

## SECOND ORDER CORRECTIONS TO WEAK LENSING BY LARGE-SCALE STRUCTURE

ASANTHA COORAY<sup>1,2</sup> AND WAYNE HU<sup>3</sup>

<sup>1</sup>Division of Physics, Mathematics and Astronomy, California Institute of Technology, Pasadena, CA 91125.

<sup>3</sup>Center for Cosmological Physics, Department of Astronomy and Astrophysics, and Enrico Fermi Institute, University of Chicago, Chicago, IL 60637.

E-mail: asante@caltech.edu, whu@background.uchicago.edu

*Draft version February 25, 2002*

### ABSTRACT

We calculate corrections to the power spectrum predictions of weak lensing by large scale structure due to higher order effects in the gravitational potential. Using a perturbative approach to third order in transverse displacements, we calculate a second order correction to the angular power spectra of  $E$  and  $B$  mode shear and convergence resulting from dropping the so-called Born approximation, where one integrates along the unperturbed photon path. We also consider a correction to the power spectra from the coupling between lenses at different redshifts. Both effects generate  $B$ -mode shear and the latter also causes a net rotation of the background galaxy images. We show all these corrections are at least two orders of magnitude below the convergence or  $E$ -mode power and hence relevant only to future ultra high precision measurements. These analytical calculations are consistent with previous numerical estimates and validate the use of current large scale structure weak lensing predictions for cosmological studies and future use of  $B$ -modes as a monitor of systematic effects.

*Subject headings:*

#### 1. INTRODUCTION

As experiments that measure the distortion induced in distant galaxy images by weak gravitational lensing from the large-scale structure of the universe (e.g., Blandford et al 1991; Miralda-Escudé 1991; Kaiser 1992) move from the current discovery phase (Bacon et al 2000; Kaiser et al 2000; Wittman et al 2000; Van Waerbeke et al 2000) into the precision measurement phase, it will become increasingly important to quantify and separate subtle cosmological, astrophysical and instrumental effects that alter the statistics of the lensing observables.

Current predictions of weak lensing statistics and their utility in measuring fundamental cosmological parameters are based on several assumptions. In this Letter, we discuss two of these assumptions and the extent of their validity for calculations of the power spectra of the shear and convergence. In the so-called Born approximation, one integrates the lensing distortion over the unperturbed photon paths. We relax this assumption using the transverse deflection as a perturbative parameter. We show that the correction to the Born approximation results in a generation of curl, or magnetic-like,  $B$ -modes in the weak lensing shear. Absent in first order lensing contributions,  $B$ -modes are often used as a monitor of subtle systematic errors in the data. Indeed, data from the current generation of surveys routinely show  $B$ -modes in the shear field (e.g. Pen et al 2002). It will be important for future surveys to know the level at which lensing itself produces  $B$ -modes.

An additional assumption is the neglect of the coupling between lenses at two different redshifts. Unlike the correction to the Born approximation, the lens-lens coupling results in both  $B$ -mode generation and a net rotation of galaxy images. Since the rotational effect has been measured in numerical simulations (Jain et al 2000), it provides a check on the validity of our perturbative technique.

The physical mechanism behind the two corrections discussed in the present paper were first considered by Bernardeau et al (1997) and Schneider et al (1998). In both these studies, these two second order corrections were discussed as a possible contribution to the three point statistics of weak lensing. The correction to the weak lensing convergence skewness, however, is below a few percent. In this paper, we concentrate on the two point statistics which requires a third order expansion in the potential to complete a second order expansion in the power.

For illustrative purposes, we calculate these higher order effects for a flat  $\Lambda$ CDM cosmology throughout, with parameters  $\Omega_c = 0.3$ ,  $\Omega_b = 0.05$ ,  $\Omega_\Lambda = 0.65$ ,  $h = 0.65$ ,  $n = 1$ ,  $\delta_H = 4.2 \times 10^{-5}$  and no tensor contribution.

#### 2. STATISTICAL PROPERTIES

In this section, we define the statistical properties of the lensing observables in angular Fourier space. In general, we can write the weak lensing deformation matrix that maps between the source (S) and image (I) planes,  $\delta x_i^S = A_{ij} \delta x_j^I$ , as

$$\mathbf{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 - \omega \\ -\gamma_2 + \omega & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (1)$$

where all components are functions of position on the sky  $\mathbf{n}$ . Here,  $\kappa$  is the convergence and  $\gamma_i$  are the two shear components and  $\omega$ , the antisymmetric component induces a rotation in the images (e.g., Bartelmann & Schneider 2000 and Mellier 1999 for recent reviews). The deformation component may be isolated as

$$\psi_{ab}(\mathbf{n}) = \delta_{ab} - A_{ab}(\mathbf{n}). \quad (2)$$

Following Kaiser (1998), we define the Fourier representation of the deformation under the flat-sky approximation

$$\psi_{ab}(\mathbf{l}) = \int d\mathbf{n} e^{-i\mathbf{l}\cdot\mathbf{n}} \psi_{ab}(\mathbf{n}). \quad (3)$$

<sup>2</sup>Sherman Fairchild Senior Research Fellow

Statistical homogeneity requires that its two-point correlation satisfy

$$\langle \psi_{ab}^*(\mathbf{l}) \psi_{cd}(\mathbf{l}') \rangle = (2\pi)^2 \delta(\mathbf{l} - \mathbf{l}') C_{abcd}(\mathbf{l}). \quad (4)$$

The various components of  $C_{abcd}$  define the two point statistical properties of the lensing observables. It is useful first to reexpress the shear components of  $\psi_{ab}(\mathbf{l})$  in terms of the coordinate-free  $E$  and  $B$  mode representation (see, e.g. Stebbins 1996; Kamionkowski et al 1998; Hu & White 2001)

$$\begin{aligned} \epsilon(\mathbf{l}) &= \cos 2\phi_l \gamma_1(\mathbf{l}) + \sin 2\phi_l \gamma_2(\mathbf{l}), \\ \beta(\mathbf{l}) &= \cos 2\phi_l \gamma_2(\mathbf{l}) - \sin 2\phi_l \gamma_1(\mathbf{l}). \end{aligned} \quad (5)$$

where  $\cos \phi_l = \mathbf{l} \cdot \hat{\mathbf{x}}_1$  where  $\hat{\mathbf{x}}_1$  is one of the orthogonal directions on the sky.

The power and cross spectra of the observable fields  $\alpha$ ,  $\beta = \kappa$ ,  $\epsilon$ ,  $\beta$ ,  $\omega$  are defined as

$$\langle \alpha^*(\mathbf{l}) \beta(\mathbf{l}') \rangle \equiv (2\pi)^2 \delta(\mathbf{l} - \mathbf{l}') C_l^{\alpha\beta}. \quad (6)$$

Since the power spectra are functions of the magnitude of  $\mathbf{l}$  only, one can take  $\mathbf{l} \parallel \mathbf{x}_1$  to simplify calculations of the power spectrum without loss of generality. With this restriction,  $\phi_l = 0$  and we obtain for the power spectra

$$\begin{aligned} C_l^{\kappa\kappa} &= \frac{1}{4} [C_{1111} + 2C_{1122} + C_{2222}], \\ C_l^{\epsilon\epsilon} &= \frac{1}{4} [C_{1111} - 2C_{1122} + C_{2222}], \\ C_l^{\beta\beta} &= \frac{1}{4} [C_{1212} + 2C_{1221} + C_{2121}], \\ C_l^{\omega\omega} &= \frac{1}{4} [C_{1212} - 2C_{1221} + C_{2121}], \end{aligned}$$

and similarly for the cross spectra. Cross spectra between  $\kappa$  or  $\epsilon$  with  $\beta$  or  $\omega$  vanish if the statistical properties are invariant under inversion of the coordinates.

### 3. CALCULATIONAL TECHNIQUE

It remains to evaluate the deformation power spectrum  $C_{abcd}$  to second order in perturbation theory. The deformation tensor for sources at a single redshift is given implicitly by (see e.g. Schneider et al 1998)

$$\psi_{ab}(\mathbf{n}, \chi_s) = 2 \int d\chi g(\chi, \chi_s) \Phi_{,ac}(\mathbf{x}; \chi) [\delta_{cb} + \psi_{cb}(\mathbf{n}, \chi)], \quad (7)$$

where commas represents spatial derivatives. Repeated indices are summed over the 2 transverse directions. The presence of  $\psi$  in the integral reflects a foreground lens affecting the deformation from a more distant lens - or ‘‘lens-lens coupling’’. The lensing efficiency is

$$\begin{aligned} g(\chi', \chi) &= \frac{d_A(\chi') d_A(\chi - \chi')}{d_A(\chi)}, \quad \chi' < \chi, \\ &= 0, \quad \chi' \geq \chi. \end{aligned} \quad (8)$$

Here the gravitational potential  $\Phi$  is evaluated at a deflected position<sup>2</sup>

$$\mathbf{x}(\mathbf{n}, \chi) = \mathbf{n} d_A(\chi) + \delta \mathbf{x}(\mathbf{n}, \chi),$$

<sup>2</sup>Formally  $d_A(\chi) \mathbf{n} \rightarrow d_A(\chi) \mathbf{n}_\perp + \chi \mathbf{n}_\parallel$  in an open universe but components parallel to the fiducial line of sight drop out in the Limber approximation.

$$\delta x_a(\mathbf{n}, \chi) = -2 \int d\chi' g(\chi', \chi) \frac{d_A(\chi)}{d_A(\chi')} \Phi_{,a}(\mathbf{x}; \chi'), \quad (9)$$

where the deflections are confined to the transverse plane and  $\chi$  is the conformal distance or lookback time, from the observer, given by

$$\chi(z) = \int_0^z \frac{dz'}{H(z')}, \quad (10)$$

and the analogous angular diameter distance

$$d_A(\chi) = H_0^{-1} \Omega_K^{-1/2} \sinh(H_0 \Omega_K^{1/2} \chi), \quad (11)$$

with  $\Omega_K = 1 - \Omega_{\text{tot}}$  as the space curvature parameter. We will occasionally suppress the radial/temporal argument  $\chi$  where no confusion will arise.

The familiar form of the lensing observables comes about by keeping only first order terms in the potential. In other words, they are calculated under the so-called Born approximation where the potential is evaluated on the undeflected path and with the neglect of lens-lens coupling. Higher order corrections can then be calculated by iterative correction of  $\mathbf{x}$  and  $\psi_{ab}$  in Eq. (7).

We shall see that in general the deformation may be expressed as a line-of-sight projection of a source field

$$\psi_{ab}(\mathbf{n}) = 2 \int d\chi g(\chi, \chi_s) S_{ab}(\mathbf{n} d_A; \chi), \quad (12)$$

whose power spectrum may be evaluated under the Limber approximation (Kaiser 1992) from the power spectrum of the source field

$$\begin{aligned} \langle S_{ab}^*(\mathbf{l}; \chi) S_{cd}(\mathbf{l}'; \chi') \rangle &= (2\pi)^2 \delta(\mathbf{l} - \mathbf{l}') \delta(\chi - \chi') \\ &\quad \times d_A(\chi)^{-6} P_{abcd}(\mathbf{l}; \chi), \end{aligned} \quad (13)$$

to be

$$C_{abcd}(\mathbf{l}) = 4 \int d\chi \frac{g^2(\chi, \chi_s)}{d_A^6(\chi)} P_{abcd}(\mathbf{l}; \chi). \quad (14)$$

For the first order term  $S_{ab}^{(1)} = \Phi_{,ab}$  and

$$P_{abcd}^{(11)} = l_a l_b l_c l_d P_{\Phi\Phi} \left( \frac{l}{d_A} \right), \quad (15)$$

where  $P_{\Phi\Phi}(k)$  is the spatial power spectrum of the potential fluctuations evaluated under the non-linear scaling relations for the density power spectrum (Peacock & Dodds 1996).

As discussed in the previous section, one may evaluate the power spectra of the observables under the constraint  $l_1 = l$  and  $l_2 = 0$ , from which one immediately obtains

$$C_l^{\kappa\kappa 11} = l^4 \int_0^{\chi_s} d\chi \frac{g^2(\chi, \chi_s)}{d_A(\chi)^6} P_{\Phi\Phi} \left( \frac{l}{d_A(\chi)}; \chi \right), \quad (16)$$

and  $C_l^{\epsilon\epsilon 11} = C_l^{\kappa\kappa 11}$  while  $C_l^{\beta\beta 11} = C_l^{\omega\omega 11} = 0$ . Here the subscripts  $nn$  denote  $n$ th order term expansion in the potentials. The lack of  $B$ -modes in the signal, to first order in the perturbations, has been considered as a possible test of instrumental and astrophysical systematic effects.

In general if  $S_{ab}$  is symmetric under interchange of  $a$  and  $b$ , power spectra involving the rotation  $\omega$  vanish. If  $P_{abcd}$  is symmetric under the exchange of all indices then  $C_l^{\kappa\kappa} = C_l^{\epsilon\epsilon} + C_l^{\beta\beta}$ . Finally if  $P_{1122} = P_{1221}$  then  $C_l^{\kappa\kappa} + C_l^{\omega\omega} = C_l^{\epsilon\epsilon} + C_l^{\beta\beta}$ . These relationships are useful for checking consistency in the higher order terms.

### 3.1. Born approximation

Corrections to the Born approximation can be calculated by Taylor expanding the potential in Eq. (7) to second order in the deflection of Eq. (9)

$$\begin{aligned} \Phi(\mathbf{n}d_A + \delta\mathbf{x}) &= \Phi(\mathbf{n}d_A) + \delta x_a \Phi_{,a}(\mathbf{n}d_A) \\ &+ \frac{1}{2} \delta x_a \delta x_b \Phi_{,ab}(\mathbf{n}d_A) + \dots \end{aligned} \quad (17)$$

The last term is effectively third order in the potential but must be kept since it can couple with the first order deformation in the power spectrum. The corrections may be represented as second order and third order source fields

$$\begin{aligned} S_{ab}^{(2)}(\mathbf{n}) &= -2\Phi_{,abc}(\mathbf{n}d_A)B_c(\mathbf{n}), \\ S_{ab}^{(3)}(\mathbf{n}) &= 2\Phi_{,abcd}(\mathbf{n}d_A)B_c(\mathbf{n})B_d(\mathbf{n}), \end{aligned} \quad (18)$$

with the Born correction

$$B_c(\mathbf{n}; \chi) \equiv \int d\chi' g(\chi', \chi) \frac{d_A(\chi)}{d_A(\chi')} \Phi_{,c}(\mathbf{n}d_A(\chi'); \chi'). \quad (19)$$

The second-second order terms in the power spectrum become

$$P_{abcd}^{(22)} = 4 \int \frac{d^2 l'}{(2\pi)^2} l'_a l'_b l'_c l'_d (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''), \quad (20)$$

with  $\mathbf{l}'' = \mathbf{1} - \mathbf{l}'$ . The mode coupling integrand is given by

$$\begin{aligned} M(l, l'; \chi) &= \int d\chi' \frac{g^2(\chi, \chi')}{d_A^6(\chi')} P_{\Phi\Phi} \left( \frac{l'}{d_A(\chi')}; \chi' \right) \\ &\times P_{\Phi\Phi} \left( \frac{l}{d_A(\chi)}; \chi \right). \end{aligned} \quad (21)$$

In terms of the lensing observables,

$$\begin{aligned} C_l^{\alpha\beta 22} &= 4 \int d\chi \frac{g^2(\chi, \chi_s)}{d_A(\chi)^6} \int \frac{d^2 l'}{(2\pi)^2} l'^4 G_\alpha^{(22)} G_\beta^{(22)} \\ &\times (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''). \end{aligned} \quad (22)$$

The geometric factors for the specific observables are

$$\begin{aligned} G_\kappa^{(22)} &= 1, \\ G_\epsilon^{(22)} &= \cos \phi_l, \\ G_\beta^{(22)} &= \sin \phi_l, \\ G_\omega^{(22)} &= 0, \end{aligned} \quad (23)$$

where we have used the condition  $\phi_l = 0$ . Notice that the parity violating cross terms vanish after integration over the azimuthal angle.

Likewise the first-third order source term becomes

$$P_{abcd}^{(13)} = -4l_a l_b l_c l_d \int \frac{d^2 l'}{(2\pi)^2} (\mathbf{1} \cdot \mathbf{l}')^2 M(l, l'). \quad (24)$$

Note that there is no mode coupling in the third order term. For the lensing observables,

$$\begin{aligned} C_l^{\alpha\beta 13} &= -4 \int d\chi \frac{g^2(\chi, \chi_s)}{d_A(\chi)^6} \int \frac{d^2 l'}{(2\pi)^2} l'^4 G_\alpha^{(13)} G_\beta^{(13)} \\ &\times (\mathbf{1} \cdot \mathbf{l}')^2 M(l, l'), \end{aligned} \quad (25)$$

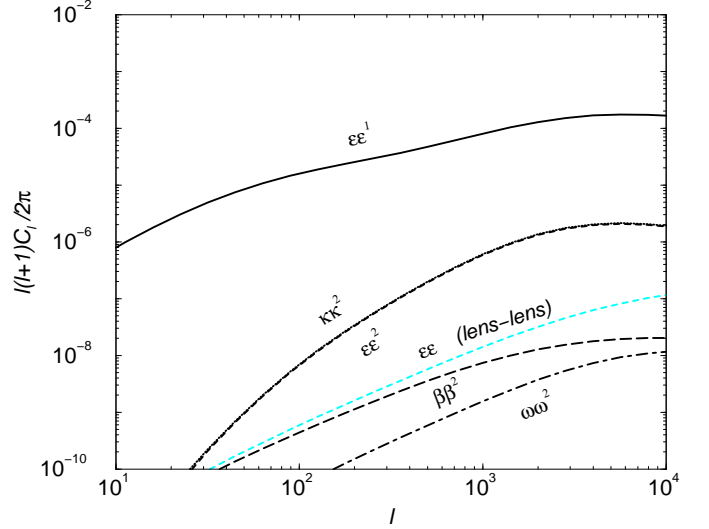


FIG. 1.— Angular power spectra of weak lensing. The solid line is the well known first order result. The dot, dashed, long-dashed and dot-dashed lines are the total, Born approximation plus lens-lens coupling, second order contribution to lensing observables involving convergence, shear  $\epsilon$ -modes, shear  $\beta$ -modes, and rotation, respectively. For comparison, the gray dashed line shows the contribution to shear  $\epsilon$ -modes from lens-lens coupling only;  $\kappa\kappa$  and  $\beta\beta$  scale similarly. Note that the rotational power spectrum,  $\omega\omega$ , is also generated by lens-lens coupling only.

where

$$\begin{aligned} G_\kappa^{(13)} &= G_\epsilon^{(13)} = 1, \\ G_\beta^{(13)} &= G_\omega^{(13)} = 0. \end{aligned} \quad (26)$$

Notice that contributions to the power from modes with  $\mathbf{l}' = \mathbf{1} - \mathbf{l}'' \approx \mathbf{1}$  in Eq. (22) is cancelled by contributions from Eq. (13) for  $\kappa\kappa$  and  $\epsilon\epsilon$ . Finite  $\beta\beta$  power is generated by the Born corrections but is also highly suppressed geometrically. There is also no contribution to the rotational power from the second order Born approximation. The rotational power spectrum as measured in simulations by Jain et al (2000) should not be interpreted as a test of the Born approximation.

### 3.2. Lens-Lens coupling

Lens-lens coupling involves the iterative correction of the lensing deformation of distant lenses due to the deformation from foreground lenses in Eq. (7). The second and third order corrections become

$$\begin{aligned} S_{ab}^{(2)}(\mathbf{n}) &= -\Phi_{,ac}(\mathbf{n}d_A; \chi) \\ &\times 2 \int d\chi' g(\chi', \chi) \Phi_{,cb}(\mathbf{n}d_A(\chi'); \chi'), \\ S_{ab}^{(3)}(\mathbf{n}) &= \Phi_{,ac}(\mathbf{n}d_A; \chi) \\ &\times 2 \int d\chi' g(\chi', \chi) \Phi_{,cd}(\mathbf{n}d_A(\chi'); \chi') \\ &\times 2 \int d\chi'' g(\chi'', \chi') \Phi_{,db}(\mathbf{n}d_A(\chi''); \chi''). \end{aligned} \quad (27)$$

The second-second order terms in the power spectrum become

$$P_{abcd}^{(22)} = 4 \int \frac{d^2 l'}{(2\pi)^2} l'_a l'_b l'_c l'_d (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''), \quad (28)$$

with  $\mathbf{l}'' = \mathbf{1} - \mathbf{l}'$  and recall that the mode coupling integrand was defined in Eq. (21). The first-third order term vanishes under the Limber approximation since lenses at the same redshift cannot lens each other.

The results for the power spectra are

$$C_l^{\alpha\beta 22} = 4 \int d\chi \frac{g^2(\chi, \chi_s)}{d_A(\chi)^6} \int \frac{d^2 l'}{(2\pi)^2} l'^2 l''^2 G_\alpha^{(22)} G_\beta^{(22)} \times (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''), \quad (29)$$

with the geometric factors

$$\begin{aligned} G_\kappa^{(22)} &= \cos(\phi_{l'} - \phi_{l''}), \\ G_\epsilon^{(22)} &= \cos(\phi_{l'} + \phi_{l''}), \\ G_\beta^{(22)} &= \sin(\phi_{l'} + \phi_{l''}), \\ G_\omega^{(22)} &= \sin(\phi_{l'} - \phi_{l''}). \end{aligned} \quad (30)$$

Recall we have set  $\phi_l = 0$ . As derived, lens-lens coupling generates power in all the lensing observables. Again, notice that the parity violating terms oscillate leaving negligible contribution after azimuthal integration.

### 3.3. Born-Lens coupling

There are also cross terms between corrections to the Born approximation and lens-lens coupling. The second-second order term follows directly from Eqs. (18) and (27)

$$P_{abcd}^{(22)} = 4 \int \frac{d^2 l'}{(2\pi)^2} (l'_a l'_b l'_c l'_d + l'_a l'_b l'_c l'_d) (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''), \quad (31)$$

where again  $\mathbf{l}'' = \mathbf{1} - \mathbf{l}'$ .

The second-second order terms for the lensing observables are likewise simply related to the pure Born and lens-lens corrections

$$C_l^{\kappa\kappa 22} = 4 \int d\chi \frac{g^2(\chi, \chi_s)}{d_A(\chi)^6} \int \frac{d^2 l'}{(2\pi)^2} l'^3 l'' G_\alpha^{(22)} G_\beta^{(22)} \times (\mathbf{l}' \cdot \mathbf{l}'')^2 M(l', l''), \quad (32)$$

where the geometric factors are simply products of the individual Born (B) and lens-lens (LL) factors

$$G_\alpha^{(22)} G_\beta^{(22)} = G_\alpha^{(22)} \Big|_B G_\beta^{(22)} \Big|_{LL} + G_\alpha^{(22)} \Big|_{LL} G_\beta^{(22)} \Big|_B, \quad (33)$$

given in Eqns. (23) and (30).

For the third order term

$$S_{ab}^{(3)}(\mathbf{n}) = 2\Phi_{,acd}(\mathbf{n}d_A) B_d(\mathbf{n}) \times 2 \int d\chi' g(\chi', \chi) \Phi_{,cb}(\mathbf{n}d_A(\chi'); \chi'), \quad (34)$$

and

$$P_{abcd}^{(13)} = 16 \int \frac{d^2 l'}{(2\pi)^2} (l_a l_b l_c l'_d + l_a l'_b l_c l_d) (\mathbf{l} \cdot \mathbf{l}')^2 M(l, l'). \quad (35)$$

This implies that all power spectra of lensing observables vanish after integration over the azimuthal angle  $\phi_{l'}$ .

## 4. RESULTS & DISCUSSION

Here, we presented a general discussion of the accuracy of first order calculations of statistical properties of weak lensing by large-scale structure. Using a perturbative approach to third order in transverse displacements, we calculated the leading order corrections to angular power spectra of the shear and convergence that results from dropping the Born approximation. We also considered the coupling between lenses at two different redshifts in its contribution to the power spectra of lensing observables. Both effects generate power in the  $B$ -mode shear which places an ultimate limit on its use as a monitor of systematics or for the search for weak lensing by gravitational waves. In addition, lens-lens coupling results in a net rotation of the background galaxy images.

In Fig. 1, we summarize our results and show the total contribution to the lensing observables at the second order level in power from corrections involving the Born approximation and lens-lens couplings. As shown, these corrections are at least two orders of magnitude below the power in the convergence, or  $E$ -mode shear, at the first order level. While these corrections are larger than cosmic variance at  $\ell \gtrsim 100$ , they are unlikely to affect the interpretation of the next generation of surveys. Our analytical calculations are consistent with previous numerical estimates: the rotational power spectrum generated by the coupling between two lenses agree with numerical measurements by Jain et al (2000). These results also validate the use of current large scale structure weak lensing predictions for cosmological studies. The relatively small contribution to the lensing observables suggest that higher order corrections related to the Born approximation and coupling between two lenses are unlikely to affect the current estimates of weak lensing statistics and place only very mild limitations on the use of  $B$ -modes as a monitor of systematics in future surveys.

*Acknowledgments:* WH was supported by NASA NAG5-10840 and the DOE OJI program. AC was supported by the Sherman Fairchild foundation and the DOE. We thank Marc Kamionkowski for useful discussions.

## REFERENCES

- Bacon, D., Refregier, A., Ellis R. 2000, MNRAS, 318, 625
- Bartelmann, M., Schneider, P. 2000, Phys. Rep., 340, 291
- Bernardeau, F., van Waerbeke, L., Mellier, Y. 1997, A&A, 322, 1
- Blandford, R. D., Saust, A. B., Brainerd, T. G., Villumsen, J. V. 1991, MNRAS 251, 60
- Hu W., White M. 2001, ApJ 554, 67
- Jain, B., Seljak, U. & White, M. 2000, ApJ 530, 547
- Kaiser, N. 1992, ApJ, 388, 286
- Kaiser, N. 1998, ApJ, 498, 26
- Kaiser, N. Wilson, G., Luppino G. 2000, astro-ph/0003338
- Kamionkowski, M., Babul, A., Cress, C. M., Refregier, A. 1998, MNRAS, 301, 1064.
- Limber, D. 1954, ApJ, 119, 655
- Mellier, Y., 1999, ARA&A, 37, 127.
- Miralda-Escudé J. 1991, ApJ 380, 1
- Peacock, J.A., Dodds, S.J. 1996, MNRAS, 280, L19
- Pen, U.-L., van Waerbeke, L., Mellier, Y. 2002, ApJ, in press (astro-ph/0109182)
- Schneider P., van Waerbeke, L., Jain, B., Guido, K. 1998, MNRAS, 296, 873
- Stebbins, A. 1996, preprint, astro-ph/9609149
- Van Waerbeke, L., Mellier, Y., Erben, T. et al. 2000, A&A, 358, 30.
- Wittman, D. M., Tyson, J. A., Kirkman, D., Dell'Antonio, I., Bernstein, G. 2000, Nature submitted, astro-ph/0003014