Observational Limits on Patchy Reionization: Implications for B-modes

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The recent detection of secondary CMB anisotropy by the South Pole Telescope places a conservative bound on temperature fluctuations from the optical depth-modulated Doppler effect of $T_{3000} < \sqrt{13}\,\mu$K at multipoles $\ell \sim 3000$. This bound is the first empirical constraint on reionization optical depth fluctuations at arcminute scales, $\tau_{3000} = 0.001\,T_{3000}/\mu$K, implying that these fluctuations are no more than a few percent of the mean. Optical depth modulation of the quadrupole source to polarization generates B-modes that are correspondingly bounded as $B_{3000} = 0.003\,T_{3000}$. The maximal extrapolation to the $\ell \sim 100$ gravitational wave regime yields $B_{100} = 0.1\,T_{3000}$ and remains in excess of gravitational lensing if the effective comoving size of the ionizing regions is $R \gtrsim 80\,$Mpc. If patchy reionization is responsible for much of the observed arcminute scale temperature fluctuations, current bounds on B_{100} already require $R \lesssim 200\,$Mpc and can be expected to improve rapidly. Frequency separation of thermal Sunyaev-Zel'dovich contributions to the measured secondary anisotropy would also substantially improve the limits on optical depth fluctuations and B-modes from reionization.

I. INTRODUCTION

Recent observations by the South Pole Telescope (SPT) [1, 2] and the Atacama Cosmology Telescope (ACT) [3] are ushering in a new era in which our understanding of secondary anisotropy in the cosmic microwave background (CMB) will be revolutionized. Secondary anisotropy is generated after recombination by gravitational and scattering processes. It is thus more dependent on astrophysical processes than the primary anisotropy that has been so useful in determining fundamental cosmological parameters. However, certain relationships between the various CMB secondary observables can be used to scale out and, in principle, determine the unknown astrophysics.

In this *Brief Report*, we discuss the example of these scaling relations provided by patchy reionization. Taken as an upper bound to account for contributions from other secondary effects, SPT measurements limit optical depth fluctuations on arcminute scales. These same fluctuations generate B-mode polarization [4] and so the implied limits at arcminute scales are relatively free of both cosmological and ionization model assumptions.

By making a maximal extrapolation to the degree scales relevant for gravitational wave detection, we place upper limits on the contamination by patchy reionization B-modes. Conversely, observational limits on degree scale B-modes constrain the ionization model, in particular the size distribution of the ionized regions, when combined with arcminute scale temperature measurements.

II. SCALING RELATIONS

Thomson scattering of CMB photons off free electrons in linear velocity flows generates temperature fluctuations via the Doppler effect. The first order effect from the mean optical depth during reionization is highly suppressed on subhorizon scales and the dominant contributions on arcminute scales reflect optical depth modulation. Optical depth modulations can arise from linear density fluctuations (Ostriker-Vishniac effect), non-linear objects (kinetic Sunyaev-Zel'dovich effect), or ionization fluctuations (patchy reionization).

Given a measurement or an upper limit on the temperature anisotropy due to this effect, one can constrain optical depth fluctuations and the corresponding effect on B-mode polarization. For notational convenience, we call the temperature power assigned to the modulated Doppler effect at a multipole of $\ell = 3000$

$$T_{3000}^{2(v)} \equiv \frac{\ell(\ell+1)}{2\pi} C_\ell^{TT(v)}\Big|_{\ell=3000}. \tag{1}$$

Likewise, we use the general shorthand notation

$$X_\ell^{2(s)} \equiv \frac{\ell(\ell+1)}{2\pi} C_\ell^{XX(s)} \tag{2}$$

for optical depth ($X = \tau$) and B-mode polarization ($X = B$) fluctuations; for B-modes, s denotes the contribution from a particular source field.

A conservative interpretation of the SPT detection of secondary anisotropy is that it places an upper limit of $T_{3000}^{2(v)} < 13\,\mu\mathrm{K}^2$ at 95% CL. This limit assumes that all of the measured anisotropy is assigned to the modulated Doppler effect and considers the detection as an upper limit [1]. For reasonable cosmological models, one would expect the thermal Sunyaev-Zel'dovich fluctuations from unresolved clusters and groups to dominate the signal. Thus $T_{3000}^{2(v)} \lesssim 5\,\mu\mathrm{K}^2$ might serve as a more typical, albeit model dependent, limit [2]. For this reason, we will preserve the dependence of our main results on $T_{3000}^{2(v)}$.

Optical depth fluctuations with a power spectrum $C_\ell^{\tau\tau}$ modulate the Doppler effect from a velocity field with

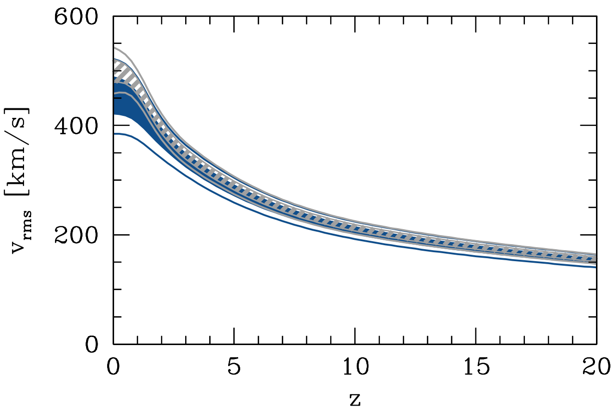

FIG. 1: Predicted range of $v_{\rm rms}(z)$ (shading: 68% CL region; curves: 95% CL region) for flat ΛCDM (light gray) and non-flat quintessence (dark blue) models constrained by current data.

rms $v_{\rm rms}$ to produce a temperature power spectrum [4]

$$C_\ell^{TT(v)} \approx \frac{1}{3} C_\ell^{\tau\tau} v_{\rm rms}^2 \tag{3}$$

below the coherence scale of the flows. The factor of 3 here accounts for the line-of-sight nature of the Doppler effect. Note that for simplicity we assume optically thin conditions throughout and ignore the 10–20% effects from the finite mean opacity during reionization. Given the linear theory matter power spectrum today, $P_{\rm lin}(k)$, the rms velocity is computed as

$$v_{\rm rms}^2(z) = \left[\frac{H(z)}{1+z}\frac{dD_1}{d\ln a}\right]^2 \int \frac{dk}{2\pi^2} P_{\rm lin}(k), \tag{4}$$

where D_1 is the linear growth function for density fluctuations. Figure 1 shows the range of $v_{\rm rms}(z)$ allowed by current constraints on flat ΛCDM models and non-flat quintessence models with arbitrary variations in the dark energy equation of state at $z < 1.7$, including measurements of the CMB, supernovae, baryon acoustic oscillations, and the Hubble constant as described in Ref. [5]. Note that predictions for $v_{\rm rms}$ at $z > 2$ have little uncertainty from cosmological parameters including variations in curvature and dark energy, with the possible exception of the presence of a substantial component of the energy density in dark energy or massive neutrinos at high redshift.

The same optical depth fluctuations modulate the generation of polarization from the primordial quadrupole with rms $Q_{\rm rms}$. In the Sachs-Wolfe approximation [4],

$$Q_{\rm rms}^2(z) \approx \frac{1}{60} A_s(k_0)[k_0(\eta(z)-\eta_*)]^{1-n_s}\Gamma_{\rm SW}(n_s)\,, \tag{5}$$

where the initial curvature spectrum normalized at the scale k_0 is $\Delta^2_{\mathcal{R}}(k) = A_s(k/k_0)^{n_s}$, η is conformal time with η_* evaluated at recombination, and

$$\Gamma_{\rm SW}(n_s) = 3\sqrt{\pi}\,\frac{\Gamma[(3-n_s)/2]\Gamma[(3+n_s)/2]}{\Gamma[(4-n_s)/2]\Gamma[(9-n_s)/2]}\,. \tag{6}$$

At plausible redshifts for reionization, $z \sim 10$, the predicted quadrupole for flat ΛCDM models is a nearly constant $Q_{\rm rms} = 18.2 \pm 0.5\ \mu$K, using the same data as for the $v_{\rm rms}$ predictions in Fig. 1. The uncertainty in $Q_{\rm rms}$ is negligible compared with $v_{\rm rms}$.

Modulation destroys the symmetry that produces only E-modes from scalar perturbations, generating equal power in E and B polarization. When scaled to the temperature spectrum, the B-mode power spectrum from the modulated quadrupole becomes [4]

$$C_\ell^{BB(Q)} \approx \frac{9}{100}\left(\frac{Q_{\rm rms}}{v_{\rm rms}}\right)^2 C_\ell^{TT(v)}\,. \tag{7}$$

Note that this scaling relation remains true regardless of whether the optical depth fluctuations are due to density effects like the Ostriker-Vishniac anisotropy [6] or ionization effects from inhomogeneous reionization. The only difference between these effects is the effective redshift at which $Q_{\rm rms}/v_{\rm rms}$ is evaluated, with $v_{\rm rms}$ dominating the variations. To maintain generality, we keep both $Q_{\rm rms}$ and $v_{\rm rms}$ in the relations but scale their values to $z \sim 10$.

Modulation of the $e^{-\tau}$ screening of the primary E-modes also generates B-modes below the coherence scale of the E-modes, i.e. the $\ell \sim 10^3$ damping scale of the primary anisotropy. On these scales, the screening B-modes take the form [7]

$$C_\ell^{BB(E)} \approx \frac{3}{2}\left(\frac{E_{\rm rms}}{v_{\rm rms}}\right)^2 C_\ell^{TT(v)}\,, \tag{8}$$

where

$$E_{\rm rms}^2 \approx \sum_\ell \frac{2\ell+1}{4\pi} C_\ell^{EE}\,. \tag{9}$$

$E_{\rm rms} = 6.4\,\mu$K for the maximum likelihood ΛCDM model and varies little with cosmological parameters.

From Eq. (3), optical depth fluctuations are related to the temperature anisotropy $T_{3000}^{(v)}$ as

$$\tau_{3000} \approx 0.00095\,\frac{T_{3000}^{(v)}}{\mu{\rm K}}\,\frac{200\,{\rm km/s}}{v_{\rm rms}}\,. \tag{10}$$

Hence for the strict upper limit of $T_{3000}^{(v)} < \sqrt{13}\,\mu$K, the rms fluctuation in τ at a few arcminutes is $\tau_{3000} < 0.003$, i.e. no more than a few percent of the mean optical depth (e.g. $\bar\tau = 0.10 \pm 0.02$ for general reionization histories at $6 < z < 30$ constrained by 5-year WMAP data [8]). Models that predict a percent optical depth rms or $\sim 10\%$ of the mean as a typical fluctuation are observationally unviable (cf. [9]).

For B-modes from the modulated quadrupole [Eq. (7)],

$$B_{3000}^{(Q)} \approx 0.003\,T_{3000}^{(v)}\,\frac{Q_{\rm rms}}{18\,\mu{\rm K}}\,\frac{200\,{\rm km/s}}{v_{\rm rms}}\,. \tag{11}$$

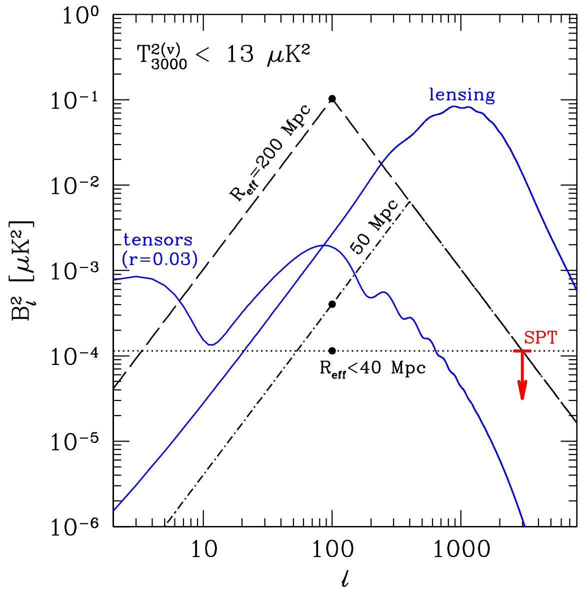

FIG. 2: Extrapolation of the inferred SPT limit on modulated quadrupole B-modes (red arrow at $\ell = 3000$ with $T^{2(v)}_{3000} < 13\,\mu\text{K}^2$, $z \approx 10$) to limits at $\ell \sim 100$ (black dots) using the sharp peak model with a single bubble size of $R_{\rm eff} = 200$ Mpc (black dashed lines) or $R_{\rm eff} = 50$ Mpc (black dot-dashed lines). For $R_{\rm eff} \lesssim 40$ Mpc, we take a more conservative upper limit shown by the horizontal dotted line. For reference, solid blue curves show the contributions to B^2_ℓ expected from lensing and gravitational waves with tensor-to-scalar ratio $r = 0.03$ for the best fit flat ΛCDM model.

Combining quadrupole and E-mode screening modulation yields

$$B^{2(Q+E)}_{3000} \approx \left(0.003\, T^{(v)}_{3000} \frac{200\,\text{km/s}}{v_{\rm rms}}\right)^2 \times \left[\left(\frac{Q_{\rm rms}}{18\,\mu\text{K}}\right)^2 + 2.1\left(\frac{E_{\rm rms}}{6.4\,\mu\text{K}}\right)^2\right] \quad (12)$$

as the total B-modes from reionization at $\ell = 3000$. Thus upper limits on $T^{(v)}_{3000}$ place stringent bounds on patchy reionization B-modes given predictions for $v_{\rm rms}$, $Q_{\rm rms}$, and $E_{\rm rms}$. For the strict upper limit $T^{(v)}_{3000} < \sqrt{13}\,\mu$K, $B^{(Q)}_{3000} < 0.01\,\mu$K.

III. LARGE ANGLE B-MODE LIMITS

To relate the B-mode contributions at $\ell = 3000$ to the large angle regime at $\ell \sim 100$ relevant for gravitational wave studies we require a model for the optical depth fluctuations. Note that the screening contributions die off as white noise for $\ell < \ell_A \sim 300$ given the acoustic scale and cannot generate large contributions at $\ell \sim 100$ [7]. We thus need to relate the modulated quadrupole contributions between the two scales.

To place upper limits on the B-mode contribution we begin by assuming all of the temperature power is due to ionization fluctuations since the B-mode signal from density fluctuations is expected to be well below that of gravitational lensing at all scales [10]. For a variety of simple analytic models of patchy reionization that assume completely ionized, spherical bubbles with a lognormal distribution of bubble radii [11, 12], the modulated quadrupole contribution to B-modes can be approximated as a single peak in B^2_ℓ that rises as ℓ^2 at $\ell \ll \ell_{\rm peak}$ and falls as ℓ^{-2} at $\ell \gg \ell_{\rm peak}$, where $\ell_{\rm peak} \approx 2 \times 10^4 (R_{\rm eff}/\text{Mpc})^{-1}$ [10]. The effective ionized bubble radius $R_{\rm eff}$ is determined by both the volume-averaged radius of bubbles R_V and the lognormal width of the bubble size distribution $\sigma_{\ln R}$: $R_{\rm eff} = R_V \exp(2.5\sigma^2_{\ln R})$.

Let us start by taking a single bubble size, i.e. a delta function in the distribution with $\sigma_{\ln R} \to 0$. Then the shape of the patchy reionization signal, shown in Fig. 2, is approximately described as a sharp peak[1]

$$B^{2(Q)}_\ell \approx \begin{cases} \left(\frac{\ell}{\ell_{\rm peak}}\right)^2 B^{2(Q)}_{\ell_{\rm peak}}\,, & \ell \le \ell_{\rm peak}\,, \\ \left(\frac{\ell}{\ell_{\rm peak}}\right)^{-2} B^{2(Q)}_{\ell_{\rm peak}}\,, & \ell > \ell_{\rm peak}\,. \end{cases} \quad (13)$$

The B-mode rms at $\ell = 100$ scales with $R_{\rm eff}$ as

$$B^{(Q)}_{100} \approx 2.2\times10^{-6}\, T^{(v)}_{3000} \frac{Q_{\rm rms}}{18\,\mu\text{K}} \frac{200\,\text{km/s}}{v_{\rm rms}} \left(\frac{R_{\rm eff}}{\text{Mpc}}\right)^2 \quad (14)$$

for $7 \lesssim R_{\rm eff}/\text{Mpc} \lesssim 200$ and saturates outside this range, as shown in Fig. 3.

This assumption of a sharply peaked spectrum provides a conservative extrapolation of upper limits from $\ell \sim 3000$ to $\ell \sim 100$ if $\ell_{\rm peak} \lesssim 500$ (or $R_{\rm eff} \gtrsim 40$ Mpc). In this case, realistic bubble size distributions would produce a flatter spectrum and thus less power at $\ell = 100$ (see [10], Fig. 7).

In the opposite limit of $R_{\rm eff} \lesssim 40$ Mpc, a flatter distribution would produce more power at $\ell \sim 100$ than a sharp peak and so we conservatively assume $B^{(Q)}_{100} \ge B^{(Q)}_{3000}$. Using this model to extrapolate from the SPT limit at $\ell \sim 3000$ to $\ell \sim 100$ produces an upper bound shown in Fig. 3 as the shaded excluded region.

Note that as the bubble size increases, the upper limit at $\ell = 100$ saturates. This saturation is essentially model independent and corresponds to a maximal slope of ℓ^{-2} connecting the power at $\ell = 100$ and 3000. This strict upper bound is given by

$$B^{(Q)}_{100} < 0.09\, T^{(v)}_{3000} \frac{Q_{\rm rms}}{18\,\mu\text{K}} \frac{200\,\text{km/s}}{v_{\rm rms}}\,. \quad (15)$$

[1] Ringing in ℓ produced by a delta function distribution is smoothed out by any more realistic bubble distribution and by the width in redshift of the reionization transition.

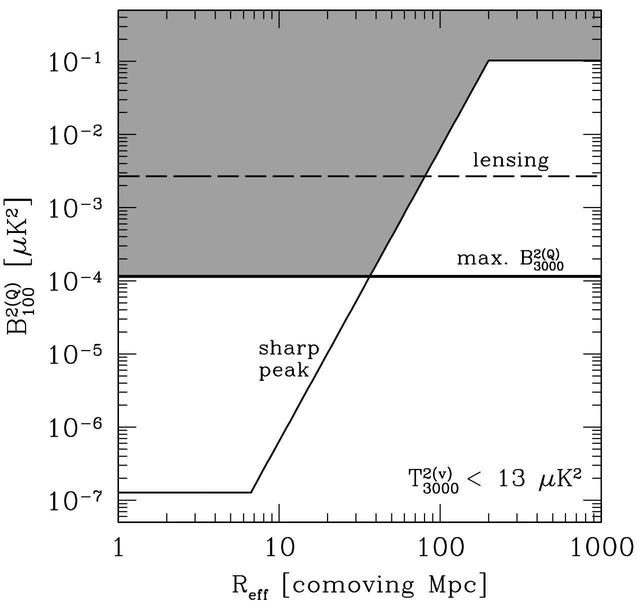

FIG. 3: Upper limits on the B-mode power from patchy reionization at $\ell = 100$ as a function of the effective radius of ionized bubbles, using $T^{2(v)}_{3000} < 13\,\mu\text{K}^2$ and assuming $z \approx 10$. The thin solid line extrapolates from the upper limit on $B^{2(Q)}_{3000}$ to $\ell = 100$ using the scaling of Eq. (14), and the thick solid line shows the extrapolation assuming a flat spectrum. The shaded region above both of these curves is excluded. The B-mode power at $\ell = 100$ from lensing in the best fit flat ΛCDM model is shown as a dashed line.

With $T^{(v)}_{3000} < \sqrt{13}\,\mu\text{K}$, this limit still allows more power than the lensing B-modes, $B^{2(L)}_{100} \approx (2.68 \pm 0.25) \times 10^{-3}\,\mu\text{K}^2$ (with fractional errors approximately scaled as $25\sigma(\Omega_c h^2)$ [13]).

Conversely, observational limits on B_{100} can be combined with measurements of $T^{(v)}_{3000}$ to constrain the bubble size distribution. If the distribution is dominated by a single effective comoving radius, Eq. (14) provides an estimate of the required precision to place an upper bound on this radius of

$$R_{\text{eff}} \sim 200(B^{(Q)}_{100}/0.1\,T^{(v)}_{3000})^{1/2}\,\text{Mpc} \qquad (16)$$

Current bounds of $B^2_{100} < 0.1\,\mu\text{K}^2$ (95% CL) from BICEP [14] in fact exclude $R_{\text{eff}} \gtrsim 200\,\text{Mpc}$ for the maximal $T^{(v)}_{3000} = \sqrt{13}\,\mu\text{K}$.

IV. DISCUSSION

We have shown that the SPT detection of secondary temperature anisotropy interpreted as a limit on modulated Doppler contributions from scattering of $< \sqrt{13}\,\mu\text{K}$ provide the first empirical bounds on optical depth fluctuations during reionization at a scale of a few arcminutes. Models that produce fluctuations on these scales in excess of a few percent of the mean are no longer viable. This limit on optical depth variations in turn produces a nearly model- and cosmology-independent limit on the B-mode polarization from reionization of $< 0.01\,\mu\text{K}$ at similar scales. We expect both limits to improve dramatically once more frequency channels of the data and larger regions of the sky have been analyzed so that the presumably dominant thermal Sunyaev-Zel'dovich contribution can be better separated.

Such upper limits can be turned into constraints on degree scale B-mode contamination of the gravitational wave signal. The maximal allowed B-modes at $\ell \approx 100$ can still exceed those from gravitational lensing, but only if the effective radius of the ionized regions is greater than $\sim 80\,\text{Mpc}$. In fact, current direct limits on degree scale B-mode polarization require ionized regions $\lesssim 200\,\text{Mpc}$ for the maximal allowed fluctuations at arcminute scales.

Hence expected improvements in both small angle temperature measurements and large angle B-mode polarization measurements should rapidly advance our empirical understanding of reionization.

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