rho + p)U^\mu U_\nu + p\delta^\mu_\nu, \quad (3)$$

with the association of a 4-velocity [25]

$$U_\mu = \frac{\nabla_\mu Q}{(2X)^{1/2}}, \quad (4)$$

which is timelike for the nearly homogeneous cosmological initial conditions. We shall show in §VI that it remains so throughout structure formation. Here, the proper energy density and pressure, or energy density and isotropic stress as measured by an observer comoving with the “fluid”, is

$$\rho = 2X\mathcal{L}_{,X} - \mathcal{L}, \quad p = \mathcal{L}. \quad (5)$$

Although the mapping from a scalar field to a perfect fluid is exact, the fluid equations are not closed without a specification of  $\mathcal{L}$  to provide the equation of state.

Scalar fields provide fertile ground for unified dark matter models. When combined, the kinetic and potential terms possess sufficient degrees of freedom to alter the equation of state of the fluctuations separately from the background. Both the potential and the kinetic versions of UDM posit a separable Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_m, \quad (6)$$

where  $\mathcal{L}_e$  and  $\mathcal{L}_m$  behave respectively as dark energy and dark matter. Mimicry of a cosmological constant is possible if  $\mathcal{L}_e$  is constant or, more generally, slowly varying with the field degrees of freedom in comparison to the matter piece  $\mathcal{L}_m$ . In what follows, we shall assume that the former case holds and set

$$-\mathcal{L}_e = \rho_\Lambda = \frac{3H_0^2 \Omega_\Lambda}{8\pi G}. \quad (7)$$

The stress energy tensors  $\mathcal{L}_e$  and  $\mathcal{L}_m$  are then separately conserved. Other splittings are of course possible (e.g.  $\rho_e = -\mathcal{L}$  [4]) but all choices yield the same observable predictions. Our choice simplifies the calculation in that the dark matter and dark energy pieces interact only through gravity.

The prototype for this sort of unification is an axion-like matter component

$$\mathcal{L}_m = X - m^2(Q - Q_{\min})^2 \quad (8)$$

in a potential whose minimum gives the dark energy density  $\mathcal{L}_e = -V(Q_{\min})$ . We will refer to this option as potential UDM (pUDM).

Scherrer [19] introduced a similar but alternate ansatz based on a modified kinetic term

$$\mathcal{L}_m = F(X - X_{\min})^2; \quad (9)$$

we call this kinetic UDM (kUDM). The constant  $F$  can be eliminated in favor of the average matter density and kinetic term

$$\bar{\rho}_m = F X_{\min}^2 \epsilon(4 + 3\epsilon), \quad \bar{\rho}_{m0} = \frac{3H_0^2 \Omega_c}{8\pi G}, \quad (10)$$

where  $\epsilon = (\bar{X} - X_{\min})/X_{\min}$  and “0” denotes evaluation at the present epoch.

The fine tuning between the mass and offset scale in pUDM discussed in §I still exists in kUDM as the coincidence problem  $\bar{\rho}_{m0} \sim \rho_\Lambda$ . We shall see that kUDM also requires  $\epsilon_0 \lesssim 10^{-18}$  (see §V), which is similar to the pUDM requirement of a purely quadratic potential near the minimum or the neglect of self interaction terms in the potential.

Nonetheless, both the pUDM and kUDM model satisfy the basic requirement for a successful UDM model in that their Lagrangian reaches a non-vanishing minimum value  $\mathcal{L}_e$  that can serve as a dark energy component that cannot be spatially perturbed.

### III. EQUATION OF STATE

Given the mapping between scalar fields and perfect fluids described in the previous section, the dynamics of the kUDM model are determined by its equation of state, the relationship between its pressure and energy density in its rest frame. An important property of this purely kinetic model is that there is only one degree of freedom, the kinetic term  $X$ , and therefore the equation of state is barotropic to all orders in the density fluctuation, i.e.  $p_m$  is a function of  $\rho_m$  only.

We begin with the background equation of state

$$w_m(a) \equiv \frac{\bar{p}_m}{\bar{\rho}_m} = \frac{\epsilon(a)}{4 + 3\epsilon(a)}. \quad (11)$$

To obtain  $\epsilon(a)$  note that energy conservation provides an explicit relation for

$$a(\epsilon) = \left[ \frac{\epsilon^2(1 + \epsilon)}{\epsilon_0^2(1 + \epsilon_0)} \right]^{-1/6}. \quad (12)$$

A convenient fitting formula for the required inverse relation

$$\epsilon(a) \approx \epsilon_0 a^{-3} \left\{ 1 + \left[ (1 + \epsilon_0^{-1})^{1/3} a \right]^{-\gamma} \right\}^{-1/\gamma}, \quad (13)$$

achieves  $\sim 1\%$  accuracy with  $\gamma = 5/2$ . Thus, the ‘‘matter’’ component behaves as radiation in the background for  $a \ll \epsilon_0^{1/3}$  and non-relativistic dark matter thereafter.

Since the matter component depends only on  $X$ , the pressure is adiabatic and the fully non-linear equation of state

$$p_m(\rho_m) = \rho_m \left[ \frac{1}{3} + \frac{2}{3f} (1 - \sqrt{1+f}) \right], \quad (14)$$

$$f = \frac{3\rho_m}{4\bar{\rho}_m} \epsilon(4 + 3\epsilon),$$

is barotropic. The adiabatic sound speed

$$\frac{dp_m}{d\rho_m} = \frac{p_{m,X}}{\rho_{m,X}} = \frac{1}{3} - \frac{1}{3} \frac{1}{\sqrt{1+f}}, \quad (15)$$

then quantifies the pressure response to a density fluctuation in both the linear and non-linear regimes. Note that the pressure is a monotonically increasing function of the energy density. Hence, unlike the Chaplygin gas model [12, 13], the matter cannot be destabilized by the appearance of high density fluctuations. We will exploit this fact in §VI in calculating the abundance of collapsed objects.

Note that the simplification of a purely kinetic UDM field is an important feature in the model that is critical in the sections that follow. In the more general case (including pUDM), where  $\mathcal{L}$  is a function of both  $Q$  and  $X$ , the pressure fluctuations  $p(X, Q)$  cannot be expressed as a function of  $\rho$ , except in the background. In particular,

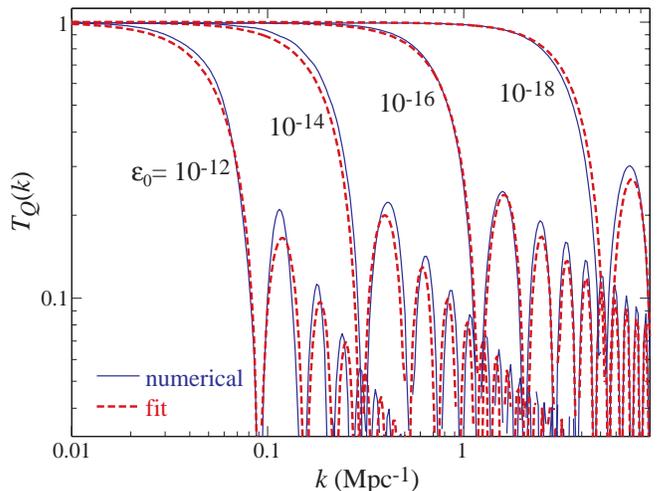


FIG. 1: Modification to the matter transfer function introduced by kUDM as a function of  $\epsilon_0$ , which quantifies the deviation of  $\bar{X}$  from  $X_{\min}$  today. The fluctuations oscillate acoustically and decay at small scales in a manner that can be conveniently fit by Eqn. (18).

they may be highly time variable. Regardless, the scalar field convention is to define the effective sound speed as that of the kinetic degree of freedom [15, 25], i.e.

$$c_e^2 \equiv \left. \frac{\delta p}{\delta \rho} \right|_{U_i=0} = \frac{p_{,X}}{\rho_{,X}}, \quad (16)$$

since it quantifies the stresses in the frame comoving with the field. However, this mapping of field variables onto fluid variables is merely formal in the general case. For example, the matter component of pUDM acts as collisionless non-relativistic dark matter even though  $c_e^2 = 1$  (see §VII). In the kUDM model, no such subtlety exists and the matter component behaves purely hydrodynamically.

### IV. LINEAR REGIME

For the linear fluctuations it is sufficient to note that though there is a single kUDM Lagrangian, the matter field component is decoupled from the dark energy component and behaves as a fluid. It possesses an adiabatic sound speed given by Eqn. (15) with  $f = 3\epsilon(1 + 3\epsilon/4)$  and negligible anisotropic stress, as the latter scales quadratically with spatial field derivatives [see Eqn. (3)].

Given that the only gravitational degrees of freedom involve the parametrization of the stresses, mimicry of  $\Lambda$ CDM for all linear theory observables above the sound horizon, including the cosmic microwave background (CMB) and large-scale structure, is guaranteed by construction [14]. Perfect mimicry follows from  $\mathcal{L}_e$  being strictly constant, since the spatial fluctuations  $\delta\rho_\Lambda$  and  $\delta p_\Lambda$  vanish independently of the scalar field sound speed. More generally, allowing  $\mathcal{L}_e$  to depend weakly on  $Q$  or  $X$

can modify the predictions on the largest scales by introducing a relativistic dark energy sound speed [15, 26, 27].

Below the sound horizon, perturbations in the matter component undergo Jeans or acoustic oscillations instead of growth, effectively cutting off the linear power spectrum. This behavior places an upper limit on  $\epsilon_0$  such that the matter component acts as non-relativistic matter during the usual matter dominated epoch  $a > a_{\text{eq}} = 3 \times 10^{-4} (\Omega_m h^2 / 0.14)^{-1}$ . Scherrer placed a conservative upper limit by demanding that  $\epsilon(a_{\text{eq}}) < 1$  or, equivalently,  $\epsilon_0 < 4 \times 10^{-11} (\Omega_m h^2 / 0.14)^{-3}$  [19]. This constraint would ensure that the kUDM behaves as  $\Lambda$ CDM above the horizon at matter radiation equality and thus only deep in the linear regime today. To ensure mimicry for non-linear structure requires a much more stringent constraint on  $\epsilon_0$ .

These considerations can be usefully quantified by noting that the equation of state  $w_m$  and sound speed drop rapidly and monotonically from their relativistic values  $w_m = dp_m/d\rho_m = 1/3$  once  $a > a(\epsilon = 1)$ . After a scale exits the sound horizon, matter fluctuations grow as CDM independently of scale. Thus, on all larger scales, linear fluctuations can be described by a modification to the usual transfer function (e.g. [28])

$$T(k) = T_{\text{CDM}}(k)T_Q(k). \quad (17)$$

To characterize  $T_Q$ , we solve the coupled Einstein-Boltzmann equations that include photons, baryons and neutrinos in the usual way, following [15, 29]. The resulting transfer function, shown in Fig. 1, is well fit by

$$T_Q(k) \approx \frac{3j_1(x)}{x} \left[ 1 + \left( \frac{x}{3.4} \right)^2 \right]^{1/(\beta+1)}, \quad (18)$$

with

$$x = \left( \frac{k\eta_*}{7.74} \right), \quad \beta = 0.21 \left[ \frac{\epsilon_0}{10^{-18}} \left( \frac{\Omega_m h^2}{0.14} \right)^3 \right]^{0.12}. \quad (19)$$

Here,  $\eta_*$  is the conformal time evaluated at  $a_* = 14\epsilon_0^{1/3}$ . Note that the power drops by at least an order of magnitude at  $x = 5.9$ , the first extremum of the oscillation. For  $\epsilon_0 \rightarrow 0$ , this drop reflects the lack of the usual logarithmic growth of the matter fluctuations in the radiation dominated era. For larger  $\epsilon_0$ , the fluctuations are further suppressed as a function of  $k$  due to the change in the growth rate in the matter dominated epoch.

In Fig. 2 we show that the CMB power spectrum predictions remain unchanged from  $\Lambda$ CDM to the cosmic variance limit for  $\ell \lesssim 10^3$  and  $\epsilon_0 \lesssim 10^{-16}$ . The main effect of higher  $\epsilon_0$  is an enhancement of the radiation driving and a reduction of the baryonic modulation of the peaks.

## V. HALO ABUNDANCE

Given that the background pressure of the matter component of kUDM drops as the universe expands, the

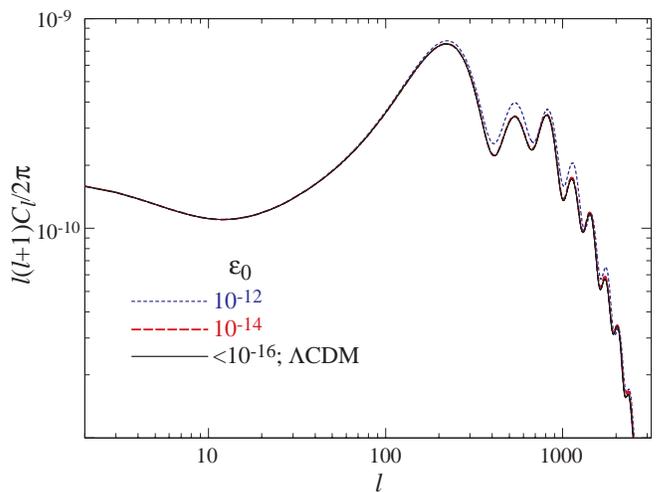


FIG. 2: CMB power spectrum in the fiducial cosmological model as a function of  $\epsilon_0$ . For the small  $\epsilon_0 < 10^{-16}$  required to match early structure, CMB predictions are indistinguishable from  $\Lambda$ CDM.

largest qualitative difference between this model and  $\Lambda$ CDM in the non-linear regime comes simply from the suppression of small scale linear power that is frozen in early on.

This suppression in the initial fluctuations reduces the number of low mass dark matter halos in a manner that has been well quantified by simulations. For Gaussian initial conditions, the number density of halos of mass  $M$  is controlled by the variance of linear density field [30]

$$\sigma^2(M; z, \epsilon_0) = \int \frac{d^3k}{(2\pi)^3} P_{\text{CDM}}(k; z) T_Q^2(k; \epsilon_0) |W(k; M)|^2. \quad (20)$$

where  $W(k; M)$  is the Fourier transform of the top hat window that encloses the mass  $M$  at the mean density. Specifically, their comoving number density is described by the mass function [31]

$$\frac{dn}{d \ln M} = \frac{\bar{\rho}_{m0}}{M} f(\nu) \frac{d\nu}{d \ln M}, \quad (21)$$

where  $\nu = \delta_c / \sigma(M)$  and

$$\nu f(\nu) = A \sqrt{\frac{2}{\pi}} a \nu^2 [1 + (a\nu^2)^{-p}] \exp[-a\nu^2/2]. \quad (22)$$

A choice of  $\delta_c = 1.69$ ,  $a = 0.75$ ,  $p = 0.3$ , and  $A$  such that  $\int d\nu f(\nu) = 1$  fits the results of simulations well.

At high redshift, where  $\sigma$  is small and  $\nu$  large, collapsed objects of a given mass are rare. In this regime, the number density is exponentially sensitive to  $\sigma(M; z, \epsilon_0)$  and hence the amplitude of initial fluctuations. Thus, the requirement that there are sufficient low mass halos at high redshift to reionize the universe places a constraint on kUDM models.

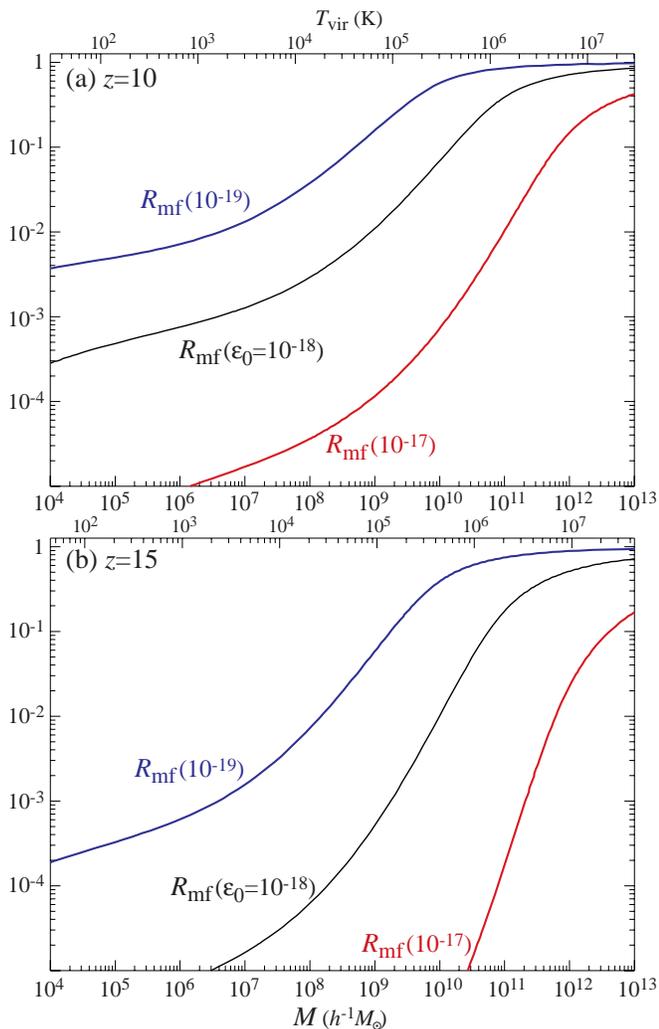


FIG. 3: Ratio of mass functions  $R_{\text{mf}}$  in kUDM and  $\Lambda$ CDM. The comoving number density of halos of mass  $M$  or virial temperatures of  $T_{\text{vir}}$  is suppressed due to the cut off in the power spectrum from linear theory. The suppression increases with decreasing mass and increasing redshift.

Let us define the suppression in kUDM for a given  $\epsilon_0$  relative to  $\Lambda$ CDM in the mass function as

$$R_{\text{mf}}(\epsilon_0) \equiv \frac{dn/d \ln M(\epsilon_0)}{dn/d \ln M(0)}. \quad (23)$$

In Fig. 3 we show this suppression as a function of mass for two redshifts representative of reionization and several choices of  $\epsilon_0$ . Note that the suppression increases with decreasing mass due to the small-scale cut off in the transfer function  $T_Q$  and with increasing redshift due to the exponential sensitivity.

For the specific question of reionization, the relevant quantity for comparison is the fraction of the universe that has collapsed into objects of sufficient mass that they can cool and form stars. For atomic line cooling,

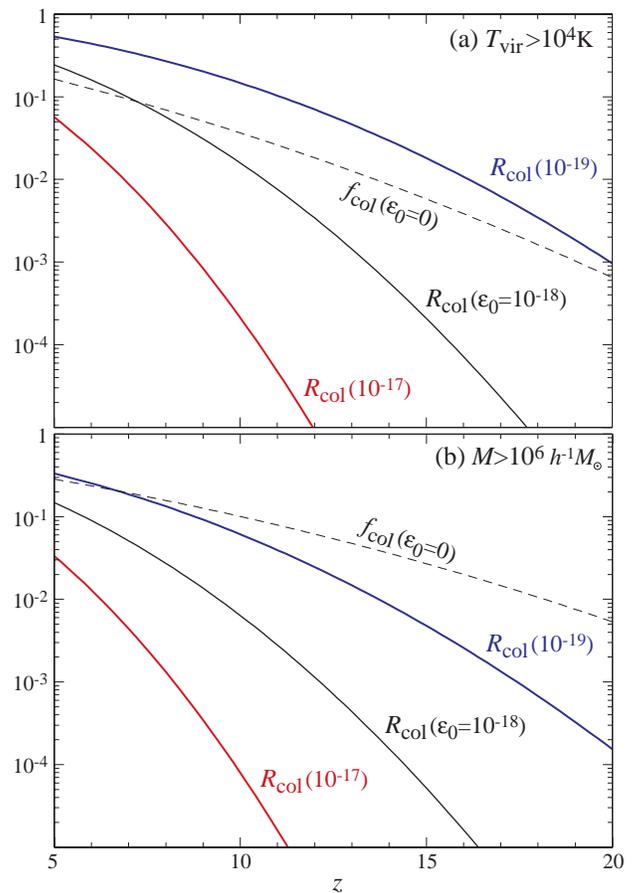


FIG. 4: Ratio of collapse fractions  $R_{\text{col}}$  between kUDM and  $\Lambda$ CDM in halos of mass relevant for reionization. Top panel: halos that can cool by atomic line transitions  $T_{\text{vir}} > 10^4 \text{K}$ . Bottom panel: by molecular hydrogen cooling  $M \gtrsim 10^6 h^{-1} M_{\odot}$ . Also shown for reference is the collapse fraction itself for  $\Lambda$ CDM  $f_{\text{col}}(0)$ .

the virial temperature of the halo

$$T_{\text{vir}} = 1.3 \times 10^4 \text{K} \left( \frac{\Omega_m}{0.27} \right)^{1/3} \left( \frac{1+z}{10} \right) \left( \frac{M}{10^8 h^{-1} M_{\odot}} \right)^{2/3} \quad (24)$$

must be greater than  $\sim 10^4 \text{K}$ ; for molecular hydrogen cooling,  $M \gtrsim 10^6 h^{-1} M_{\odot}$  but the reionization efficiency is lower [32, 33, 34]. The collapse fraction is given by the integral over the mass function

$$f_{\text{col}} = \int d \ln M \frac{M}{\bar{\rho}_{m0}} \frac{dn}{d \ln M}. \quad (25)$$

We will again define the collapse fraction relative to  $\Lambda$ CDM as

$$R_{\text{col}}(\epsilon_0) \equiv \frac{f_{\text{col}}(\epsilon_0)}{f_{\text{col}}(0)}. \quad (26)$$

We show this suppression factor in Fig. 4.

Since even  $\Lambda$ CDM has difficulty reionizing the universe sufficiently early, this ratio cannot be smaller than  $R_{\text{col}} \sim 10^{-2}$  around  $z \sim 10$ . Comparing requirements in the very similar context of broken scale invariance and warm dark matter [32, 33, 35] we place a conservative bound of  $\epsilon_0 \lesssim 10^{-18}$  for atomic cooling. If reionization occurs through molecular hydrogen cooling, then the limit is even more stringent. Furthermore if indications from the CMB polarization measured by WMAP of a high reionization redshift  $z > 10$  [36] are confirmed then even this regime will be ruled out, given reasonable assumptions on star formation and the escape fraction of ionizing photons [33]. The only loophole is if scale invariance in the initial power spectrum is strongly broken at coincidentally similar scales to compensate.

## VI. HALO SUBSTRUCTURE

The kUDM model differs from  $\Lambda$ CDM in the structure of dark matter halos both because of the reduction in initial small scale power and the fact that it behaves as a nearly pressureless fluid instead of a set of collisionless particles. These issues will require full cosmological simulations to resolve quantitatively, which is beyond the scope of this work. However, we shall see that both aspects of the modifications can be reduced to cases that have been previously solved in the literature. The latter is in qualitative disagreement with some recent observations.

A suppression of small scale power may in fact be desirable to reduce the claimed excess present-day substructure in galactic sized halos of the  $\Lambda$ CDM model. Of course, there is tension between these beneficial effects and the inability to form enough high redshift halos to reionize the universe. Saturating the conservative bound by setting  $\epsilon_0 \sim 10^{-18}$  one recovers a suppression of small scale power at a similar scale and form to the well-studied broken scale invariance model [37, 38]. This model was explicitly constructed to alleviate the substructure problem and so there may remain a small window in kUDM parameter space that satisfies these conflicting requirements. Cosmological simulations will be required to address this question quantitatively. Note, however, that the substructure problem may have an astrophysical origin.

For the evolution of small scale structure, it is useful to bear in mind that Eqn. (14) implies that the pressure remains small compared with the energy density until  $f = 1$ . Given the low upper bound for  $\epsilon_0$ , this corresponds to extremely large overdensities today  $\rho_{m0}/\bar{\rho}_{m0} \sim 1/\epsilon_0 \sim 10^{18}$ . For the relevant  $f \ll 1$  case, the non-linear equation of state simplifies to

$$p_m = \frac{\rho_m^2}{4\bar{\rho}_m} \epsilon \quad (27)$$

and the sound speed to

$$\frac{dp_m}{d\rho_m} = \frac{1}{2} \frac{\rho_m}{\bar{\rho}_m} \epsilon. \quad (28)$$

Note that in this limit the fluctuation to the kinetic term is still small, since

$$\frac{\delta X}{X_{\text{min}}} = \epsilon \frac{\delta \rho_m}{\bar{\rho}_m} \ll 1. \quad (29)$$

Hence, the fluid 4-velocity [see Eqn. (4)] remains timelike (see [39] for a discussion of spacelike kinetic scalar fields) and the 3-velocity non-relativistic. Thus, the non-linear evolution for kUDM may be calculated with non-relativistic hydrodynamic techniques, such as smoothed particle hydrodynamics (e.g. [40, 41]).

Qualitatively, density fluctuations will stabilize below the Jeans scale, which is the distance sound can travel in a dynamical time. The comoving Jeans wavenumber becomes

$$\begin{aligned} k_J &= \sqrt{\frac{4\pi G \rho_m a^2}{dp_m/d\rho_m}} = \sqrt{\frac{8\pi G \bar{\rho}_m a^2}{\epsilon}} \\ &= 0.216a \left(\frac{\epsilon_0}{10^{-18}}\right)^{-1/2} \left(\frac{\Omega_m h^2}{0.14}\right)^{1/2} \text{pc}^{-1}, \end{aligned} \quad (30)$$

or a physical Jeans scale that is fixed in the matter dominated epoch for both linear and non-linear density fluctuations. For a model that has sufficient linear theory power to explain high redshift structure and reionization, this scale is well below cosmologically interesting scales.

Given these constraints, the matter component of kUDM acts as a pressureless fluid on cosmologically relevant scales. This type of dark matter has been studied extensively in the literature in the context of collisional or self-interacting dark matter with a vanishingly small mean free path [23, 24]. Qualitatively, cold dark matter behaves differently from fluid dark matter as soon as there exist multiple streams in the dark matter. Since cold dark matter is collisionless, it occupies a phase space distribution  $f(\mathbf{x}, \mathbf{q}, \eta)$ , defined in a locally orthonormal basis. Its gravitational influence comes through the stress energy tensor

$$T^\mu_\nu = g \int \frac{d^3q}{(2\pi)^3} \frac{q^\mu q_\nu}{E(q)} f, \quad (31)$$

where  $q^\mu$  is the 4-momentum of the CDM particles and  $g$  is the degeneracy factor. For CDM in the non-linear regime, there can be multiple streams or momentum states occupied for a given position state. On the other hand, kUDM trajectories cannot cross and instead kUDM will behave as a fluid and shock.

Specifically, kUDM or fluid dark matter is challenged by observations of a galaxy cluster system observed to be undergoing a high velocity merger [42]. Here, the collisional baryonic gas lags behind the collisionless stars and

lensing-observed dark matter due to ram pressure stripping. Likewise, kUDM is in potential conflict with the shape of gravitational wakes of galactic halos moving in a cluster halo [43] and the fundamental plane of ellipticals in clusters versus the field [44]. Finally, kUDM would not predict as large a triaxiality of halos as measured by strong lensing [23, 24, 45, 46], although some ambiguities remain due to projection effects [47]. Note that the kUDM is fluid-like even down to scales where the pressure becomes relevant so that there is no equivalent to an adjustable mean free path or cross section to alleviate these difficulties [48].

## VII. DISCUSSION

Building a successful unified dark matter model is a useful exercise, even if its end role is only as a foil to  $\Lambda$ CDM that highlights the successes of the standard model. Observations already sharply delineate the basic properties of a successful unified model.

Compared with Chaplygin gas models of unified dark matter, kinetic unified dark (kUDM) [19] is more successful as it can explain the well-established observations both of the background expansion rate and phenomena in the linear regime, such as the cosmic microwave background and large-scale structure of the universe. It is not without its problems however in the non-linear regime. If recent observations in clusters of galaxies are confirmed then even this type of model will be ruled out.

We have shown that the kUDM behaves as a fluid with a closed-form barotropic equation of state plus a constant term that acts as a cosmological constant to all orders in structure formation. In linear theory, the high density of the matter component in the early universe introduces a relativistic pressure and hence a cut off to the linear power spectrum from Jeans oscillations. We have provided a convenient fitting function to the numerical results that should assist in future investigations of the model.

In order to produce halos that reionize the universe, kUDM must behave like CDM in linear theory on scales above  $k \sim \text{few Mpc}^{-1}$ . This requirement reduces the viable kUDM parameter space to the limiting case of highly

collisional, or self-interacting, non-relativistic dark matter with a negligible mean free path [23, 24]. kUDM is then also phenomenologically equivalent to a Chaplygin gas UDM, where the pressure is independent of the density [10] ( $\alpha \rightarrow 0$ , see §I). Such models are in moderate observational conflict with the observed shape of cluster halos and effects like ram pressure stripping in cluster substructure. Hydrodynamic simulations of the dark matter with the initial conditions provided by the linear theory given here may be used to quantitatively address these issues in the specific kUDM context. Note that these qualitative features are likely to remain valid even if the quadratic assumption for the kinetic Lagrangian fails far away from the minimum, so long as the implied pressure is still of order the energy density once  $\epsilon \gtrsim 1$ .

If these recent observations of clusters of galaxies are confirmed only the potential variant of scalar field UDM remains viable. Here although the field can still be mapped onto a perfect fluid, it possesses a phase space structure and mimics CDM down to the de-Broglie wavelength of the field, i.e. the kiloparsec regime for  $m > 10^{-22}$  eV [18]. In this case, the rapidly oscillating equation of state makes a direct solution of the fluid equations intractable [49, 50]. It can be shown through a WKB approximation that the momentum information is carried by a spatially varying phase to the field, similar to a quantum phase space for the wavefunction [51]. The collisionless nature of CDM is then reflected by the linearity of the wave equation or the quadratic nature of the potential. Self-interactions for the pUDM model are included by adding anharmonic terms to the potential [52].

From these examples, it is clear that a successful UDM model will likely be as complex phenomenologically as the well-tested  $\Lambda$ CDM model. Complexity reduces the appeal of the UDM hypothesis, unless a fundamental theory can explain the disparate scales introduced by the components that masquerade as the dark matter and dark energy. Neither the kUDM or pUDM models in the form presented here do so.

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