

**Model-independent reionization observables in the CMB**

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We represent the reionization history of the universe as a free function in redshift and study the potential for its extraction from cosmic microwave background (CMB) polarization spectra. From a principal component analysis, we show that the ionization history information is contained in 5 modes, resembling low-order Fourier modes in redshift space. The amplitudes of these modes represent a compact description of reionization observables in the CMB, easily predicted given a model for the ionization fraction. Measurement of these modes can ultimately constrain the total optical depth, or equivalently the initial amplitude of fluctuations to the 1% level regardless of the true model for reionization.

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**I. INTRODUCTION**

The Wilkinson Microwave Anisotropy Probe (WMAP) experiment [1] has recently detected the reionization of the Universe through the large-angle polarization of the cosmic microwave background (CMB). Rescattering of CMB radiation bearing a quadrupole temperature anisotropy leads to a small linear polarization which is then correlated with the temperature anisotropy itself. It was through this cross-correlation that WMAP measured the total optical depth to Thomson scattering,  $\tau = 0.17 \pm 0.04$  [1].

The CMB power spectra potentially contain more information than the optical depth integrated over the whole ionization history [2]. Unfortunately, the nature of the ionizing sources is not currently well understood (e.g. [3], cf. [4]), and therefore the detailed ionization history should not be treated in the traditional CMB approach of adding well-motivated model parameters to a parameter estimation chain. Attempts to do so can result in biases and conflicting results. For example, using an ionization history derived from a particular numerical prescription [5] it was found that the partially ionized epoch was undetectable in the CMB, while analyses using semianalytic models [6,7] showed that the predicted multiple epochs of reionization and partial recombination can be detected [8,9]. As another example, by assuming an overly simplistic step function model for reionization, the total optical depth  $\tau$  in the semianalytic models can be misestimated by up to a few  $10^{-2}$  [9]; while not a large amount it is already approaching the statistical errors in the first year WMAP data.

These uncertainties argue for a more model-independent approach to the phenomenology of reionization in the CMB. In this paper, we begin by considering a complete basis for the ionization history in Sec. II. In Sec. III, we employ a principal component analysis to isolate the CMB reionization observables. Throughout we work in a flat cold dark matter model with a cosmological constant ( $\Lambda$ CDM) cosmology with  $h = 0.72$ ,  $\Omega_m h^2 = 0.145$ ,  $\Omega_b h^2 = 0.024$  and  $n = 1$ . In the absence of systematic errors, most of the ionization history

information comes from the power spectrum of the  $E$ -mode polarization  $C_\ell^{EE}$  as opposed to the temperature-polarization cross correlation measured by WMAP [10] and for simplicity we will here consider only this power spectrum. Nevertheless the main results are applicable to the cross power spectrum as well.

**II. MODEL-INDEPENDENCE**

To zeroth order, the total Thomson optical depth parameterizes the effects of reionization independently of the specific model for the ionization history. Firstly, the acoustic peaks in the CMB power spectra are lowered by  $e^{-\tau}$  in amplitude. Because the effect is degenerate with a change in the initial normalization of the curvature fluctuations  $\delta_\zeta$ , it is absorbed into fixing the combination  $\delta_\zeta e^{-\tau}$ . Secondly, rescattering generates large angle temperature fluctuations through the Doppler effect and thirdly, it generates large angle polarization. Both generative effects have an rms that depends mainly on  $\tau$  but since the primordial polarization is free of large angle contributions, it is the better probe of reionization. Finally there are further signals from inhomogeneous scattering that appear beyond the damping tail ( $\ell \gtrsim 2000$ ) but whose phenomenology is inextricably tied to the model for the ionizing sources; we will not consider them further here.

In detail, the residual temperature and polarization effects depend on the reionization history, mainly through a sensitivity to the horizon scale at the epoch of rescattering [11]. We assume the hydrogen ionization fraction  $x$  to be a free function of redshift. A complete basis for this function can be formed from a series of delta functions in redshift. We approximate this series as Kronecker delta functions  $\delta x(z_i)$  spaced by  $\Delta z$  with linear interpolation between the points (see Fig. 1). The spacing is chosen to capture all of the information in the CMB and also be sufficiently fine to reproduce features in the ionization model. We take  $\Delta z = 1$  throughout.

A specific model can then be represented as a sum over

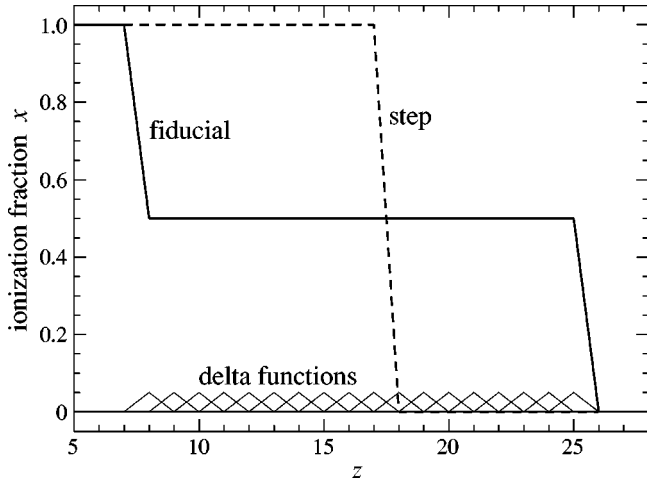


FIG. 1. Hydrogen ionization fraction  $x$  as a function of redshift  $z$  in the fiducial model (thick), traditional step function ionization (dashed) and delta-function perturbations (thin, normalized to reflect the transfer function calculation).

the delta function modes in the approximation of linear interpolation between the points. Consider a fiducial ionization history with a hydrogen ionization fraction of  $x=1$ ,  $z_i \leq 7$ ;  $x=0.5$ ,  $7 < z_i < 26$ ;  $x=x_{\text{rec}}$ ,  $z_i \geq 26$  where  $x_{\text{rec}} (\approx 0)$  is the ionization fraction coming out of standard recombination (see Fig. 1). Compare that to a step function ionization with  $x=1$ ,  $z_i \leq 17$  and  $x_{\text{rec}}$   $z_i > 17$  (Fig. 1). Both models have  $\tau \approx 0.17$  but differ in the coherence scale of the polarization. These two models are clearly distinguishable in principle (see Fig. 2, top) and so more information than the optical depth can be extracted from precise measurements [2,9].

The parameters  $x(z_i)$  may be simply appended to the usual CMB parameter estimation chain but that approach has several drawbacks. The  $\Delta z$  spacing of the points is arbitrary and moreover for  $\Delta z=1$  the parameters are highly degenerate and would cause numerical problems in a likelihood analysis. To see this, consider the effect of a delta-function perturbation to the fiducial ionization history given above and quantify the response in the power spectrum as a transfer matrix

$$T_{\ell i} \equiv \frac{\partial \ln C_{\ell}^{EE}}{\partial x(z_i)}, \quad (1)$$

where we vary  $7 < z_i < 26$  as motivated by the range in redshift over which reionization is expected to occur [3]. The derivative is calculated by a double-sided finite difference of  $\delta x=0.05$ . This matrix may be viewed as a transfer function for small perturbations from the fiducial model. The main feature is that perturbations at relatively low redshift appear at low  $\ell$  and those at high redshift at high  $\ell$ . The perturbations also introduce ringing into the power spectrum which is retained in the full model. Because neighboring redshift modes produce similar responses in the power spectrum, there exists a degeneracy in the recovery of  $\delta x(z_i)$ .

Under the linear approximation, we can pose the inverse problem as the recovery of  $\delta x(z_i)$  from the linear response

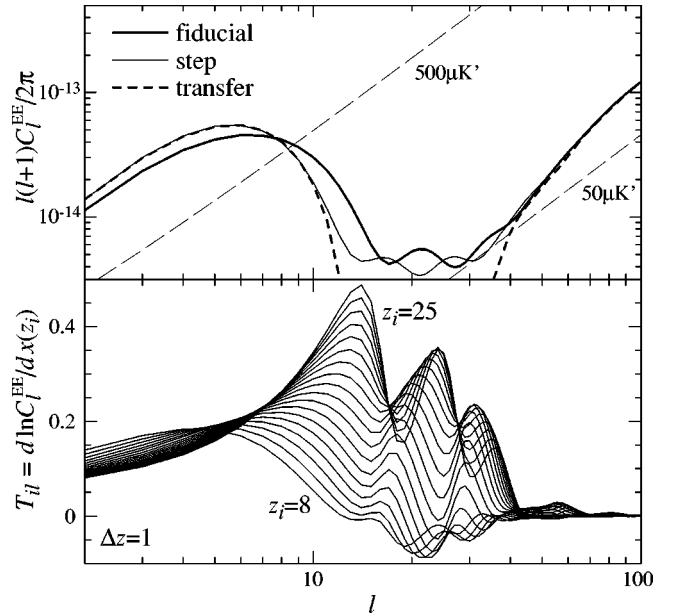


FIG. 2. Top:  $E$ -mode polarization power spectrum for: the fiducial model of Fig. 1 (thick); the step function model (thin); the step function model with deviations transferred onto the fiducial model (dashed); instrumental noise power  $w_p^{-1/2}$  (denoted in  $\mu\text{K-arcmin}$ ) that roughly brackets expectations from WMAP and Planck (long dashed). Noise and sampling errors per  $\ell$  are reduced by  $(\ell + 1/2)^{1/2}$ . Bottom: the transfer function or fractional power spectrum response to a delta function perturbation of unit amplitude at  $8 \leq z_i \leq 25$ .

$$\delta C_{\ell}^{EE} = C_{\ell}^{EE} \left|_{\text{fid}} \sum_i T_{\ell i} \delta x(z_i) + N_{\ell}, \quad (2)$$

where  $N_{\ell}$  represents noise sources. Instrumental noise powers that roughly bracket the expectations for WMAP and Planck [12] are shown in Fig. 2 but systematic effects such as residual foreground contamination will likely dominate the errors. With a characterization of the statistical properties of the noise and a regularization of the reconstructed signal, e.g. by placing a smoothness criteria on  $x(z_i)$  through a two-point prior, standard techniques such as Wiener filtering allow for model reconstruction. It is important to take a fiducial model that is close to the observed power spectrum since linear response will only hold for small perturbations. It will also be important below in that the cosmic variance of the assumed signal sets the fundamental noise in the reconstruction. We chose a fiducial model with an ionization  $x(z_i) = 0.5$  in the parametrized regime so that, neglecting helium reionization [6], the maximal excursion from the model is  $\delta x(z_i) = 0.5$ .

As a worst case scenario consider the reconstruction of the step function ionization in Fig. 1 from the fiducial model,  $\delta x(z_i)$  and the transfer matrix. Here all of the parameters  $x(z_i)$  are offset from the fiducial choice by the maximum value. The predicted power spectrum is shown in dashed lines in Fig. 2 (top). Note the agreement is still quite good at the peak of the power but fails in the low signal regime where small errors in the transfer function lead to large frac-

tional effects. Still, given realistic measurement noise and an iterated choice of fiducial model, the linear response approximation can provide a useful tool for analyzing the data. A detailed consideration is beyond the scope of this work. Instead we will show that a modification of the forward approach suffices to extract essentially all of the information in the CMB.

### III. CMB OBSERVABLES

To better quantify the information contained in the power spectrum, let us consider the ultimate limit of an all-sky experiment that is cosmic variance limited. The variance of the power spectrum is then given by

$$\langle \delta C_\ell^{EE} \delta C_{\ell'}^{EE} \rangle = \frac{2}{2\ell+1} (C_\ell^{EE})^2 \delta_{\ell\ell'} \quad (3)$$

and hence the covariance of the ionization parameters  $\langle \delta x(z_i) \delta x(z_j) \rangle \approx (\mathbf{F}^{-1})_{ij}$ , where

$$F_{ij} = \sum_\ell (\ell + 1/2) T_{\ell i} T_{\ell j} \quad (4)$$

is the Fisher matrix. The structure of the transfer matrix implies a large covariance between estimates of  $\delta x(z_i)$  and renders the delta-function representation difficult to visualize.

Consider instead the principal component representation based on the orthonormal eigenvectors of the Fisher matrix, decomposed as

$$F_{ij} = \sum_\mu S_{i\mu} \sigma_\mu^{-2} S_{j\mu}. \quad (5)$$

For a fixed  $\mu$ , the  $S_{i\mu}$  specify linear combinations of the  $\delta x(z_i)$  for a new representation of the data

$$m_\mu = \sum_i S_{i\mu} \delta x_i, \quad (6)$$

where the covariance matrix of the mode amplitudes is given by

$$\langle m_\mu m_\nu \rangle = \sigma_\mu^2 \delta_{\mu\nu}. \quad (7)$$

In other words, the eigenvectors form a new basis that is complete and yields uncorrelated measurements with variance given by the inverse eigenvalue. The largest eigenvalues correspond to the minimum variance directions and the first 5 are shown in the upper panel of Fig. 3. The first two correspond essentially to the average ionization at high redshift and low redshift respectively. The lower panel shows the directions with the 5 highest variances. Here neighboring delta modes with similar responses compensate each other to leave the observable power spectrum unchanged. The rms of each mode is shown in Fig. 4. Because the ionization fraction cannot be negative and the amplitude of each mode is  $\sim 0.5$ , only the first 5 modes with  $\sigma_\mu \lesssim 1$  have useful information. An added benefit of the principal component repre-

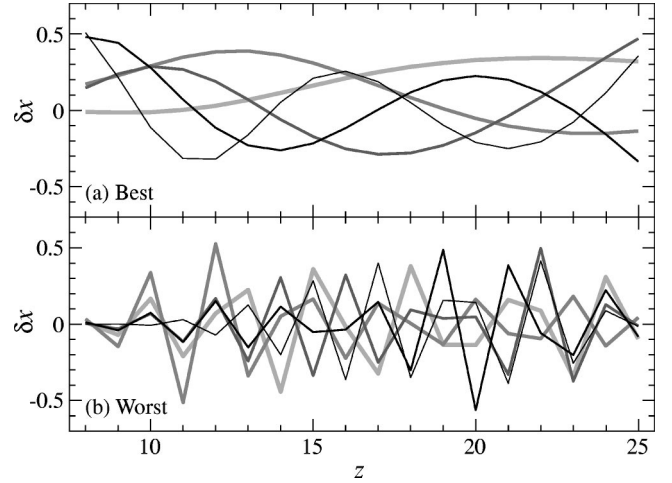


FIG. 3. Eigenmodes. (a) 5 best (decreasing eigenvalue thick to thin) constrained eigenmodes or linear combinations of ionization history. (b) 5 worst constrained eigenmodes.

sentation is that the structure in the lowest modes is invariant under refinement of the binning scheme  $\Delta z$ . In  $\ell$  space, the first mode controls the high  $\ell$  power, the second the low  $\ell$  power and the third through fifth adjust the ringing in the spectrum.

These eigenmodes provide a good meeting ground between observations and models. The amplitude  $m_\mu$  of these 5 best modes may be added to the usual CMB parameter estimation chain and the results compared to model predictions for  $m_\mu$  without significant loss of information. As an example, in Fig. 5 we have represented a complex ionization history (inset, thick-dashed) through its first 1 through 5 eigenmodes. For this ionization history the first 3 eigenmodes suffice to recover the observable power spectrum. The temperature polarization cross spectra converge similarly.

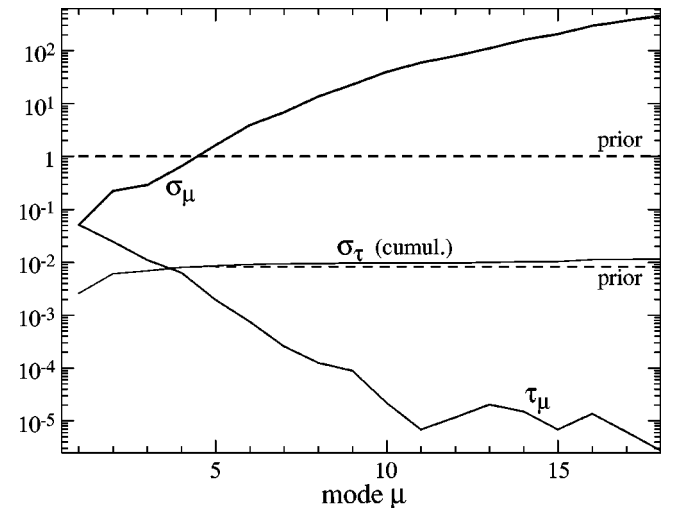


FIG. 4. Eigenmode statistics. Top curve: rms error  $\sigma_\mu$  on mode amplitude; dashed line represents a physicality prior on  $x$ ; only the first 5 modes contain interesting information. Bottom curve: optical depth per unit-amplitude mode  $\tau_\mu$ . Middle curve: rms error on total optical depth shown as the cumulative contribution from modes  $\leq \mu$ ; dashed line represents the physicality prior on  $x$ .

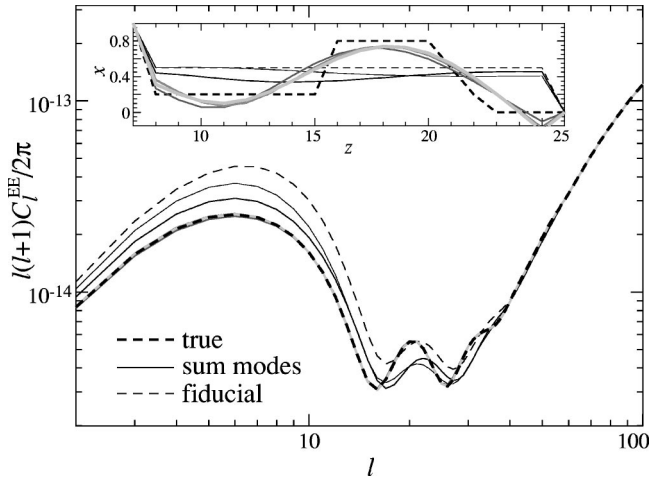


FIG. 5. Representation of an arbitrary ionization history with the first 5 eigenmodes. Inset: ionization history in the true model (thick dashed) compared with representation with 1 to 5 eigenmodes (solid, increasing thickness) away from the fiducial model (thin dashed). Main panel: resulting prediction for the power spectrum. With three or more modes the prediction is indistinguishable from the true model.

Note that in the eigenmode analysis the assumed ionization fraction can go unphysically negative but this causes no difficulty for a Boltzmann code and no ambiguity once interpreted as a mode amplitude.

Finally it is interesting to consider the implications for the total optical depth or equivalently errors on the initial amplitude of fluctuations  $\sigma_{\ln \delta_{\zeta}} \approx \sigma_{\tau}$ . A precise normalization of the initial conditions is crucial to dark energy studies utilizing the evolution of structure [13]. Each principal component mode perturbs the total optical depth by

$$\delta\tau_{\mu} = \sum_i S_{i\mu} \frac{\delta\tau_i}{\delta x(z_i)}, \quad (8)$$

where  $\delta\tau_i$  is the contribution to the optical depth from bin  $z_i$ . This quantity is shown in Fig. 4 (lower curve). The best

constrained mode bears a  $\tau$  uncertainty of 0.0026. Note that although the rms uncertainty in the higher modes is large, the optical depth contributed by them is compensatingly small. Given that the total optical depth variation is  $\delta\tau = \sum_{\mu} \tau_{\mu} m_{\mu}$ , the total variance is given by  $\sigma_{\tau}^2 = \sum_{\mu} \tau_{\mu}^2 \sigma_{\mu}^2$ . We plot in Fig. 4 the cumulative variance from increasing the number of modes included in the sum. The contributions are again dominated by the first few modes and placing a prior of  $\sigma_{\mu} < 1$  against unphysical values of  $x$  is enough to eliminate the small contribution from the higher modes. Thus, even with an arbitrary ionization history, the total optical depth can in principle be measured to  $\sigma_{\tau} \approx 0.01$  by an ideal experiment. Note that this is an uncertainty due to cosmic variance and not a bias. In the example of Fig. 5, the total optical depth with 0, 2 and 4 modes is  $\tau = 0.1732, 0.1498, 0.1379$  compared with a true optical depth of  $\tau = 0.1375$ .

#### IV. DISCUSSION

We have studied the effect of an arbitrary ionization history on the CMB polarization power spectrum and shown that the information lies in 5 broadly distributed modes in redshift. We have taken a complete basis for a range of redshifts  $7 < z < 26$  but this can be extended as needed. The amplitude of these modes represents a compact description of the observable properties of reionization in the CMB and can be easily predicted as a filtered version of any given model for the ionization fraction. They can therefore serve as a model-independent tool for data analysis and model testing. They also can ultimately remove any bias in the measurement of the total optical depth, or equivalently the initial amplitude of fluctuations, leaving a residual uncertainty from cosmic variance at the 1% level.

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