

Weighing Neutrinos with Galaxy Cluster Surveys

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Large future galaxy cluster surveys, combined with cosmic microwave background observations, can achieve a high sensitivity to the masses of cosmologically important neutrinos. We show that a weak lensing selected sample of $\gtrsim 100,000$ clusters could tighten the current upper bound on the sum of masses of neutrino species by an order of magnitude, to a level of 0.03 eV. Since this statistical sensitivity is below the best existing lower limit on the mass of at least one neutrino species, a future detection is likely, provided that systematic errors can be controlled to a similar level.

Recent experiments have placed both stringent lower and upper bounds on the masses of neutrinos. The lower bounds derive from neutrino oscillations experiments, which measure the difference in the squared masses of the coupled species. Atmospheric neutrino oscillations showed that at least one neutrino species has a mass $\gtrsim 0.05$ eV [1], implying that neutrinos make a non-negligible contribution to dark matter. Solar neutrino oscillations showed smaller mass splittings (~ 0.008 eV [2]) (see Fig. 1 for the allowed masses of individual species).

The upper bounds come from cosmological large scale structure measurements, which are sensitive to the sum of the masses of all neutrinos. By combining CMB anisotropies from the Wilkinson Microwave Anisotropy Probe (WMAP) [5] and the clustering properties of galaxies in the Sloan Digital Sky Survey (SDSS), with no prior assumptions about the amplitude of galaxy bias, $\sum m_\nu < 1.7$ eV at 95% CL was obtained [4]. The limit can be tightened to 0.42 eV [6] by incorporating weak lensing, Ly α forest and Supernovae data, but at the expense of introducing possible new systematics [34].

The above bounds have narrowed the allowed range for the sum of the masses $\sum m_\nu$ to within an order of magnitude. In this *Letter*, we discuss the prospect of closing the gap, utilizing large, future surveys of galaxy clusters. Such surveys, covering large fractions of the sky to impressive depths, are being planned in several wavelength bands. There are several advantages of such a cluster sample. First, clusters correspond to massive dark matter potential wells, whose abundance and spatial distribution are dictated by gravity alone, and have been simulated to high accuracy [7].

Second, galaxy clusters are highly clustered relative to the mass distribution. This bias can, at least principle, be determined from simulations, and it increases the signal-to-noise in power measurements by a factor of $\sim 10-100$ (for the relevant cluster masses below). Third, a galaxy

cluster survey delivers several observables. As we will find below, the power spectrum $P_c(k)$ and abundance evolution dN/dz provide complementary constraints: the free-streaming of neutrinos suppresses fluctuation power on small scales, affecting the shape of $P_c(k)$, and also the growth rate of perturbations in the linear regime $g(z)$, affecting dN/dz . The main limitation of using galaxy clusters is that the relation between cluster mass and the actual observables needs to be known to high precision, at least statistically. As we discuss below, this should be achievable using empirical calibrations and numerical simulations.

Neutrino Signature. — Massive neutrinos cluster on very large scales, but free-stream out of small-scale dark matter potential wells and thus suppress density fluctuations on small scales. In addition, the evolution of perturbations is no longer independent of scale. The late-time evolution of perturbations in a cosmology with cold dark matter (CDM), baryons and massive neutrinos, accounting for all post-recombination effects, can be accurately treated using a product of a scale-dependent growth function (Eq. 25 in [8]) and a time-independent transfer function, reflecting conditions at the drag epoch. The latter is obtained numerically from CMBFAST [9] and includes acoustic baryonic features, which contain information comparable to the broader features at the matter-radiation equality and the sound horizon [10].

Cluster Abundance and Distribution. — Since our analysis closely follows the treatment of [11], here we only outline the methods and refer the reader to [11] for details. We use the results of numerical simulations from Jenkins *et al.* [7] for the differential comoving number density $dN/dM(M, z)$ of clusters of total mass M at redshift z (the fitting formula in their Eq. B3). Correlations in the spatial distribution of clusters are described by the power spectrum, $P_c(k)$, taking into account redshift space distortions [12] and bias [13].

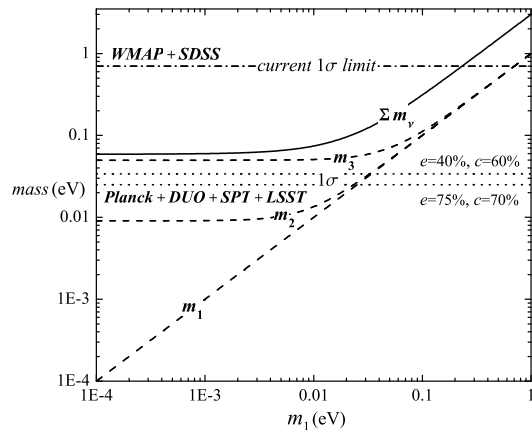


FIG. 1: The total mass and three mass eigenvalues of neutrinos as a function of the unknown smallest mass m_1 [3]. A normal hierarchy is assumed. In our analysis, we consider the scenario of three families of neutrinos with one massive and two massless. The upper dotted line shows the current most robust 1σ limit [4]. The lower dotted lines show our predicted 1σ constraints, for two different sets of weak lensing selection efficiency and completeness, from combining all three cluster surveys and Planck.

Future Cluster Surveys. — Galaxy clusters can be identified and cataloged using several different observational signatures. We consider three methods that use the X-ray emission of the hot cluster gas, the up-scattering of CMB photons in energy by this gas (Sunyaev-Zel’dovich effect, hereafter SZE), or the distortion of the images of background galaxies by weak gravitational lensing (WL). To determine the detection mass limit $M_{\min}(z)$ in each case, we adopt a flux limited X-ray survey (guided by plans of the Dark Universe Observatory, DUO [14], yielding a total of $\approx 11,500$ clusters in $6,150 \text{ deg}^2$), a temperature-decrement limited SZE survey (as planned with the South Pole Telescope, SPT [15], yielding $\approx 20,000$ clusters in $4,000 \text{ deg}^2$), and a shear-limited WL survey (planned with the Large Synoptic Survey Telescope, LSST [16], yielding $\approx 200,000$ clusters in $18,000 \text{ deg}^2$). We have additionally imposed a floor for the minimum mass of $10^{14} h^{-1} M_{\odot}$ in the X-ray and SZE surveys, since less massive halos correspond to small clusters or groups in which these two observables are subject to strong non-gravitational effects. If we imposed the same minimum mass floor for LSST, it would yield $\approx 143,000$ clusters. The mean redshifts are 0.36 for DUO, 0.58 for SPT and 0.43 for LSST (see [11], Fig. 1 for the redshift distribution of clusters). To parameterize these effects for the clusters above the detection threshold, we introduce three additional non-cosmological parameters in predicting both the X-ray flux and the SZ decrement, of the form $f_x(z) \propto A_x M^{\beta_x} (1+z)^{\gamma_x}$ and

$f_{sz} \propto A_{sz} M^{\beta_{sz}} (1+z)^{\gamma_{sz}}$, respectively. Deviations from these power-law scalings are not captured by our parameterization and would have to be addressed by allowing more general forms of cluster structure and evolution (see discussion below). For the WL survey, we assume that clusters (even at low-mass) produce shear signals with a well-defined mass-shear relation and do not introduce analogous free parameters. The main systematic error for WL surveys comes from false detections and incompleteness (discussed below).

Fisher Matrix Forecasts. — We estimate the constraints on cosmological parameters using the Fisher matrix formalism that has become a standard tool [17], allowing easy combination of different independent data sets and/or methods, by summing individual matrices. The Fisher matrix for the redshift distribution and power spectrum are described in detail in [11]. The Fisher matrix for the temperature and E-mode polarization anisotropy of the cosmic microwave background (CMB) is constructed as given by [18]. In this paper, the covariance matrix of WMAP is from the first year data, and the expected parameters of the Planck satellite [19] are listed in Table I of [11].

The Fisher formalism requires a fiducial model around which variations are considered. For the cosmological parameters, we adopt a spatially-flat, CDM model, dominated by a cosmological constant. The set of parameters included in our analysis is $\{\Omega_b h^2, \Omega_m h^2, \Omega_\nu h^2, \Omega_{\text{DE}}, w, n_s, \sigma_8, \tau\}$, where all the symbols have their standard meaning. The values are adopted from recent measurements by WMAP, as summarized in Table 1 of [5]: $\{0.024, 0.14, 0, 0.73, -1, 1, 0.9, 0.17\}$. We find that choosing $\Omega_\nu h^2 = 0.00058$ – the current lower limit – would not change our results below.

Our analysis takes into account the dominant systematic errors for the three surveys. For X-ray and SZE surveys, these come from uncertainties in the mass-observable relation due to structure and evolution of clusters. As proposed in [20, 21], however, one can use cluster surveys not only as a source of cosmological information, but also to constrain the mass-observable relation, thereby making the survey self-calibrating. To account for this, we include the parameters of the mass-observable relation in the Fisher matrix analysis: A_x ($= 10^{-4.159}$), β_x ($= 1.807$) and γ_x ($= 0$) for X-ray survey; A_{sz} ($= 10^{8.9}$), β_{sz} ($= 1.68$) and γ_{sz} ($= 0$) for SZE survey, where numbers in parentheses indicate the fiducial values for these parameters [21].

The main systematic limitation for WL surveys comes from false detections and incompleteness. In our analysis of the LSST-like WL survey, we used a constant shear S/N threshold to select clusters. False detections or missing clusters result from statistical fluctuations in these ellipticities and from projections of physical structures along the line of sight. Several papers [22, 23] have done a comprehensive study of mass-selected clusters using

TABLE I: Estimated Constraints on Total Mass of Neutrinos (in unit of eV). The DUO-like and SPT-like surveys are self-calibrated by including three non-cosmological parameters in our analysis. The errors in the LSST-like survey incorporate calibration with numerical simulations, assuming an efficiency of $e = 40\%$ and completeness $c = 60\%$; numbers in parentheses assume $e = 75\%$ and $c = 70\%$. 'W' and 'P' stand for WMAP and Planck respectively.

	DUO (6,150 deg ²)	SPT (4,000 deg ²)	LSST (18,000 deg ²)
$P_c(k)$	1.4	1.1	0.71 (0.42)
$P_c(k)+dN/dz$	0.70	0.72	0.53 (0.32)
$P_c(k)+C_\ell(W)$	0.22	0.20	0.15 (0.11)
$P_c(k)+C_\ell(W)+dN/dz$	0.16	0.15	0.12 (0.10)
$P_c(k)+C_\ell(P)$	0.11	0.10	0.086 (0.061)
$P_c(k)+C_\ell(P)+dN/dz$	0.071	0.062	0.040 (0.027)
DUO+SPT+LSST+Planck	0.034 (0.025)		

N-body simulations. They point out that because the simulations depend only on gravity, the expected cluster distribution, including false detections and missing clusters, can be reliably calculated for any cosmological model. Thus, false detections and missing clusters can be accounted for, and their presence serves only to increase the statistical error. We here adopt the efficiency $e = 40\%$ and the completeness $c = 60\%$ for a 4.5 standard deviation detection threshold [22]. To account for this, we multiply our parameter error bars for the WL survey by a correction factor of $\sqrt{[(1/e - 1) + 1/e]/c} \sim 2.6$.

Results and Discussion. — Table I shows the neutrino mass constraints from our Fisher matrix analysis for CMB, and three types of cluster surveys, including self-calibration for X-ray and SZE surveys, as well as false detection and completeness for the WL survey. The constraint from the forecast for Planck alone is $\sigma(\sum m_\nu) = 0.23$ eV [24].

For all three cluster surveys, the power spectrum, $P_c(k)$, is a much more sensitive probe of the neutrino mass than the counts, dN/dz , as expected, and, by itself, yields a constraint of $\sigma(\sum m_\nu) \sim 1$ eV. Combining with WMAP and Planck improves this by factors of ≈ 5 and ≈ 10 , respectively. Similarly, combining $P_c(k)$ with dN/dz yields a factor of 2 improvement for DUO and SPT, but only a modest change for LSST. (This is because LSST measures a smaller fraction of clusters at higher redshift, as seen from Fig. 1 of [11], therefore making dN/dz less relevant for this survey.) Hence we find that each cluster survey, combined with WMAP, yields $\sigma(\sum m_\nu) \sim 0.1$ eV. When combined with Planck, each survey gives $\sigma(\sum m_\nu) = 0.04 - 0.07$ eV, very near the interesting limit of 0.05 eV from the Super-Kamiokande atmospheric neutrino experiment [1].

For the X-ray and SZE surveys, one can ask how much could be gained by completely eliminating the systematics due to cluster structure and evolution. We can

estimate this by dropping the corresponding parameters, A_x, β_x , etc. from the Fisher matrix. When combined with Planck, each survey would give $\sigma(\sum m_\nu) = 0.02 - 0.03$ eV, which is well below the Super-K bound. While a complete elimination of systematic errors is unrealistic, it is pointed out in [25] that including the additional information from the shape of the mass function (dN/dM) allows one to largely eliminate ambiguities caused by a wide range of possible evolutions of the mass-observable relation. Here we consider reducing the degradation of constraints due to self-calibration by combining DUO and SPT together (i.e. statistically; no overlap of the survey areas is then needed). Once again with Planck, we now find $\sigma(\sum m_\nu) \approx 0.054$ eV, back below the lower limit of $\sum m_\nu \approx 0.058$ eV implied by the combination of Super-K and solar oscillation experiments.

For the WL survey, it has been demonstrated [23] that a tomographic analysis can significantly reduce projections. Also, [26] has shown that red galaxies can be used to selected clusters optically with a $< 5\%$ false detection rate. LSST will contain both lensing and optical observations of clusters, and projection effects and false detections can be removed rather than statistically subtracted. A detailed study of these and other possible enhancements, such as applying a series (rather than a single) smoothing filter to the WL shear maps, which were not considered by [22], is needed. If a completeness of 70% and efficiency of 75% can be attained without introducing a new systematic bias, then a 0.027eV sensitivity of neutrinos from LSST will be achievable. In Table I, numbers in parentheses in the LSST column correspond to $c = 70\%$ and $e = 75\%$.

We have checked the effects of several other systematics, and found them to be small: possible redshift evolution in the bias $b(z)$ [10, 11]; scatter in the mass-observable relation [27]; baryon cooling altering the density profile of clusters [28]. One important systematic error is the uncertainty of photometric redshifts. The redshift uncertainty of one single red galaxy is 0.03 for low redshifts and approximately twice worse for high redshifts [29]. On the other hand, a cluster of mass as large as $10^{14}h^{-1}M_\odot$ should have at least 25 red galaxies [30], and the redshift uncertainty of each cluster as a whole would be reduced after averaging the redshifts of the constituent galaxies. We find that an error of $\Delta z = 0.03$ would degrade the constraint on $\sum m_\nu$ from $P_c(k)$ alone by a factor of ~ 2 , as modes with a relatively large k_\parallel will be swamped by shot noise and therefore give no leverage [31]. However, after combining with dN/dz and CMB, the effect is much smaller: a $\approx 20\%$ increase on the final errors.

There are other cosmological parameters that could be added in our analysis, for example, running of the scalar spectral index (dn_s/dk) or an evolution of the dark energy equation of state (dw/dz). However, the constraint on neutrino mass derives from the evolving

scale-dependence of the growth function; an unevolving curvature in $P_c(k)$ as a function of scale, or a smooth evolution of the equation of state, can not mimic such a scale-dependent evolution. As an example, we have explicitly verified that adding an additional free parameter, $\alpha \equiv dn_s/d\ln k$ [5], increases our final error on the neutrino mass by less than 2 percent.

To conclude, taking into account self-calibration for X-ray and SZE surveys, as well as completeness and efficiency for WL surveys, we find a combined constraint for DUO + SPT + LSST + Planck of $\sigma(\sum m_\nu) = 0.034$ eV. This implies at least a 1.7σ detection of neutrino dark matter; improving WL selection efficiency and completeness would increase the significance to 2.3σ (see Table 1).

We consider how our constraints could be improved by a measurement of the power spectrum from the Ly α forest. Using the one-dimensional power spectrum $k_{\parallel}P_{1D}(k_{\parallel})$ as an observable, measured at the single value of line-of-sight wavenumber $k_{\parallel} = 1h$ Mpc $^{-1}$, we find that our constraints on $\sum m_\nu$ for all cluster surveys improve by a factor of 2 if the 1D power spectrum can be measured to an accuracy, including control of systematic errors, better than $\Delta P/P \sim 1\%$ (note this accuracy would allow a more modest improvement of 30% if a $w = -1$ prior was adopted). This yields an improved prediction of a $\gtrsim 2\sigma$ detection of the neutrino mass. We find that adding constraints from 2,000 SNe between $0 < z < 1.7$, whose magnitudes are measured to an accuracy of 0.15 mag, following the Fisher matrix analysis of [32], yields only a modest 20% improvement on $\sum m_\nu$. An additional complementary probe is tomographic cosmic shear statistics. Comparable bounds on $\sum m_\nu$ were found for an LSST-like survey combined with Planck [33].

In conclusion, our results suggest that detection of the effect of neutrino dark matter is likely to be possible. Systematic errors such as projections and false detections in the WL survey may be controlled using self-calibration and comparisons to numerical simulations. However, we emphasize that additional biases will be introduced by selection effects in real surveys. These must be carefully studied and controlled in analyzing any future survey data. Finally, we note that at the current lower limit on their mass, neglecting neutrinos from the type of analysis we outlined would result in a bias of other cosmological parameters. We find, for example, that $w \approx -0.95$ would be inferred if neutrinos are ignored in an $w = -1$ universe. Hence, it is important to include neutrinos in any analysis that aims to derive dark energy parameters to $\sim 1\%$ precision.

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- [1] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1158 (1998); E.T. Kearns, hep-ex/0210019; Y. Ashi *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **93**, 101801 (2004); E. Aliu *et al.*, hep-ex/0411038.
 - [2] K.N. Abazajian, Astropart. Phys. **19**, 303 (2003); S. Hannestad, JCAP **0305**, 004 (2003); J.N. Bahcall *et al.* JHEP **0408**, 016 (2004).
 - [3] J.F. Beacom and N.F. Bell, Phys. Rev. D **65**, 113009 (2002).
 - [4] M. Tegmark *et al.*, Phys. Rev. D **69**, 103501 (2004).
 - [5] D.N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).
 - [6] U. Seljak *et al.*, astro-ph/0407372.
 - [7] A. Jenkins *et al.*, MNRAS **321**, 372 (2001).
 - [8] W. Hu and D.J. Eisenstein, Astrophys. J. **498**, 497 (1998).
 - [9] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996); see also www.cmbfast.org.
 - [10] W. Hu and Z. Haiman, Phys. Rev. D **68**, 063004 (2003).
 - [11] S. Wang, J. Khoury, Z. Haiman and M. May, Phys. Rev. D **70**, 123008 (2004).
 - [12] N. Kaiser, MNRAS **227**, 1 (1987).
 - [13] R.K. Sheth and G. Tormen, MNRAS **308**, 119 (1999).
 - [14] See, K. Jahoda, Astron. Nachr. **324**, 132 (2003), for description of a similar, previously proposed survey, DUET.
 - [15] SPT Collaboration, J.E. Ruhl, *et al.*, Proc. SPIE-Int. Soc. Opt. Eng. **5498**, 11 (2004); see also astro.uchicago.edu/spt.
 - [16] See, www.lsst.org.
 - [17] M. Tegmark, A.N. Taylor and A.F. Heavens, Astrophys. J. **480**, 22 (1997).
 - [18] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997); M. Zaldarriaga, D.N. Spergel and U. Seljak, Astrophys. J. **488**, 1 (1997).
 - [19] See www.rssd.esa.int/index.php?project=PLANCK.
 - [20] S. Majumdar and J.J. Mohr, Astrophys. J. **585**, 603 (2003).
 - [21] S. Majumdar and J.J. Mohr, Astrophys. J. **613**, 41 (2004).
 - [22] T. Hamana, M. Takada and N. Yoshida, MNRAS **350**, 893 (2004).
 - [23] J.F. Hennawi and D.N. Spergel, astro-ph/0404349.
 - [24] D.J. Eisenstein, W. Hu and M. Tegmark, Astrophys. J. **518**, 2 (1998).
 - [25] W. Hu, Phys. Rev. D **67**, 081304 (2003).
 - [26] H.K.C. Yee, M.D. Gladders, astro-ph/0111431.
 - [27] M. Lima and W. Hu, astro-ph/0503363.
 - [28] M. White, Astropart. Phys. **22**, 211 (2004).
 - [29] N. Padmanabhan *et al.*, astro-ph/0407594.
 - [30] A. Cooray and R. Sheth, Phys. Reports, **372**, 1 (2002).
 - [31] H.-J. Seo and D.J. Eisenstein, Astrophys. J. **598**, 720 (2003).
 - [32] E.V. Linder and D. Huterer, Phys. Rev. D **67**, 081303 (2003).
 - [33] Y.-S. Song and L. Knox, Phys. Rev. D **70**, 063510 (2004).
 - [34] Although we note that prior work on neutrinos have assumed a fixed equation of state $w = -1$ for the dark energy, whereas we allow w to vary. Our results on the neutrino mass would improve by a factor of 1.5 for DUO and SPT, a factor of 2 for LSST if we had also fixed $w = -1$.