

Isocurvature mechanism for structure formation

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We examine a novel mechanism for structure formation involving initial number density fluctuations between relativistic species, one of which then undergoes a temporary downward variation in its equation of state and generates superhorizon-scale density fluctuations. Isocurvature decaying dark matter (IDDM) models provide concrete examples. This mechanism solves the phenomenological problems of traditional isocurvature models, allowing IDDM models to fit the current cosmic microwave background and large-scale structure data, while still providing novel behavior. We characterize the decaying dark matter and its decay products as a single component of “generalized dark matter.” This simplifies calculations in decaying dark matter models and others that utilize this mechanism for structure formation. [S0556-2821(98)50124-5]

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Conventional models for structure formation utilize either initial density fluctuations that then grow by gravitational instability or initial stress fluctuations between the matter and radiation which push the matter into gravitationally unstable configurations. We call these adiabatic and traditional isocurvature models, respectively. The possibility remains that the origin of structure lies elsewhere. We consider here an origin from a temporary change in the equation of state of the dark matter and give a concrete example in the form of an isocurvature decaying dark matter (IDDM) model for structure formation.

TRADITIONAL ISOCURVATURE MODELS

In traditional isocurvature models, the initial density balance comes at the expense of number density or “entropy” fluctuations between the matter and radiation. These models suffer from various problems that stem from the lack of a natural time scale other than the expansion time in the radiation-dominated early universe. Phenomenologically, models with scale-invariant entropy fluctuations differ from their adiabatic counterparts with scale-invariant curvature fluctuations through: (1) an under-production of large-scale structure relative to large-angle cosmic microwave background (CMB) anisotropies [1] and (2) diminished acoustic peaks appearing at small angles [2]. The former is in conflict with the observed large-scale structure, once normalized to large-scale anisotropies, and the latter is in conflict with recent measurements of degree-scale anisotropies.

Variants of the basic isocurvature model have been proposed to alleviate these problems. Peebles [3] introduced a blue-tilt to the entropy power spectrum to address the lack of large-scale structure and the falling spectrum of CMB anisotropies. Unfortunately, such models are in moderate conflict with the slope of the cosmic background explorer (COBE) anisotropy spectrum [4].

The basic problem with the spectrum is one of time scale. This model forms structure through the residual stress perturbation remaining when the density fluctuations in a fluid with relativistic pressure $p_r = \rho_r/3$ are balanced off those in a pressureless fluid. These stresses move matter around until

the perturbation crosses the horizon or the radiation density becomes negligible. For wavelengths smaller than the horizon at matter-radiation equality, the amount of time the stresses have to act is a decreasing function of wave number k , leading to a falling spectrum of CMB anisotropies and large-scale structure.

Cosmological defects provide an alternative means of generating structure from isocurvature initial conditions that reduce these problems of spectral shape. These models balance seed fluctuations in the defects off ordinary matter to establish isocurvature initial conditions. Unlike the matter-radiation isocurvature models, the temporal behavior of the stresses scale with the horizon crossing time [5]. Thus, the effect of the stresses on the ordinary matter is the same for all scales. The other problems of isocurvature models remain. For example, these models tend to underpredict large-scale structure (see [6,7] for recent assessments). Part of this problem would seem to be common to all isocurvature models. It comes from the relationship between the Sachs-Wolfe temperature anisotropy $\Delta T/T$ and the Newtonian curvature Φ , $\Delta T/T \approx -2\Phi$, which should be compared with $\Delta T/T \approx \Phi/3$ in adiabatic models. Defect models fare even worse, since their vector and tensor modes provide additional sources of anisotropy. Furthermore, the superhorizon growth of the curvature leads to forced acoustic oscillations whose features are shifted to smaller angles relative to adiabatic models. This would also seem to be a generic feature of isocurvature models, since the curvature can only grow from its vanishing initial condition. The observed rise of the anisotropy power spectrum at degree scales would be difficult to explain in defect or other similar isocurvature variants [6].

NOVEL ISOCURVATURE MECHANISM

Nonetheless, these problems are not fundamental to isocurvature initial conditions, but rather to the choice of the stress perturbation history that generates the structure. In both examples above, the problem is that horizon crossing for the perturbations sets the time scale over which the stresses act. If the stresses on all superhorizon scales could be turned on and then off uniformly, then the universe would

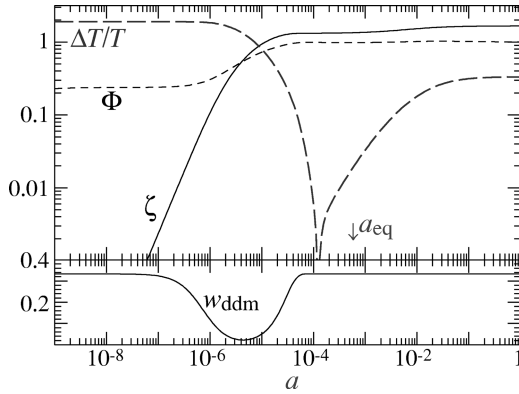


FIG. 1. Time evolution of the comoving curvature ζ , Newtonian curvature Φ , effective temperature perturbation $\Delta T/T$, and equation of state w_{ddm} in $m = 5$ keV, $\tau = 5$ yr, $\Omega_m = 1$, $h = 0.5$, $\Omega_b h^2 = 0.02$ universe.

be left with constant scale-invariant curvature fluctuations similar to an adiabatic model.

A dark matter species that undergoes a variation in its equation of state provides such a mechanism. Consider the case where a dark matter species goes non-relativistic and then decays: the equation of state for this matter and its decay products begins at $w_{\text{ddm}} = p_{\text{ddm}}/\rho_{\text{ddm}} = 1/3$, dips toward zero, and returns to $1/3$ (see Fig. 1 lower panel). Another important aspect of this model is that since the perturbations are in a species that is originally ultra-relativistic, isocurvature conditions require balancing perturbations from the other relativistic species, including the photons in the CMB. Because the balance is through species with the same equation of state initially, vanishing density perturbations imply vanishing stress perturbations as well. The initial stress-energy tensor of the total matter is completely homogeneous and isotropic.

As the dark matter becomes non-relativistic, the initially counterbalancing stress fluctuations become unbalanced as δp_{ddm} drops below $\delta \rho_{\text{ddm}}/3$. They move matter around and form density or curvature fluctuations. When the dark matter then decays back into radiation, the stresses regain their balance and stop forming curvature fluctuations. The process that generates curvature in this model in fact has an exact solution in the simple case that the fluctuations in the decaying dark matter are balanced off a single radiation component (r). A self-contained derivation of this solution can be found in [8] Sec. VII B; here we summarize the essential results. The curvature in the comoving gauge ζ is directly related to the stresses [11] and is given by

$$\zeta(a) = \frac{\sigma}{3} \frac{p_{\text{ddm}} + \rho_{\text{ddm}}}{p + \rho} \Big|_0^a + \zeta(0), \quad (1)$$

where a is the scale factor and

$$\sigma = \left(\frac{\delta \rho_{\text{ddm}}}{\rho_{\text{ddm}} + p_{\text{ddm}}} - \frac{\delta p_r}{\rho_r + p_r} \right) \Big|_0. \quad (2)$$

Isocurvature models have $\zeta(0) = 0$ by definition. The generation of curvature is most effective if the DDM species comes

to dominate the total energy density before the decay. The curvature remains constant after the decay in all cases, since the decay products and ordinary radiation redshift in the same way leaving ρ_{ddm}/ρ constant until other matter species become important.

This mechanism solves the problems isocurvature models have with the acoustic peaks in the CMB. The constancy of the potential outside the horizon during radiation domination insures acoustic phenomenology that is similar to the adiabatic model: scale-invariant oscillations before diffusion damping and a cosine series of harmonic peaks. As noted by many authors in the adiabatic variant of the decaying dark matter (DDM) scenario (e.g. [9,10]), the decay into relativistic species also delays matter-radiation equality and allows a high- Ω_m model to look like a low- Ω_m model with respect to the large-scale structure power spectrum and CMB anisotropies.

Unfortunately, this mechanism does not automatically solve the other problem of isocurvature models: the ratio of large-scale anisotropies to large-scale structure. In fact, it can actually make the agreement worse. If the DDM fluctuations are balanced by CMB fluctuations initially, then the CMB temperature will be lower in regions where the DDM density is higher. Since these are the sites of potential wells when the DDM become non-relativistic, those cold photons will lose more energy climbing out of the potential wells after last scattering and become colder. The complete gravitational redshift effect is

$$\frac{\delta T}{T}(1) \approx -2\Phi|_0^1 + \frac{\delta T}{T}(0). \quad (3)$$

The solution is clear. The initial conditions must involve the photons and DDM together compensating the other species of radiation. The photons will then be initially hotter at the sites that form potential wells.

IDDM MODELS

For definiteness, let us now examine the phenomenology of specific models. The important aspect of particle decays is that it generates the required variation in the equation of state. To highlight this property, we model the decaying particle and its decay products as a single component of ‘‘generalized’’ dark matter of the type introduced in [12]; we label it DDM here. The DDM is described by an equation of state w_{ddm} , a sound speed c_{ddm} , and a viscosity parameter α_{ddm} . The equation of state of the DDM is given implicitly in terms of the mass m and lifetime τ of the particle,

$$\omega_{\text{ddm}} = \frac{3(1+w_1)\rho_1 - 4\rho_2}{\rho_1 + \rho_2}$$

$$\frac{d\rho_1}{d \ln a} = -3(1+w_1)\rho_1 - \frac{1}{H\tau}\rho_1$$

$$\frac{d\rho_2}{d \ln a} = -4\rho_2 + \frac{1}{H\tau}\rho_1, \quad (4)$$

where $H = a^{-1}(da/dt)$,

$$w_1 = \frac{1}{3} [1 + (a/a_{\text{nr}})^{2p}]^{-1/p}, \quad (5)$$

with $p = 0.872$ and $a_{\text{nr}} = 8.3 \times 10^{-7} (m/\text{keV})^{-1} (T/T_\gamma)$. Here T/T_γ is the ratio of temperatures of the DDM and photons, while the DDM was relativistic. We assume here that the DDM accounts for one of the usual neutrino species so that $T/T_\gamma = (4/11)^{1/3}$. The sound speed is given by $c_{\text{ddm}}^2 = w_{\text{ddm}} - (dw_{\text{ddm}}/d\ln a)/(3 + 3w_{\text{ddm}})$ and the viscosity parameter $\alpha_{\text{ddm}} = w_{\text{ddm}}$ following [12]. We have checked this approximation against explicit calculations of an adiabatic DDM model with the techniques of [9]. The approximation has the practical benefit of being simple to implement (c.f. [13]) and the pedagogical value of highlighting the important aspects of this mechanism.

The critical parameter is $m^2\tau$ [10], since it determines the fractional increase in the total radiation density due to the decay $\rho_r \rightarrow \rho_r(1 + 0.15m_{\text{keV}}^{4/3}\tau_{\text{yr}}^{2/3})$; as we have seen changes in the energy density of the DDM relative to the other components is what drives curvature generation.

We establish isocurvature initial conditions of the form

$$\frac{\delta\rho_{\text{ddm}}}{\rho_{\text{ddm}}} = \frac{\delta\rho_\gamma}{\rho_\gamma} = \frac{3}{4} \frac{\delta\rho_m}{\rho_m}, \quad (6)$$

$$\delta\rho_\nu = -(\delta\rho_{\text{ddm}} + \delta\rho_\gamma + \delta\rho_m),$$

where ρ_ν is the density in the remaining two neutrino species and ρ_m is the density in baryons and CDM with all other metric and matter perturbations zero. More complicated initial conditions can alter the ratio between $\Delta T/T$ and Φ . Despite these simple initial conditions, some care must be taken to insure numerical stability in the evolution. In the synchronous gauge, we recommend the use of the variables H_T and $-H_L - H_T/3$ in the notation of [11].

The time evolution of superhorizon scale perturbations in an example model with $m = 5$ keV and $\tau = 5$ yr is shown in Fig. 1 (upper panel). As discussed above, the comoving curvature ζ grows rapidly as the equation of state w_{ddm} dips below its relativistic value. On the other hand, the curvature in the Newtonian gauge Φ is finite and constant *before* the decay. In traditional isocurvature and adiabatic models, the ζ and Φ are simply proportional. The difference here is that the radiation possesses substantial density perturbations which leads to anisotropic stress perturbations in the neutrinos of order $\pi = (k\eta)^2 \mathcal{O}(\delta\rho_{\text{ddm}}/\rho_{\text{ddm}})$ where η is the conformal time; balancing anisotropies in the photons are prevented by Compton scattering. The relativistic Poisson equation involves curvature sources of order $\pi/(k\eta)^2$ leading to constant Newtonian curvature perturbations. However once the dark matter decays, the Newtonian and comoving curvatures again become proportional to each other.

The second interesting feature is that, unlike adiabatic models, the comoving curvature ζ grows at the second transition to matter domination a_{eq} . The initial conditions (6) imply an entropy fluctuation between the non-relativistic matter ρ_m and the combined radiation after the decay. This entropy fluctuation causes stress fluctuations when the non-

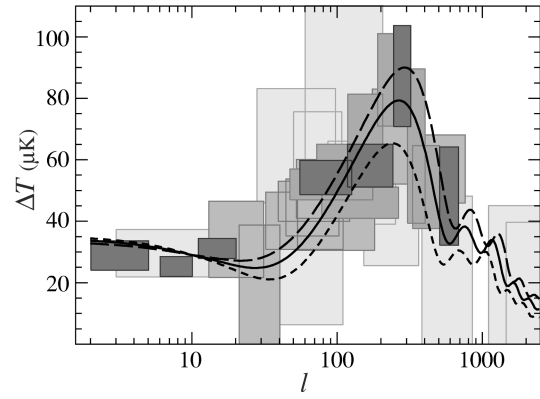


FIG. 2. CMB anisotropies for models $m = 5$ keV and $\tau = 3, 5, 8$ yr (short-dashed, solid, long-dashed lines) in an $\Omega_m = 1$, $h = 0.5$, and $\Omega_b h^2 = 0.02$ universe compared with the current CMB data [17] with 1σ error boxes shaded according to area.

relativistic matter comes to dominate the universe. This behavior is identical to matter-radiation isocurvature models and implies that the phenomenology of the complete model will be a combination of traditional isocurvature and adiabatic models. Finally, as desired, the initial temperature perturbation $\Delta T/T$ cancels part of the redshift effect from the growing Newtonian potential leaving smaller CMB anisotropies for a given matter density fluctuation.

The CMB anisotropies of models with $m = 5$ keV and $\tau = 3, 5, 8$ yr are shown in Fig. 2 and compared with the current data. Models with the same $m^2\tau$ are approximately degenerate. The large angle anisotropies have the spectral shape of traditional isocurvature models with a decline toward smaller angles due to the matter-radiation transition. However, the acoustic peaks show a distinctly adiabatic pattern with peak positions in a classic 1:2:3... series and high odd peaks. The result is a dip in the anisotropies at intermediate scales that is an interesting signature of such models. The CMB polarization however mimics adiabatic models predictions.

The power spectrum of the large scale structure in the same models are given in Fig. 3 and compared with the data

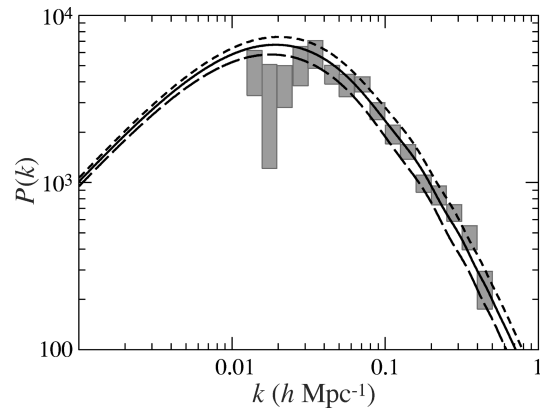


FIG. 3. Matter power spectrum for the same models as Fig. 2 compared with galaxy survey data from [14] with 1σ error boxes. Note that the data may be shifted up or down to account for the unknown galaxy bias.

[14]. The shape of the power spectrum matches the data adequately; for example, the ratio of power at the $50h^{-1}$ Mpc to $8h^{-1}$ Mpc scale $\sigma_{50}/\sigma_8=0.16$ in the $\tau=3$ yr model. Relative to an adiabatic model with the same parameters, these have more large compared with small scale power due to growth at the second matter-radiation transition. Note that the observations may be slid up and down to account for an unknown galaxy bias. On the other hand, the model curves are COBE-normalized through the fitting form of [16] and predict $\sigma_8=0.62$ for the 3 yr model. This value would reproduce the present-day cluster abundance adequately (e.g. [18]) and at high-redshift marginally (e.g. [19]). The high value of Ω_m in these examples is observationally disfavored by recent determinations of the luminosity distance to high-redshift supernovae [20,21]; if these preliminary indications are borne out by future measures, lower Ω_m variants of these models can be considered.

DISCUSSION

Perhaps more interesting than the details of any specific model of this type is the lesson their mere existence teaches us. That the current CMB and large-scale structure data is consistent with a high-density model with scale-invariant initial isocurvature perturbations warns us against overinterpreting the current data. It also suggests that if future observations reveal phenomenology that is close to, but not precisely predicted by standard adiabatic models, we should not necessarily abandon the search for new paradigms for structure formation. Our example shows that the acoustic peak locations do *not* actually discriminate between adiabatic and isocurvature initial conditions in general. They do, however, relate to the role of causality in the generation of perturbations as discussed in [15]. To be explained causally, the

scale-invariant isocurvature “initial” conditions employed here still require a period of superluminal expansion, like that provided by inflation. The models considered in the last section require that fluctuations in the chemical potential of say the e and μ neutrinos balance temperature fluctuations and may be difficult to arrange in inflationary models. However the general mechanism is of wider interest. Exploration of alternatives such as these helps to sharpen the questions that can be asked of the data.

In summary, the proposed isocurvature mechanism for structure formation solves the problems with the spectra and relative amplitude of the matter and CMB fluctuations associated with traditional isocurvature models. The key aspects of this mechanism are that (1) joint fluctuations in the energy density of some initially relativistic component *and* the photons are balanced by the remaining radiation and (2) the equation of state of this new component drops and then returns to the ultrarelativistic value of $w_{\text{ddm}}=1/3$ before the scales relevant to large-scale structure and CMB anisotropies enter the horizon. We have shown that this situation can be realized with a dark matter particle in the keV mass range that decays on the time scale of a year. Such models are in agreement with the current data, but have novel features that are testable with the upcoming generation of experiments. Their existence calls into question widely-held beliefs about isocurvature models and cautions against overinterpretation of current and future data sets.

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