CMB Cluster Lensing: Cosmography with the Longest Lever Arm

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We discuss combining gravitational lensing of galaxies and the cosmic microwave background (CMB) by clusters to measure cosmographic distance ratios, and hence dark energy parameters. Advantages to using the CMB as the second source plane, instead of galaxies, include: a well-determined source distance, a longer lever arm for distance ratios, typically up to an order of magnitude higher sensitivity to dark energy parameters, and a decreased sensitivity to photometric redshift accuracy of the lens and galaxy sources. Disadvantages include: higher statistical errors, potential systematic errors, and the need for disparate surveys that overlap on the sky. Ongoing and planned surveys, such as the South Pole Telescope in conjunction with the Dark Energy Survey, can potentially reach the statistical sensitivity to make interesting consistency tests of the standard cosmological constant model. Future measurements that reach 1% or better precision in the convergences can provide sharp tests for future supernovae distance measurements.

I. INTRODUCTION

Gravitational lensing depends on the distances between the observer, lens, and source. These distances provide geometric measurements of the expansion history of the universe in much the same way as distant supernovae. Measurements of the same lens with multiple source planes can be used to construct distance ratio estimates that are, in principle, independent of the mass distribution (e.g. [1, 2, 3, 4]). In the weak lensing regime, where measurement and projection errors on individual lenses are large, these ratios can be measured statistically by stacking multiple lenses [5, 6] or equivalently by measuring correlation functions [7, 8, 9].

In this *Brief Report* we examine the use of recently developed CMB cluster mass reconstruction techniques [10] (see also [11, 12, 13, 14, 15, 16]) for measuring distance ratios. We discuss the benefits and drawbacks to using the CMB as a lensing source plane, and assess the impact future surveys may have on dark energy parameter measurements. For illustrative purposes, we describe the cosmology with the following parameters (values in square brackets denote our adopted fiducial choices). On the low redshift side: the dark energy density in units of the critical density $\Omega_{\rm DE}\,[=0.76]$, dark energy equation of state $w(a)=w_0+(1-w_a)a\,[=-1]$, and spatial curvature $\Omega_K\,[=0]$. On the high redshift side: matter density $\Omega_m h^2\,[=0.128]$, baryon density $\Omega_b h^2\,[=0.0223]$, optical depth $\tau\,[=0.092]$, tilt $n\,[=0.958]$, and scalar amplitude $\delta_\zeta\,[=4.52\times10^{-5}]$ at $k=0.05{\rm Mpc}^{-1}$.

II. COSMOGRAPHIC DISTANCES

Gravitational lensing of galaxy images or the CMB at a redshift $z_{\rm S}$ by an object at redshift $z_{\rm L}$ with comoving surface mass density Σ can be phrased in terms of the convergence

$$\kappa(\boldsymbol{\theta}, z_{\rm L}, z_{\rm S}) = 4\pi G \mathcal{D}_{\rm L}\frac{\mathcal{D}_{\rm LS}}{\mathcal{D}_{\rm S}}(1+z_{\rm L})\Sigma(\mathcal{D}_{\rm L}\boldsymbol{\theta}, z_{\rm L})\,, \tag{1}$$

where $\mathcal{D}_{\rm L}$, $\mathcal{D}_{\rm S}$, and $\mathcal{D}_{\rm LS}$ are the comoving angular diameter distances from observer to lens, observer to source, and lens to source respectively. Here $\boldsymbol{\theta}$ denotes the angular position on the sky.

In the idealization of perfect measurements at all angular positions and all the lensing being generated by a single lensing plane, the ratio of the measured convergence for two different source planes, $z_{\rm S}$ for the galaxies and z_* for the CMB, depends only on the distance ratio [1, 2, 3, 4, 5]:

$$R(z_{\rm L}, z_{\rm S}) \equiv \frac{\kappa(\boldsymbol{\theta}, z_{\rm L}, z_{\rm S})}{\kappa(\boldsymbol{\theta}, z_{\rm L}, z_*)} = \frac{\mathcal{D}_{\rm LS}}{\mathcal{D}_{\rm L*}}\frac{\mathcal{D}_*}{\mathcal{D}_{\rm S}}\,. \tag{2}$$

One virtue of using the CMB for the second source plane is that $\mathcal{D}_*$ is measured to high precision from the positions of the acoustic peaks. For example, in the projections for the Planck satellite (see below), the fractional error in distance $\sigma(\ln\mathcal{D}_*)=0.002$.

The second virtue of using the CMB as a source plane is that the large separation between it and typical galaxy source planes boosts the sensitivity of the ratio to cosmological parameters. In Fig. 1 we show the sensitivity of R to w_0, w_a, and Ω_K assuming that the high redshift parameters and $\mathcal{D}_*(\Omega_{\rm DE})$ are fixed. A percent level determination of R with $z_{\rm L}<1$ and $z_{\rm S}\sim1$ would provide interesting constraints on the dark energy and the curvature. Contrast this with the sensitivity of the convergence ratio between two galaxy source planes (z_1, z_2)

$$G(z_{\rm L}, z_1, z_2) = \frac{\kappa(\boldsymbol{\theta}, z_{\rm L}, z_1)}{\kappa(\boldsymbol{\theta}, z_{\rm L}, z_2)} = \frac{R(z_{\rm L}, z_1)}{R(z_{\rm L}, z_2)}\,, \tag{3}$$

which is typically an order of magnitude less since it requires a measurement of the much smaller change in R

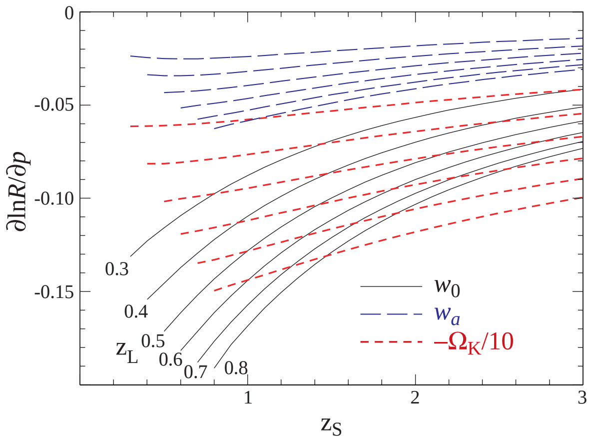

FIG. 1: Sensitivity to cosmological parameters of the convergence ratio R between the CMB last scattering surface and a galaxy source redshift $z_{\rm S}$. Utilizing the CMB as a source plane can boost the sensitivity to parameters typically by up to an order of magnitude.

with galaxy source redshift

$$\frac{\partial \ln G}{\partial p} = \frac{\partial \ln R}{\partial p}\Big|^{z_{\rm S}=z_1}_{z_{\rm S}=z_2} . \qquad (4)$$

The insensitivity of R to galaxy source redshifts around $z_{\rm S} \sim 1$ also implies that the requirements on measuring galaxy photometric redshifts is much less stringent than for G (*cf.* [6]). For example, the sensitivity of the ratio to redshift around $z_{\rm L} = 0.7$ and $z_{\rm S} = 1$ is

$$\frac{\partial \ln R}{\partial z_{\rm L}}\Big|_{z_{\rm L}=0.7} = -3.4 , \qquad \frac{\partial \ln R}{\partial z_{\rm S}}\Big|_{z_{\rm S}=1.0} = 2.3 , \qquad (5)$$

so that a measurement of R to a few percent requires photometric redshifts that are unbiased to 1%. Furthermore, given the weak dependence of R on redshift, high precision in the photometric redshifts of individual galaxies is not required.

III. FORECASTS

In practice, due to measurement errors and projection effects, cosmographic distances for individual objects like clusters of galaxies are too noisy to be useful. Instead multiple clusters can be stacked in order to measure a cluster-mass correlation function or average profile [5, 6, 7]. Projection effects from mass along the line of sight that is not associated with the cluster, which can introduce $\sim 30\%$ scatter in the mass estimates of individual clusters (e.g. [17]), averages away in this measurement [9]. Given the weak sensitivity of R to the lens and source redshift distribution compared with expected photometric redshift measurements, we can treat this statistical measurement as providing the average κ at the median lens and source redshifts for forecasting purposes. Furthermore, dividing up the distribution into multiple lens

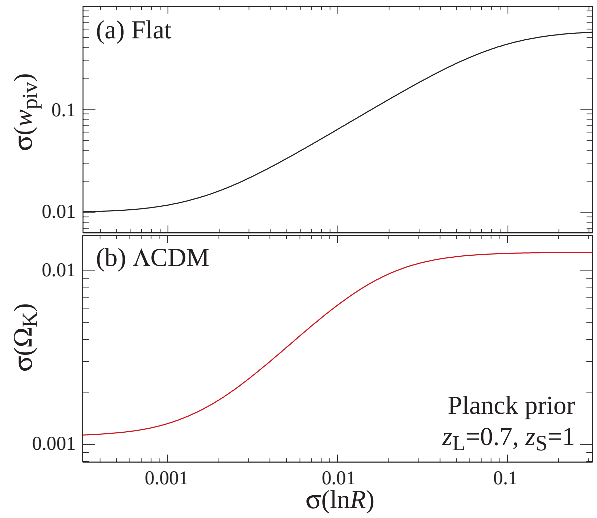

FIG. 2: Impact on parameter errors given Planck CMB power spectrum prior for (a) the equation of state at the best constrained redshift $w_{\rm piv}$ in a flat cosmology and (b) the spatial curvature in a cosmological constant (ΛCDM) cosmology. Lens and source redshifts here are $z_{\rm L} = 0.7$ and $z_{\rm S} = 1$.

and source planes does not provide much leverage for parameter estimation (see Fig. 1). For simplicity we will thus treat each pair separately.

With upcoming weak lensing surveys such as the Dark Energy Survey (DES), the expected statistical errors on R will be dominated by the CMB measurements. Hu et al. [10] estimate that the statistical errors for clusters above a mass of $10^{14.2} h^{-1} M_\odot$ at $z \approx 0.7$ equate to a $\sim 10\%$ rms error for κ at the $\sim 1'$ scale radius per 1000 clusters. This assumes a survey with $10\mu{\rm K}'$ instrument noise, comparable to the statistical sensitivity of the ongoing South Pole Telescope (SPT) experiment, but with no foreground contamination from the cluster. With an expected yield of $\sim 10^4$ clusters, the statistical precision can reach $\sim 3\%$ in κ or R. Furthermore, with longer integration times an experiment can improve on these numbers by a factor of 3–4 as the sample variance limit of temperature based estimators is reached. Lower mass objects such as the luminous red galaxies selected in DES can also serve as lenses. Finally, polarization measurements with sensitivity in the $\sim$ 1–3$\mu{\rm K}'$ range, comparable to SPTpol, can provide the means for achieving further improvements and checks for systematic errors [10, 18].

Since an actual measurement will likely be dominated by systematic errors and foregrounds, we will phrase our forecasts in an experiment-independent manner. Given a measurement of R to a certain fractional precision $\sigma(\ln R)$, the information on a set of parameters p_i is quantified by the Fisher matrix

$$F^R_{ij} = \frac{\partial \ln R}{\partial p_i} \frac{1}{\sigma^2(\ln R)} \frac{\partial \ln R}{\partial p_j} . \qquad (6)$$

The inverse of the Fisher matrix provides an estimate of

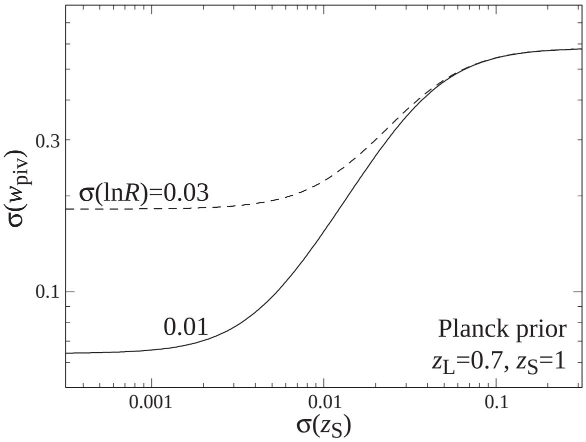

FIG. 3: Requirements on photometric redshift accuracy imposed by demanding that $w_{\rm piv}$ measurements not degrade substantially for 1% and 3% measurements of R at $z_{\rm L} = 0.7$ and $z_{\rm S} = 1$. A flat w_0–w_a cosmology is assumed.

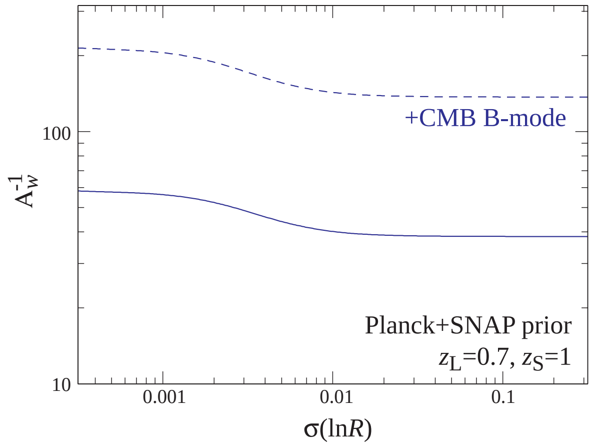

FIG. 4: Impact on the inverse area statistic A_w^{-1} of the error ellipse for the equation of state parameters w_0, w_a given SNAP supernova and Planck priors with curvature marginalized. Solid line: R measurement only. Dashed lined: including CMB B-mode power spectrum measurements of gravitational lensing comparable to SPTpol.

the covariance matrix between the parameters such that $\sigma(p_i) \approx [\mathbf{F}^{-1}]_{ii}$. Given multiple cosmological parameters and a single R, the Fisher matrix is degenerate and only one direction in the parameter space can be constrained. While multiple lens and source planes provides some opportunity to break the degeneracies, it is more useful to examine how a measurement of R will complement other measurements in the future.

We first combine the measurement of R with those of the CMB power spectrum expected from the Planck satellite. These measurements are also required to fix the distance to last scattering $\mathcal{D}_*$ in Eq. (2). Details for the construction of the Planck Fisher matrix are given in [19]; we assume 80% sky and 3 channels: FHWM $5.0'$ with temperature noise $\Delta_T = 51\mu{\rm K}'$ and polarization noise $\Delta_P = 135\mu{\rm K}'$; $7.1'$ with $\Delta_T = 43\mu{\rm K}'$, $\Delta_P = 78\mu{\rm K}'$; and $9.2'$ with $\Delta_T = 51\mu{\rm K}'$, $\Delta_P = \infty$.

Fig. 2 shows the errors in the equation of state at the best measured redshift $w_{\rm piv}$ in a flat cosmology (see e.g. [7]) and Ω_K in a $w = -1$ ΛCDM cosmology as a function of $\sigma(\ln R)$ for $z_{\rm L} = 0.7$ and $z_{\rm S} = 1.0$. These two parameters benchmark how well the standard flat ΛCDM cosmology can be tested or excluded. Note that improvements in parameter estimation begin with 10% measurements of R. Strong consistency checks are possible with 1% measurements. To utilize 0.1% measurements, improvements beyond Planck on the high redshift parameters will be required.

Other choices of lens and source redshifts in this range provide similar results. Increasing the source redshift to $z_{\rm S} = 1.2$ degrades the errors on $w_{\rm piv}$ by 12%. Decreasing the lens redshift to $z_{\rm L} = 0.6$ with $z_{\rm S} = 1$ degrades them by 9%. Increasing the lens redshift to $z_{\rm L} = 0.8$ with $z_{\rm S} = 1$ improves the measurement of $w_{\rm piv}$ by 6%.

In Fig. 2 we assumed that the redshifts of lens and source were perfectly determined. To assess the precision with which they need to be measured we add them as parameters in the Fisher matrix. In Fig. 3 we show the degradation of errors on $w_{\rm piv}$ with imperfect knowledge of the mean of the source photometric redshifts. To fully utilize 1% measurements of R, one requires a redshift accuracy of $\sigma(z_{\rm S}) \sim 0.003$ whereas 3% requires only ~ 0.01 accuracy. Source redshifts are more problematic than lens redshifts due to the large number density of sources and their higher redshift. Furthermore, with cluster lenses, multiple red galaxy cluster members can be used to estimate the redshifts. Nonetheless, sensitivity to lens redshift measurements can also be inferred from Fig. 2 by rescaling with the ratio of derivatives in Eq. (5).

Finally we assess how well CMB lensing measurements complement the combination of future supernovae (SNe) distance measures and Planck. For the SNe, we assume a sensitivity comparable to the proposed SNAP satellite and adopt the prescription described in [19]; we take 2800 SNe distributed in redshift out to $z = 1.7$ according to [20], 300 local supernovae uniformly distributed in the $z = 0.03$–0.08 range, statistical magnitude errors of $\sigma_m = 0.15$ per SN, and a systematic floor of $\sigma_{\rm sys} = 0.02\,(1 + z)/2.7$ per $\Delta z = 0.1$.

In Fig. 4 (solid curve) we show the impact on the area statistic of the w error ellipse, $A_w = \sigma(w_{\rm piv})\sigma(w_a)$ [21], with Ω_K marginalized. Until errors reach below 1%, R measurements do not provide significant parameter error improvements. Nonetheless, R measurements in the $\sim 1\%$ range do provide strong, purely geometrical consistency tests on supernovae measurements.

CMB lensing can improve A_w more significantly but the leverage comes mainly from lensing by large-scale structure. In the dashed lines we show the further improvement by including the forecasted constraints from B-mode polarization power spectrum measurements by SPTpol [19]. With both sets of lensing information combined, the improvement in A_w can ultimately reach a factor of 5.5.

IV. DISCUSSION

We have assessed the potential of joint cluster gravitational lensing measurements from the CMB and weak galaxy lensing surveys for determining distance ratios. These distance ratios are in turn sensitive to dark energy parameters and can be used to test the flat ΛCDM model. Benefits to using the CMB as a source plane include a well-determined source distance, a longer lever arm and thus higher signal, and a decreased sensitivity to photometric redshift errors of the lens and galaxy sources.

We show that if convergence ratios can be measured at percent level accuracy, the dark energy equation of state can be measured to $\sim 6\%$ when combined with CMB information from Planck in a flat universe. Such a measurement would provide an interesting consistency check on inferences from supernovae distance measures. Statistical errors of a few percent should be achievable with existing and planned cluster surveys, such as the SPT in combination with DES. However, the measurement will likely be limited by systematic errors, mainly on the CMB side. Minimum requirements include a high signal-to-noise CMB map of at least $10'$ resolution that is cleaned of the thermal Sunyaev-Zel'dovich effect in clusters [22]. Although a full assessment is beyond the scope of this *Brief Report*, we have shown that the combination of galaxy and CMB source planes have the potential to provide strong constraints on cosmological distance ratios, and thus make interesting contributions to our knowledge of the dark energy.

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