Quantum Stability of Chameleon Field Theories

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Chameleon scalar fields are dark energy candidates which suppress fifth forces in high density regions of the universe by becoming massive. We consider chameleon models as effective field theories and estimate quantum corrections to their potentials. Requiring that quantum corrections be small, so as to allow reliable predictions of fifth forces, leads to an upper bound $m < 0.0073(\rho/10\ \mathrm{g\,cm^{-3}})^{1/3}$eV for gravitational strength coupling whereas fifth force experiments place a lower bound of $m > 0.0042\,$eV. An improvement of less than a factor of two in the range of fifth force experiments could test all classical chameleon field theories whose quantum corrections are well-controlled and couple to matter with nearly gravitational strength regardless of the specific form of the chameleon potential.

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Introduction.– Cosmic acceleration, discovered over a decade ago, is the great mystery of modern cosmology. Since the simplest model, a cosmological constant, offers no clues as to the smallness of the acceleration or to its recent onset, the search for other explanations for the acceleration is an active area of research. The next simplest models typically involve a scalar field, which is likely to couple to matter in the absence of a symmetry forbidding such a coupling [1, 2]. Gravitational-strength fifth forces have been excluded over a large range of length scales [3, 4], so a viable scalar theory must contain a mechanism for screening such forces locally. Chameleon [5–7] fields become massive in high density regions of the universe, pushing fifth forces to length scales below the bounds of current experiments. Symmetron [8–10] and Galileon [11, 12] fields lower their effective couplings to matter in high density regions through symmetry restoration and higher-derivative interactions, respectively.

Although progress has been made toward embedding chameleon models in more fundamental theories [13, 14], for now they are best treated as effective field theories, valid only below a certain potential-dependent cutoff energy scale. Above the cutoff, quantum corrections to the potential make predictions of fifth forces unreliable. In this paper, we estimate the one-loop Coleman-Weinberg correction to the potential, arising from chameleons running in the loop. Demanding that quantum corrections remain small compared to the classical potential, we find that the resulting "classical" chameleon theories cannot acquire masses larger than $m_\phi \sim (\xi\rho/M_{\rm Pl})^{1/3}$ at a density ρ and dimensionless matter coupling ξ. (Here $M_{\rm Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass and $\hbar = c = 1$ throughout.) Viable chameleons must therefore tiptoe between being heavy enough to avoid fifth force constraints and remaining light enough to keep quantum corrections under control. We will find that independently of the specific form for the self-interactions, there is tension between keeping quantum corrections under control and satisfying laboratory constraints on fifth forces.

Numerically, our upper bound can be expressed as $m_\phi < 0.0073(\xi\rho/10\ \mathrm{g\ cm^{-3}})^{1/3}$ eV. This energy scale is interesting for dark energy models and is also accessible to upcoming fifth force experiments. Of course, there is no requirement for Nature to choose a model which remains a valid effective theory out to scales accessible to experiments. However, these classical theories are the only known chameleon models with predictive power there, so our analysis can offer guidance as to the regions of the theory parameter space on which future experiments should focus.

Chameleon fields.– Consider a chameleon scalar field with equation of motion $\Box\phi = V_{\rm eff,\phi}$, where the effective potential is [5–7]

$$V_{\rm eff}(\phi,\vec{x}) = V(\phi) + \frac{\xi\rho(\vec{x})\phi}{M_{\rm Pl}}\,. \tag{1}$$

Here ξ is a dimensionless coupling to the matter density ρ, and the bare potential $V(\phi)$ is a function of the field alone. The matter coupling is a linearization, valid for $|\phi| \ll M_{\rm Pl}/\xi$, of the general scalar-tensor form $-\exp(\xi\phi/M_{\rm Pl})T^\mu_\mu$, where the trace of the stress tensor reduces to $T^\mu_\mu \approx -\rho$ for nonrelativistic matter.

The effective mass depends on field value $m_\phi^2 = V''(\phi)$. Inside a sufficiently large bulk of constant matter density, the field settles to its equilibrium value of

$$V'(\phi_{\rm m}) = -\xi\rho/M_{\rm Pl}\,. \tag{2}$$

The potential $V(\phi)$ is chosen so that $m_\phi(\phi_{\rm m}(\rho))$ increases with ρ; thus the range of the fifth force mediated by ϕ shrinks with increasing ρ. By becoming more massive in higher-density regions such as the laboratory, the chameleon field can "hide" from small-scale fifth force tests including [3].

Quantum corrections.– Both the self-interaction and the matter coupling in (1) will give rise to quantum corrections to a chameleon theory. Matter loops will of course generate large radiative corrections to the chameleon mass. This is the origin of the hierarchy problem, and we have nothing new to add here. In this work we therefore focus on quantum corrections due to ϕ loops, and treat the matter as an external source. In particular, we treat the matter coupling ξ as a free parameter, and focus on scalar quantum corrections to the potential. Even with these optimistic assumptions about the matter, we will find that quantum corrections from ϕ alone impose stringent constraints on the potential.

The one-loop Coleman-Weinberg correction to the classical potential $V(\phi)$, neglecting spatial variations in the field, is given by

$$\Delta V_{1-\mathrm{loop}}(\phi) = \frac{m_\phi^4(\phi)}{64\pi^2} \ln\left(\frac{m_\phi^2(\phi)}{\mu_0^2}\right), \quad (3)$$

where μ_0 is some arbitrary mass scale. The one-loop corrected potential is then $V_{1-\mathrm{loop}} = V + \Delta V_{1-\mathrm{loop}}$. Even if we choose the mass scale to eliminate the correction at some density ρ_0, $\mu_0 = m_\phi(\phi_\mathrm{m}(\rho_0))$, the fact that ϕ is a chameleon field where the mass runs with field value will imply corrections at other densities. When the one-loop corrections become as large as the tree-level terms, there is no reason to believe that higher-order loop corrections will not also be significant. Thus we use the corrections arising from $\Delta V_{1-\mathrm{loop}}$ as estimates of the quantum uncertainty in the chameleon model. A given classical chameleon model is predictive only if these quantum corrections are small at densities of interest.

Since $\Delta V_{1-\mathrm{loop}} \sim m_\phi^4$, we can immediately see that quantum corrections can present problems for chameleon theories. Chameleon screening of fifth forces operates by increasing m_ϕ, so quantum corrections must become important above some effective mass. On the other hand, laboratory measurements place a lower bound on the effective mass leading to tension between a model's classical predictivity and the predictions that it makes.

Specifically, for the chameleon mechanism to be classically predictive we require both $\Delta V'_{1-\mathrm{loop}}/V'$ and $\Delta V''_{1-\mathrm{loop}}/V''$ to be small across the field range of interest. The former sets the field value ϕ_m and the latter sets the effective mass at that value.

1-loop bound on mass.– While we can always evaluate the loop bounds in the previous section and compare them with laboratory bounds for any given chameleon potential $V(\phi)$, it is useful to phrase the main physical content of the bound in a model independent manner.

Classicality imposes a limit on the effective mass which a chameleon field may attain in an experiment, which depends on the density ρ_lab of the experimental apparatus. As a simple estimate, we set the log term in (3) to unity so that our loop criteria becomes

$$\left|\frac{\Delta V'_{1-\mathrm{loop}}}{V'}\right| \approx \left|\frac{M_\mathrm{Pl}}{\xi\rho}\frac{(m_\phi^4)'}{64\pi^2}\right| < \epsilon\,;$$
$$\left|\frac{\Delta V''_{1-\mathrm{loop}}}{V''}\right| \approx \left|\frac{(m_\phi^4)''}{64\pi^2 m_\phi^2}\right| < \epsilon\,, \quad (4)$$

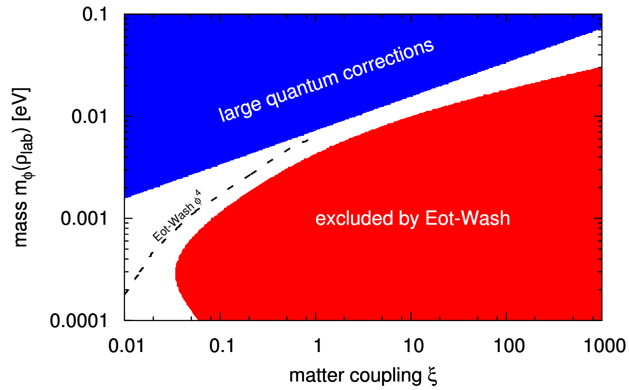

FIG. 1: Model-independent constraints on chameleon fields in the ξ, m_ϕ plane with $\rho_\mathrm{lab} = 10$ g/cm^3. Shaded regions show loop bounds from (6) and experimental constraints from Eöt-Wash [3]. The dashed curve shows the direct bound on the ϕ^4 model for $\xi < 1$ [15], converted to m_ϕ. Our bound is conservative in that it allows slightly lower values of m_ϕ.

where ϵ should not exceed unity. Using Eq. (2) the field derivatives can be replaced with density derivatives through $\mathrm{d}\phi_\mathrm{m}/\mathrm{d}\rho = -\xi M_\mathrm{Pl}^{-1} m_\phi(\phi_\mathrm{m})^{-2}$ to obtain

$$\frac{1}{\rho}\frac{\mathrm{d}m_\phi^6}{\mathrm{d}\rho}\,,\ \left|\frac{\mathrm{d}^2 m_\phi^6}{\mathrm{d}\rho^2}\right| \le \frac{96\pi^2\xi^2}{M_\mathrm{Pl}^2}\epsilon\,. \quad (5)$$

At the density ρ_lab these imply

$$m_\phi \le \left(\frac{48\pi^2\xi^2\rho_\mathrm{lab}^2\epsilon}{M_\mathrm{Pl}^2}\right)^{\frac{1}{6}} = 0.0073\left(\frac{\xi\rho_\mathrm{lab}}{10\ \mathrm{g\,cm^{-3}}}\right)^{\frac{1}{3}}\epsilon^{\frac{1}{6}}\,\mathrm{eV}\,. \quad (6)$$

For $\xi \sim \epsilon \sim 1$ and typical densities this mass scale is close to the dark energy scale of $\rho_\Lambda^{1/4} = 0.0024\,\mathrm{eV}$. This results from the numerical coincidence that $(\rho_\mathrm{lab}/M_\mathrm{Pl})^{4/3} \sim \rho_\Lambda$. Importantly, the dependence on ϵ is weak and the Compton wavelength corresponding to this maximum mass, $0.027(\xi\rho_\mathrm{lab}/10\ \mathrm{g\,cm^{-3}})^{-1/3}\epsilon^{-1/6}$ mm, is comparable to the length scales probed by the smallest-scale torsion pendulum experiments. Given this weak dependence, henceforth we set $\epsilon = 1$, the largest value at which order-unity predictions of fifth forces could reasonably be trusted.

Tension with Laboratory Bounds.– Torsion pendulum experiments such as Eöt-Wash [3] exclude fifth forces due to Yukawa scalars with constant masses m over a region of the ξ, m parameter space. Let m_max be the maximum mass of a given chameleon model in a fifth force experiment. The chameleon-mediated fifth force should be bounded from below by the force of the Yukawa scalar with mass $m = m_\mathrm{max}$. This is because the mass of the chameleon is lighter than m_max in the lower-density regions of the experiment, so the range of its fifth force is larger. Thus, approximating the chameleon's fifth force by the Yukawa force will lead to a conservative constraint on the chameleon; we refer to this as the maximum-mass

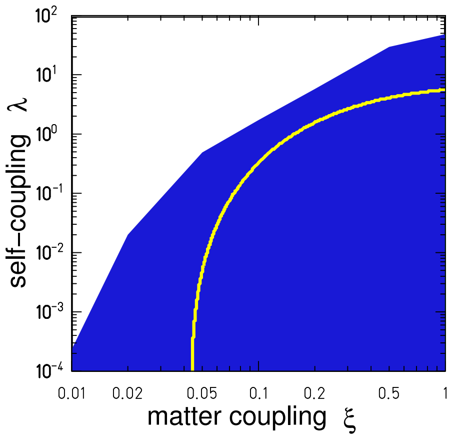

FIG. 2: Constraints on chameleon models with $V = \lambda\phi^4/4!$. The shaded region shows the models excluded by [15] while the curve shows the weaker constraints resulting from the maximum-mass approximation.

approximation. We quantify how much bounds are improved by a direct calculation for specific potentials below. Note that the maximum-mass approximation is used to place a minimum mass bound on m_ϕ.

We show this Eöt-Wash constraint [3] on the minimum mass in Fig. 1. We compare this to the maximum mass from the loop bound at the relevant density of $\rho_{\rm lab} = 10$ g/cm^3, working in the maximum-mass approximation. The tension between these two bounds is evident, especially near $\xi = 1$. A significant, but feasible, improvement in Eöt-Wash constraints over the next several years of less than a factor of 2 in the Yukawa mass or fifth force range could eliminate all chameleon fields around $\xi = 1$ whose quantum corrections are well-controlled.

Model Constraints.– Our maximum-mass approximation yields conservative but model-independent bounds on chameleon models. In the context of particular models, our approximations can be checked against direct computation.

In addition to constant-mass scalar theories, the Eöt-Wash experiment has also constrained chameleon theories with $V(\phi) = \lambda\phi^4/4!$ and $\xi < 1$ [15]. The ϕ^4 theory is also special in that the loop bound (6) is independent of ρ and ξ: since $m_\phi = \lambda^{1/6}(3\xi\rho/M_{\rm Pl})^{1/3}$ in this case, it follows that $\lambda < 32\pi^2\epsilon/3 \approx 105\epsilon$. In Fig. 1, we convert their constraints on λ, shown in Fig. 2, to a bound on m_ϕ.

As expected the direct λ bound rules out slightly more of the m_ϕ space than our mass bound for gravitational strength ξ but there is still an allowed region which satisfies both the loop and the laboratory bound. As also shown in Fig. 2, the impact of our approximation on the $\lambda - \xi$ parameter space is more pronounced since $\lambda \propto m_\phi^6$ but correspondingly the loop-compatible range appears larger and includes all of the space shown. Nonetheless it is the mass that is more closely related to the experimental observables and even with our conservative assumptions a factor of 2 there would close the $\xi \sim 1$ window entirely in this model.

We in fact expect our constraints to be conservative for generic chameleon models. To see this, consider the case of power law potentials

$$V(\phi) = \kappa M_\Lambda^{4-n}|\phi|^n\,, \tag{7}$$

where the arbitrary mass scale M_Λ is suggestively set to the dark energy scale $M_\Lambda = 0.0024\,$eV, thereby making κ a dimensionless constant. To have a chameleon model with a bounded potential requires $n < 0$ or $n > 2$. Note that our bounds would be unchanged by adding in a constant M_Λ^4 or a slowly varying piece to the potential that plays the role of a cosmological constant.

To model the experimental set up, consider a constant-density planar slab surrounded by vacuum: $\rho(x) = \rho_{\rm lab}$ for $x \le 0$ and $\rho(x) = 0$ for positive x. Using the exact solutions of Refs. [16, 17] for $V(\phi) \propto |\phi|^n$ in the vacuum $x > 0$,

$$\phi(x) = \frac{\left(1-\frac{1}{n}\right)\phi_{\rm m}(\rho_{\rm lab})}{\left(1+\sqrt{\frac{1}{2}\left|\frac{(n-2)^2(n-1)^{n-3}}{n^{n-1}}\right|}\,m_\phi(\rho_{\rm lab})x\right)^{\frac{2}{n-2}}}\,, \tag{8}$$

we can evaluate the acceleration $a_\phi = -(\xi/M_{\rm Pl}){\rm d}\phi/{\rm d}x$ of a test particle. A Yukawa scalar φ with $m = m_{\rm max} \equiv m_\phi(\rho_{\rm lab})$ and the same matter coupling ξ will cause an acceleration $a_\varphi = -(\xi/M_{\rm Pl}){\rm d}\varphi/{\rm d}x$ with

$$\varphi(x) = -\frac{\xi\rho_{\rm lab}}{2m_{\rm max}^2 M_{\rm Pl}}\exp(-m_{\rm max}x)\,. \tag{9}$$

Direct comparison shows that $|a_\phi| \ge |a_\varphi|$ at $x = 0$, and $|a_\phi|$ decreases more slowly than $|a_\varphi|$ for all $x \ge 0$. Thus $|a_\phi| \ge |a_\varphi|$ everywhere. To generalize, since $m_\phi < m_{\rm max}$ outside the highest-density part of the experiment, the fifth force due to a chameleon falls off more slowly with distance than that due to a Yukawa scalar with $m = m_{\rm max}$. The chameleon force is therefore larger and easier to exclude.

The Yukawa mass limits can then be converted into conservative constraints on the parameters of the power law potentials. Figure 3 shows models which are consistent with the data [3] in the maximum-mass approximation and whose quantum corrections satisfy (6) for various ξ. Although one can always find allowed models by tuning ξ to sufficiently small values, couplings of gravitational strength $\xi \sim 1$ and higher are the most interesting for chameleon theories.

Conclusions.– We have shown that keeping quantum corrections to chameleon theories under control imposes a density-dependent upper limit on the chameleon mass which is in tension with laboratory bounds on small-scale fifth forces. This tension can be quantified in a general, model-independent way by approximating the chameleon

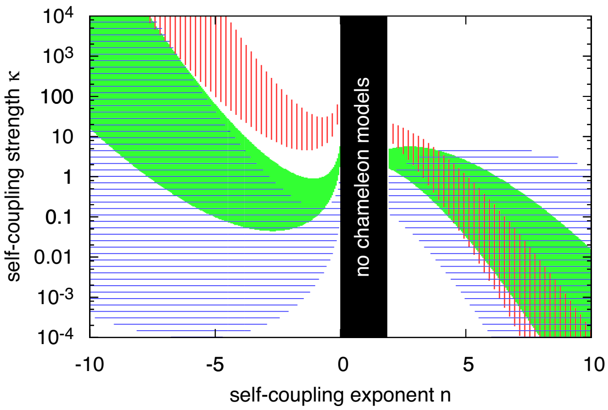

FIG. 3: Allowed classical chameleon models with power law potentials $V(\phi) = \kappa M_\Lambda^{4-n}|\phi|^n$, with $M_\Lambda = 0.0024\,\mathrm{eV}$. Blue (horizontal hatched), green (solid) and red (vertical hatched) regions show models with $\xi = 0.1$, 1, and 10, respectively, which satisfy (6) and are consistent with [3] in the maximum-mass approximation.

field by a Yukawa scalar whose constant mass equals the maximum mass of the chameleon in the experiment. Even in this conservative approximation, only a small range of viable predictive models remains for couplings around the gravitational strength, $\xi \sim 1$, which could be excluded by a factor-of-two improvement in bounds on the range of the fifth force.

Such an improvement would test all such chameleon models, regardless of the form for their self-interaction. These models include scalar-tensor theories such as the $f(R)$ model where $\xi = 1/\sqrt{6}$. Likewise they include other dark-energy motivated models where the dimensionful parameter characterizing the self-interaction is set to the dark energy scale.

In dark-energy motivated models, the chameleon may still be invoked at lower densities, e.g. to provide cosmological range forces which are sufficiently suppressed in the Solar system. At these lower densities, the loop bound is relatively easier to satisfy, e.g. at the background matter density the range $m_\phi^{-1} > 4 \times 10^5 \xi^{-1/3}$m, allowing fifth forces on cosmological scales. However such models would no longer be valid effective field theories at laboratory densities and hence would lose some of their predictive power.

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