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WEAK LENSING BY LARGE-SCALE STRUCTURE: A DARK MATTER HALO APPROACH

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ABSTRACT

Weak gravitational lensing observations probe the spectrum and evolution of density fluctuations and the cosmological parameters that govern them, but they are currently limited to small fields and subject to selection biases. We show how the expected signal from large-scale structure arises from the contributions from and correlations between individual halos. We determine the convergence power spectrum as a function of the maximum halo mass and so provide the means to interpret results from surveys that lack high-mass halos either through selection criteria or small fields. Since shot noise from rare massive halos is mainly responsible for the sample variance below 10′, our method should aid our ability to extract cosmological information from small fields.

Subject headings: cosmology: theory — gravitational lensing — large-scale structure of universe

1. INTRODUCTION

Weak gravitational lensing of faint galaxies probes the distribution of matter along the line of sight. Lensing by large-scale structure (LSS) induces correlations in the galaxy ellipticities at the percent level (e.g., Blandford et al. 1991; Miralda-Escudé 1991; Kaiser 1992). Although challenging to measure, these correlations provide important cosmological information that is complementary to that supplied by the cosmic microwave background and potentially as precise (e.g., Jain & Seljak 1997; Bernardeau, van Waerbeke, & Mellier 1997; Kaiser 1998; Schneider et al. 1998; Hu & Tegmark 1999; Cooray 1999; van Waerbeke, Bernardeau, & Mellier 1999; see Bartelmann & Schneider 2000 for a recent review). Indeed, several recent studies have provided the first clear evidence for weak lensing in so-called blank fields (van Waerbeke et al. 2000; Bacon, Refregier, & Ellis 2000; Wittman et al. 2000).

Weak lensing surveys are currently limited to small fields that may not be representative of the universe as a whole, owing to sample variance. In particular, rare massive objects can contribute strongly to the mean power in the shear or convergence but are not present in the observed fields. The problem is compounded if one chooses blank fields subject to the condition that they do not contain known clusters of galaxies. Our objective in this Letter is to quantify these effects and to understand what fraction of the total convergence power spectrum should arise from lensing by individual massive clusters as a function of scale.

In the context of standard cold dark matter (CDM) models for structure formation, the dark matter halos that are responsible for lensing have properties that have been intensely studied by numerical simulations. In particular, analytic scalings and fits now exist for the abundance, profile, and correlations of halos of a given mass. We show how the convergence power spectrum predicted in these models can be constructed from these halo properties. The critical ingredients are the Press-Schechter (1974, hereafter PS) formalism for the mass function, the Navarro, Frenk, & White (1996, hereafter NFW) profile, and the halo bias model of Mo & White (1996). Following Seljak (2000), we modify the halo profile parameters, specifically the concentration, so that halos account for the full nonlinear dark matter power spectrum, and we generalize his treatment so as to be applicable through all redshifts relevant to current galaxy ellipticity measurements of LSS lensing. This calculational method allows us to determine the contributions to the convergence power spectrum of halos of a given mass.

Throughout this Letter, we will take ΛCDM as our fiducial cosmology, with parameters $\Omega_c = 0.30$ for the CDM density, $\Omega_b = 0.05$ for the baryon density, $\Omega_\Lambda = 0.65$ for the cosmological constant, $h = 0.65$ for the dimensionless Hubble constant, and a scale-invariant spectrum of primordial fluctuations, normalized to galaxy cluster abundances ($\sigma_8 = 0.9$; see Viana & Liddle 1999) and consistent with *COBE* (Bunn & White 1997). For the linear power spectrum, we take the fitting formula for the transfer function given in Eisenstein & Hu (1999).

2. LENSING BY HALOS

2.1. *Halo Profile*

We model dark matter halos as NFW profiles with a density distribution

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} . \tag{1}$$

The density profile can be integrated and related to the total dark matter mass of the halo within r_v:

$$M = 4\pi\rho_s r_s^3 \left[\log(1 + c) - \frac{c}{1 + c}\right] , \tag{2}$$

where the concentration c is defined as r_v/r_s. Choosing r_v as the virial radius of the halo, spherical collapse tells us that $M = 4\pi r_v^3 \Delta(z)\rho_b/3$, where $\Delta(z)$ is the overdensity of collapse (see, e.g., Henry 2000) and ρ_b is the background matter density today. We use comoving coordinates throughout. By equating these two expressions, one can eliminate ρ_s and describe the halo by its mass M and concentration c. Finally, we can determine a relation between M and c such that halo distribution produces the same power as the nonlinear dark matter power spectrum, as outlined in Seljak (2000).

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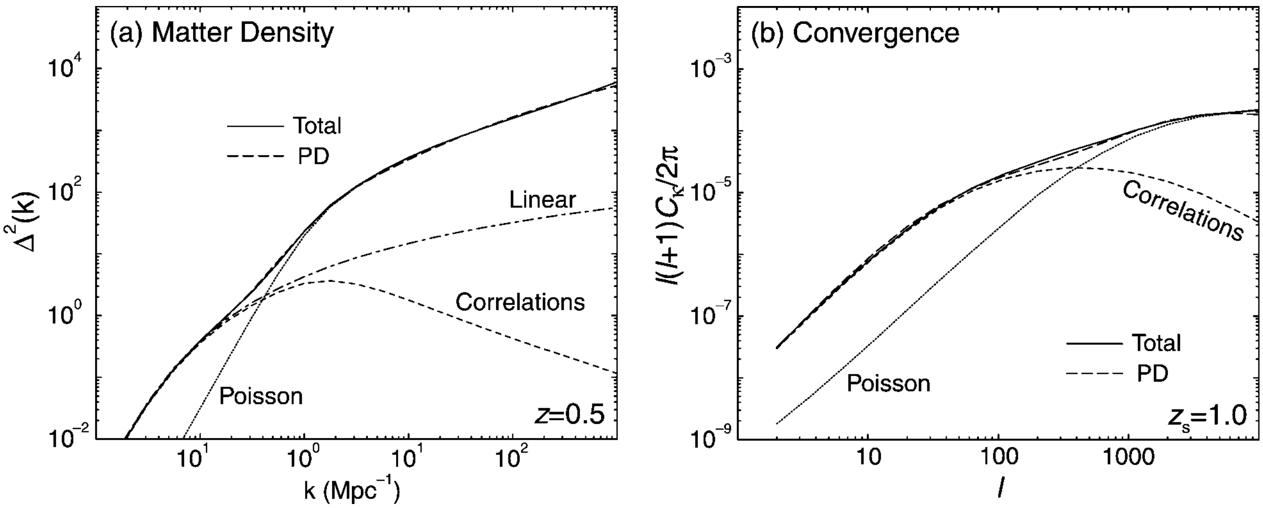

FIG. 1.—Halo model power spectra. (*a*) Dark matter power spectrum at redshift of 0.5. (*b*) Convergence power spectrum with $z_s = 1$. The sum of the Poisson (*dotted line*) and correlation (*dashed line*) contribution (*solid line*) compares well with that predicted by the nonlinear power spectrum based on the PD fitting function (*long-dashed line*). In (*a*), the linear matter density power spectrum is shown with a dot-dashed line.

2.2. *Convergence Power Spectrum*

For lensing convergence, we need the projected surface mass density, which is the line-of-sight integral of the profile

$$\Sigma(r_\perp) = \int_{-r_v}^{+r_v} \rho(r_\perp, r_\parallel) dr_\parallel, \quad (3)$$

where $r_\parallel$ is the line-of-sight distance and $r_\perp$ is the perpendicular distance. As in equation (2), the cutoff here at the virial radius reflects the fact that we only account for mass contributions out to r_v (see Bartelmann 1996 for an analytical description when $r_v \to \infty$). The convergence on the sky $\kappa(\theta)$ is related to surface mass density through

$$\kappa(\theta) = \left(\frac{4\pi G}{c^2}\frac{d_l d_{ls}}{d_s}\right)(1 + z_l)\Sigma(d_l\theta), \quad (4)$$

where the extra factor of $(1 + z_l)$ from the familiar expression comes from the use of comoving coordinates to define densities and distances; e.g., d_l, d_s, and d_{ls} are the *comoving* angular diameter distances from the observer to lens, from the observer to source, and from the lens to source, respectively.

The total convergence power spectrum due to halos $C_\kappa^{\rm tot}$ can be split into two parts: a Poisson term $C_\kappa^{\rm P}$ and a term involving correlations between individual halos C_κ^C. This split was introduced by Cole & Kaiser (1988) to examine the power spectrum of the Sunyaev & Zeldovich (1980, hereafter SZ) effect due to galaxy clusters (see Komatsu & Kitayama 1999 and references therein for more recent applications).

The Poisson term due to individual halo contributions can be written as

$$C_\kappa^{\rm P}(l) = \int_0^{z_s} dz \frac{d^2V}{dz\, d\Omega} \int_{M_{\rm min}}^{M_{\rm max}} dM \frac{dn(M, z)}{dM} [\kappa_l(M, z)]^2. \quad (5)$$

where z_s is the redshift of background sources, $d^2V/dzd\Omega$ is the comoving differential volume, and

$$\kappa_l = 2\pi \int_0^{\theta_v} \theta d\theta\, \kappa(\theta) J_0\left[\left(l + \frac{1}{2}\right)\theta\right] \quad (6)$$

is the two-dimensional Fourier transform of the halo profile in the flat-sky approximation. The halo mass distribution as a function of redshift $[dn(M, z)/dM]$ is determined through the PS formalism.

Here we have assumed that all sources are at a single redshift; for a distribution of sources, one integrates over the normalized background source redshift distribution. The minimum $M_{\rm min}$ and maximum $M_{\rm max}$ masses can be varied to study the effects of rare and excluded high-mass halos.

The clustering term arises from correlations between halos of different masses. By assuming that the linear matter density power spectrum $P(k, z)$ is related to the power spectrum of halos over the whole mass range via a redshift-dependent linear bias term $b(M, z)$, we can write the correlation term as

$$C_\kappa^C(l) = \int_0^{z_s} dz \frac{d^2V}{dz\, d\Omega} P\left(\frac{l}{d_l}, z\right) \times \left[\int_{M_{\rm min}}^{M_{\rm max}} dM \frac{dn(M, z)}{dM} b(M, z)\kappa_l(M, z)\right]^2. \quad (7)$$

Here we have utilized the Limber approximation (Limber 1954) by setting $k = l/d_l$. Mo & White (1996) find that the halo bias can be described by $b(M, z) = 1 + [\nu^2(M, z) - 1]/\delta_c$, where $\nu(M, z) = \delta_c/\sigma(M, z)$ is the peak-height threshold, $\sigma(M, z)$ is the rms fluctuation within a top-hat filter at the virial radius corresponding to mass M, and δ_c is the threshold overdensity of spherical collapse (see Henry 2000 for useful fitting functions).

3. RESULTS

Following the approach given in Seljak (2000), we first test the halo prescription against the full nonlinear density power

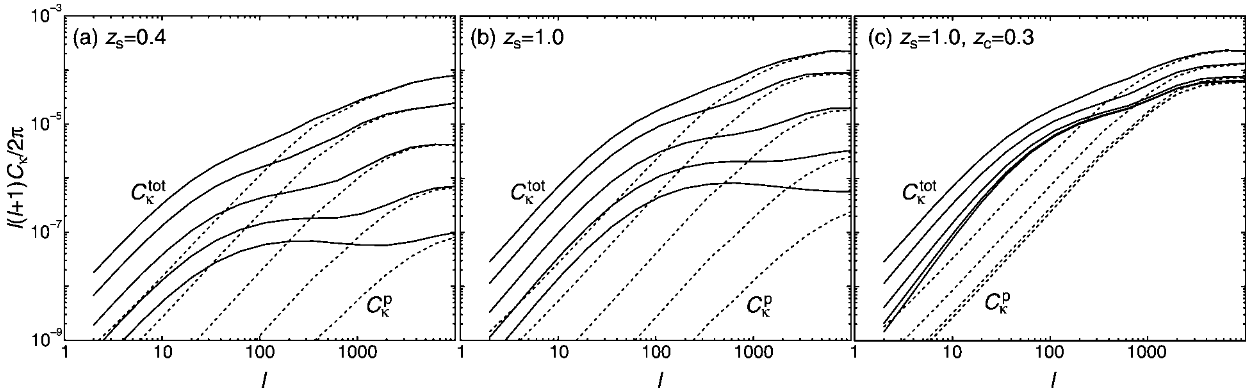

FIG. 2.—Lensing convergence as a function of maximum halo mass. In increasing order, for both Poisson (*dotted lines*) and total contributions (*solid lines*), the maximum mass is 10^{11}, 10^{12}, 10^{13}, 10^{14}, and 10^{15} $M_\odot$. The sources are at (*a*) $z_s = 0.4$, (*b*) $z_s = 1$, and (*c*) $z_s = 1$, with mass cutoff applied only out to $z = 0.3$.

spectrum found in simulations and fitted by Peacock & Dodds (1996, hereafter PD). In Figure 1*a*, as an example, we show the comparison at $z = 0.5$. A good match between the two power spectra was achieved by slightly modifying the concentration relation of Seljak (2000) as

$$c(M, z) = a(z)\left[\frac{M}{M_\star(z)}\right]^{-b(z)} . \quad (8)$$

Here $M_\star(z)$ is the nonlinear mass scale at which $\nu(M, z) = 1$, while $a(z)$ and $b(z)$ can be considered as adjustable parameters. The dark matter power spectrum is well reproduced, to within 20% for $0.0001 < k < 500$ Mpc^{-1}, out to a redshift of 1 with the parameters $a(z) = 10.3(1+z)^{-0.3}$ and $b(z) = 0.24(1+z)^{-0.3}$, which agree with the values given by Seljak (2000) for the NFW profile at $z = 0$. The two power spectra differ increasingly with scale at $k > 500$ Mpc^{-1}, but the PD power spectrum is not reliable there because of the resolution limit of the simulations from which the nonlinear power spectrum was derived. Note that the above $c(M, z)$ relation is only valid for the cosmology used here and for the NFW profile; changing the inner slope of the profile requires a compensating change in the concentration (Seljak 2000). Moreover, the adopted halo profile should not necessarily be interpreted as the true mean profile since other effects not considered in our prescription, such as halo substructure, would affect the relation between the dark matter power spectrum and the spatial distribution and mean density profiles of halos. A detailed study of $c(M, z)$ as generally applied to all cosmologies, profile shapes, and power spectra is currently in progress (U. Seljak 2000, private communication). These uncertainties, however, do not change our main conclusions on the halo mass dependence of the convergence in the ΛCDM model.

In general, the behavior of a dark matter power spectrum that is due to halos can be understood in the following way. The linear portion of the dark matter power spectrum, $k < 0.1$ Mpc^{-1}, results from the correlation between individual dark matter halos and reflects the bias prescription. The fitting formulae of Mo & White (1996) adequately describe this regime for all redshifts. The midportion of the power spectrum, around $k \sim 0.1$–1 Mpc^{-1}, corresponds to the nonlinear scale $M \sim M_\star(z)$, where the Poisson and correlated terms contribute comparably. At higher k, the power arises mainly from the contributions of individual halos (see Seljak 2000 for a discussion of the detailed properties of the density and galaxy power spectra due to halos).

In Figure 1*b*, we show the same comparison for the convergence power spectrum. The LSS power spectrum was calculated following Hu & Tegmark (1999) using the PD power spectrum for the underlying mass distribution and using the same Limber approximation as the correlation calculation presented here. The lensing power spectrum that is due to halos has the same behavior as the dark matter power spectrum. At large angles ($l \lesssim 100$), the correlations between halos domi-

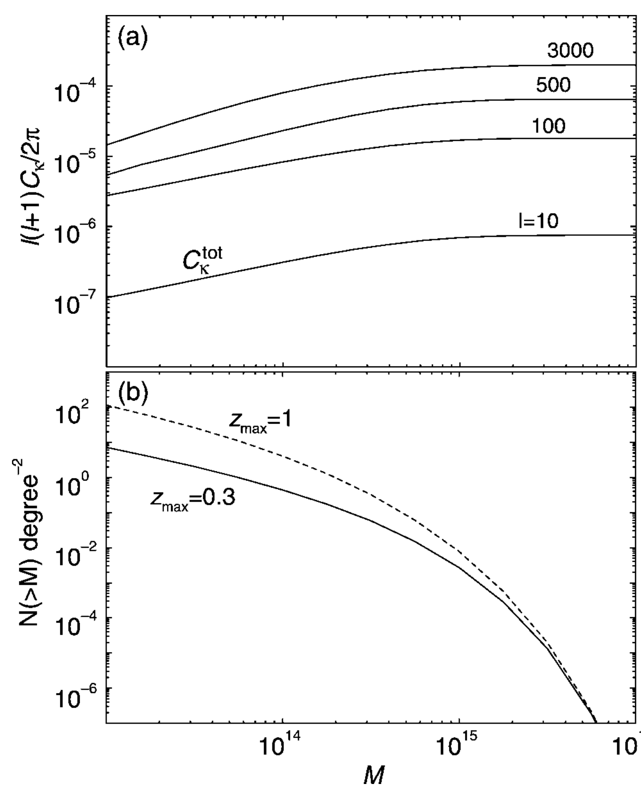

FIG. 3.—(*a*) Total lensing convergence C_κ^{tot} as a function of maximum mass for several *l*-values and sources at $z_s = 1$. As shown, contributions from halos with masses greater than 10^{15} $M_\odot$ are negligible. (*b*) Surface density of halo masses as a function of minimum mass using PS formalism out to $z_{max} = 0.3$ and $z_{max} = 1$. This determines the survey area needed to ensure a fair sample of halos greater than a given mass.

nate. The transition from linear to nonlinear is at $l \sim 500$, where halos of mass similar to $M_\star(z)$ contribute. The Poisson contributions start dominating at $l > 1000$.

In order to establish the extent to which massive halos contribute, we varied the maximum mass of halos $M_{\rm max}$ in the convergence calculation. The results are shown in Figure 2. We use background source redshifts of 1 and 0.4, corresponding to deep lensing surveys and to a shallower survey such as the ongoing Sloan Digital Sky Survey.[5] In Figures 2*a* and 2*b*, we exclude masses above a certain threshold at all redshifts, and in Figure 2*c*, we exclude masses only for halos below redshift $z = 0.3$, reflecting the fact that current observations of galaxy clusters are likely to be complete only out to such a low redshift. Assuming the latter, we find that a significant contribution comes from massive clusters at low redshifts (see Figs. 2*b* and 2*c*). Ignoring such masses, say above $\sim 10^{14}\ M_\odot$, can lead to a convergence power spectrum that is a factor of ~2 lower than the total. Note that such a high-mass cutoff affects the Poisson contribution of halos more than the correlated contributions and can bias the shape, not just the amplitude, of the power spectrum.

In Figure 3*a*, we show the dependence of $C_\kappa^{\rm tot}$ for several *l*-values. If halos less than $10^{15}\ M_\odot$ are well represented in a survey, then the power spectrum will track the LSS convergence power spectrum for all *l*-values of interest. The surface number density of halos determines how large a survey should be to possess a fair sample of halos of a given mass. We show this in Figure 3*b*, as predicted by the PS formalism, for our fiducial cosmological model for halos out to $z = 0.3$ and $z = 1.0$. Since the surface number density of halos above $10^{15}\ M_\odot$ out to a redshift of 0.3 and 1.0 is ~0.03 and 0.08 $\rm deg^{-2}$, respectively, a survey of order ~30 $\rm deg^2$ should be sufficient to contain a fair sample of the universe for recovery of the full LSS convergence power spectrum.

One caveat is that mass cuts may affect the higher moments of the convergence differently so that a fair sample for a quantity such as skewness will require a different survey strategy. From numerical simulations (White & Hu 2000), we know that $S_3 \equiv \langle\kappa^3\rangle/\langle\kappa^2\rangle^2$ shows substantial sample variance, implying that it may be dominated by rare massive halos. A detailed account of this work shows that even $M > 10^{15}\ M_\odot$ halos are important for skewness (Cooray & Hu 2000).

While upcoming wide-field weak lensing surveys, such as the MEGACAM experiment at the Canada-France-Hawaii Telescope (Boulade et al. 1998) and the proposed wide-field survey by J. A. Tyson et al. (2000, private communication), will cover areas up to ~30 $\rm deg^2$ or more, the surveys that have so far been published, e.g., Wittman et al. (2000), only cover at most 4 $\rm deg^2$ in areas without known clusters. The observed convergence in these fields should be biased low compared with the mean, and they should vary widely from field to field because of the sample variance from the Poisson contribution of the largest mass halos in the fields, which are mainly responsible for the sample variance below 10′ (see White & Hu 2000).

Our results can also be used proactively. If properties of the mass distribution such as the maximum-mass halo in the observed lensing fields are known, say through prior optical, X-ray, SZ, or even internally in the lensing observations (see Kruse & Schneider 1999), one can make a fair comparison of the observations with theoretical model predictions with a mass cutoff in our formalism. Even for larger surveys, the identification of massive halos can be beneficial: most of the sample variance in the fields will be due to rare massive halos. Simulations indicate that they increase the errors on the power spectrum by ~2 at $l \sim 2000$ (White & Hu 2000). Once identified, massive halos may be eliminated in order to reduce sampling errors. Developing techniques to do so optimally and quantifying the residual sampling errors are interesting topics for further study. The benefits could be substantial: a reduction in the sample errors increases the precision with which the power spectrum can be measured and hence the cosmological parameters upon which it depends.

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[5] Available at http://www.sdss.org.

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