Cosmological Limits on the Neutrino Mass from the Lyα Forest

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The Lyα forest in quasar spectra probes scales where massive neutrinos can strongly suppress the growth of mass fluctuations. Using hydrodynamic simulations with massive neutrinos, we successfully test techniques developed to measure the mass power spectrum from the forest. A recent observational measurement in conjunction with a conservative implementation of other cosmological constraints places upper limits on the neutrino mass: $m_\nu < 5.5$ eV for all values of Ω_m, and $m_\nu \lesssim 2.4(\Omega_m/0.17 - 1)$ eV, if $0.2 \le \Omega_m \le 0.5$ as currently observationally favored (both 95% C.L.).

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Experimental evidence for finite neutrino masses and flavor oscillations continues to mount. Recently the Super-Kamiokande experiment has provided strong evidence that oscillations from ν_μ to another species involve a mass greater than $\sqrt{\delta m^2} = 0.07^{+0.02}_{-0.04}$ eV [1]. The LSND experiment suggests the existence of ν_μ to ν_e oscillations with $\sqrt{\delta m^2} \sim 0.4$ eV [2]. Finally, the solar neutrino deficit requires $\sqrt{\delta m^2} \sim 10^{-5}$–$0.003$ eV [3]. These mass splitting results are consistent with one to three weakly interacting neutrinos in the eV mass range [4].

Neutrinos in this mass range are important cosmologically since, if they exist, they would represent a non-negligible contribution to the dark matter content of the Universe. In units of the critical density, neutrinos contribute $\Omega_\nu = Nh^{-2}m_\nu/94$ eV, where h is the dimensionless Hubble constant ($H_0 = 100h$ km s^{-1} Mpc^{-1}), and N is the number of degenerate mass neutrinos. As light neutrinos do not cluster on small scales, they retard the gravitational growth of density fluctuations. Any measure of small-scale clustering is thus sensitive to neutrino masses in this range. One such useful measure is the clustering of the intergalactic medium revealed by the absorption features in quasar spectra known as the Lyα forest [5]. As we will discuss below, the Lyα forest has the distinct advantage that clustering properties of the mass distribution can be inferred from it, which greatly facilitates comparisons with theory, and makes large searches of parameter space possible.

In this Letter, we make use of a recent Lyα forest measurement of the power spectrum of mass fluctuations [6] to place limits on the mass of the neutrino(s). Conversion of the power spectrum measurements into neutrino mass limits requires a framework for cosmological structure formation. There is growing evidence that structure formed by the gravitational instability of cold dark matter (CDM) with adiabatic, Gaussian initial fluctuations. Upcoming cosmic microwave background (CMB) experiments should conclusively determine whether this assumption is a good one [7]. For now, we note that this framework, which we adopt, includes all currently favored models.

Our adiabatic CDM dominated universes are described by six free parameters: the matter density Ω_m, dimensionless Hubble constant h, baryon density Ω_b, neutrino density Ω_ν, density fluctuation amplitude A, and tilt n, which define the initial density power spectrum $P_{\rm init}(k) = Ak^n$. We also initially assume that spatial geometry is flat, as implied by recent measurements of distant supernovae and CMB anisotropies [8], before investigating the consequences of relaxing this assumption.

We begin by describing the Lyα forest power spectrum measurement method and test it on hydrodynamic simulations. We then apply the observational constraint to the six dimensional CDM parameter space to find an upper limit on the neutrino mass. Given this large parameter space, we conservatively employ other cosmological constraints, notably from the abundance of galaxy clusters and the age of globular clusters, to constrain other parameters that can mimic the effects of massive neutrinos. Finally, we consider prospects for making a precise measurement of m_ν using future Lyα forest observations and upcoming CMB experiments.

Testing Lyα forest simulations with $m_\nu > 0$.—The Lyα forest of neutral hydrogen absorption seen in quasar spectra [5] arises naturally in cosmological scenarios where structure forms by the action of gravitational instability. In hydrodynamic simulations of such models [9] (also, see [10]) most of the absorption arises in gas of moderate overdensity, whose physical state is governed mainly by photoionization heating and adiabiatic cooling. The density field can then be locally related to the Lyα optical depth [11] and hence a directly observable quantity, the transmitted flux in a quasar spectrum.

The Lyα forest can therefore be used to determine the statistical properties of the density distribution, and in particular $P(k)$, the power spectrum of density fluctuations. A method for carrying this out was described in Ref. [12], whose authors also tested it on hydrodynamic simulations. An observational measurement was made in Ref. [6]. Previous tests have not included the effect of "hot particles" which are used to represent neutrinos in

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simulations of models where their rest mass is nonzero. Here we test whether, in a specific model, $P(k)$ can be correctly measured from the forest. If, with the neutrino hot particles, the analysis yields the $P(k)$ expected from linear theory, we will make the assumption that the procedure will also work for all other models in our parameter space which include massive neutrinos. The $P(k)$ recovery assumes explicitly that the initial fluctuations were Gaussian. As long as the results are expressed in observational units ($\mathrm{km\,s^{-1}}$), the effectiveness of the procedure is independent of other cosmological parameters, such as h, Ω_m, and Λ (see [12]).

The measurement of $P(k)$ from Lyα forest spectra is carried out in two stages. First, the shape of $P(k)$ is measured from the power spectrum of the Lyα forest flux. Second, normalizing simulations are used to set the amplitude of the linear mass $P(k)$. We refer the reader to Ref. [12] for details.

The hydrodynamic simulation itself is described in detail in Ref. [13]. We follow the evolution of structure in a model with two mass-degenerate neutrino species, using the Parallel TreeSPH hydrodynamic code [14]. The model parameters are $\Omega_m = 1$, $h = 0.5$, $\Omega_b = 0.075$, and $\Omega_\nu = 0.2$, so that the mass in both species combined is 5 eV. This so-called cold plus hot dark matter model (CHDM) is normalized to fit the Cosmic Background Explorer (COBE) results [15], so that the amplitude of mass fluctuations in $8h^{-1}$ Mpc spheres at $z = 0$, $\sigma_8 = 0.7$. We use a box of size $11.111h^{-1}$ Mpc, periodic boundary conditions and initial conditions taken from [16]. The CDM and gas components are represented by 64^3 particles each, and the neutrinos by 2×64^3 particles. We use the distribution and physical state of the gas at $z = 2.5$ to generate artificial Lyα spectra, for 1200 randomly chosen lines of sight through the simulation volume.

We then apply the $P(k)$ recovery method of [12] to these spectra. We use normalizing simulations run under the PM (particle-mesh) approximation [12,17] with 64^3 particles and an $11.111h^{-1}$ Mpc box. We use the same estimator for the amplitude of $P(k)$ as in the observational analysis paper [6]. The results of the test are shown in Fig. 1, where we plot the recovered $P(k)$, together with the linear theory prediction for the model. We also show the linear power spectrum of a CDM-only model (with $\Omega_m = 1$, $h = 0.5$), again normalized to COBE, so that $\sigma_8 = 1.2$.

The error bars on the $P(k)$ points are representative of the "cosmic variance" error which arises from having only one hydrodynamic simulation volume. We estimate this uncertainty by running ten additional PM approximation simulations of the CHDM model, and extracting 1200 lines of sight from each. The standard deviation of their results for each $P(k)$ point provides the error bar. There is an additional overall amplitude uncertainty associated with the normalization. We estimate this by applying the normalizing procedure to the PM CHDM simulations taken individually, finding that the additional error in this test case is a negligible 4% in $P(k)$. Also, our results are robust to reasonable variations in the gas equation of state adopted when generating spectra, with the normalization amplitude of $P(k)$ changing by $\sim$10% in the most extreme case (see also [12]).

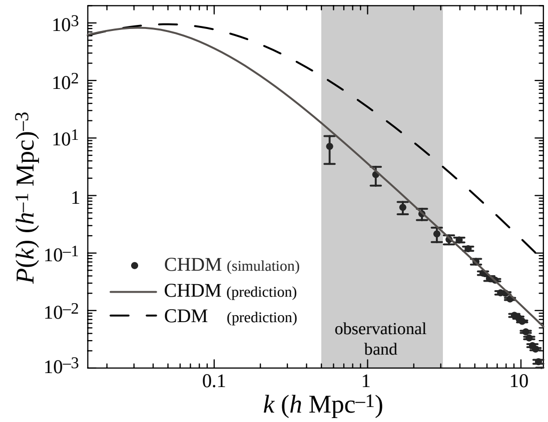

FIG. 1. A test of the $P(k)$ recovery method. The points are $P(k)$ recovered from a hydrodynamic simulation of the Lyα forest in a massive neutrino model (CHDM see text). The turndown for $k > 5h$ Mpc^{-1} is caused mainly by the limited resolution of the simulation and lies outside of the observational band (shaded). The lines show the linear $P(k)$ predictions for the massive neutrino model, and for a CDM-only model.

We use the simulation to test for systematic errors in the technique. The observational result was given in Ref. [6] in terms of a power law fit to the $P(k)$ data points with $2.7 \times 10^3 < k < 1.42 \times 10^2$ $(\mathrm{km\,s^{-1}})^{-1}$ (which corresponds to $0.5h$ Mpc^{-1} $< k < 2.7h$ Mpc^{-1} for an Einstein–de Sitter model). We fit the simulation data points to a power law over this range, finding an amplitude $\Delta^2(k) = 0.16 \pm 0.028$ (1σ) at $k = 1.5h$ Mpc^{-1} $[= 0.008\ (\mathrm{km\,s^{-1}})^{-1}]$. Here $\Delta^2(k) = k^3P(k)/2\pi^2$, the contribution to the density field variance from a unit interval in log$k$. The logarithmic slope $n = -2.18^{+0.34}_{-0.28}$ (1σ). The linear theory prediction for CHDM is $\Delta^2(k) = 0.21$, $n = -2.40$. Our recovered $P(k)$ is therefore about 2σ too low in amplitude, and has a slightly flatter slope. By examining results from the more numerous PM simulations, we find that the largest scale data point is systematically lowered by peculiar velocity distortions (as predicted by [18]), an effect which is not accounted for in our estimate of the $P(k)$ shape. Including or leaving out this point (which has the largest statistical errors) has only a small effect on the power law fit. It is possible, however, that taking these effects into account or further refining the analysis would improve the result. We note that the measurement of [6] has a statistical uncertainty of $\sim$70% (2σ), larger than any biases revealed by our test.

Constraining the neutrino mass.—Although the suppression of power in the Lyα forest due to a finite neutrino mass is large [$\sim(800\Omega_\nu/\Omega_m)$% [19]], other aspects of the model can counterbalance this effect; limits on the neutrino mass consequently depend on the range of models allowed by other cosmological constraints.

In addition to the Lyα forest power spectrum measurement of $\Delta^2(0.008\ (\mathrm{km\,s^{-1}})^{-1}) = 0.57^{+0.54}_{-0.27}$ (95% C.L.) (we do not use the Lyα slope measurement, which has no significant effect) at $z = 2.5$, we consider six other cosmological constraints. The Hubble constant is measured to be $h = 0.72 \pm 0.17$ (95% C.L.) [20]. We derive the 95% C.L. by doubling the 1σ errors that result from adding the statistical and systematic errors in quadrature. The amplitude of the fluctuations is determined by the COBE detection of large angle anisotropies [15]; we ignore the 7% measurement uncertainties on the temperature fluctuations which are substantially smaller than the other uncertainties. The abundance of galaxy clusters today constrains models at the $8h^{-1}$ Mpc scale, where the amplitude of fluctuations in top-hat spheres (at $z = 0$) is $\sigma_8 = 0.56\Omega_m^{0.47}$ with 95% C.L. of $+20\Omega_m^{0.2\log\Omega_m}\%$ and $-18\Omega_m^{0.2\log\Omega_m}\%$ [21].

Galaxy surveys measure the shape of the power spectrum; we use the measurements of [22], but only from the range $0.025h\ \mathrm{Mpc}^{-1} < k < 0.25h\ \mathrm{Mpc}^{-1}$ to avoid spurious survey volume effects and uncertainties in the nonlinear corrections. We employ a $\Delta\chi^2$ statistic and take $\Delta\chi^2 = 4$ to represent the 95% confidence limits on the shape of the galaxy power spectrum. We assume that a linear "bias" is operating so that the matter $P(k)$ is related to the galaxy $P(k)$ by a constant factor (which we find as part of our χ^2 minimization). We also employ nucleosynthesis constraints on the baryon density of $\Omega_b h^2 = 0.019 \pm 0.0024$ (95% C.L.) [23]. Finally, we place a lower limit on the age of the Universe by assuming that it must be at least as old as the oldest globular clusters (13.2 ± 2.9) Gyr (95% C.L.) [24].

Given these constraints, we could construct a joint likelihood to find the best fitting neutrino mass. We have decided to be more conservative, however, and consider a model ruled out if it violates the (95%) confidence limits on any constraint taken individually. With these "2σ" constraints on the parameter space, we use the analytic approximations of [25] to explore the remaining space rapidly and find the model that maximizes the neutrino mass as a function of Ω_m. The result for one massive species is displayed in Fig. 2a. We also show the effect of omitting the Lyα forest measurement. The measurement has a powerful constraining effect at low and high Ω_m since the amount of tilt required to match the cluster abundance and galaxy power spectrum shape violates the upper and lower Lyα forest bounds, respectively. For no value of Ω_m can m_ν be greater than 5.5 eV. We also show the results assuming two neutrino species with identical masses in Fig. 2b. These limits are roughly half the single species results since the change in the growth rate is mainly governed by Ω_ν.

To address the robustness of our upper limits, we show the effect of scaling all errors by a factor of 1.5 to approximate "3σ" constraints in Fig. 2. We also test for single point failures by dropping each constraint sequentially. Omission of either the age or cluster abundance constraint changes the maximal neutrino mass to $\sim$7 eV. While dropping the galaxy power spectrum constraint does not increase the maximal mass substantially, it does weaken the bounds by up to 2 eV for $\Omega_m \lesssim 0.5$. Omission of the h or $\Omega_b h^2$ constraint has essentially no effect. The simulation tests indicate that (at the 2σ level) the $P(k)$ results for the CHDM model are systematically underestimated (by a factor of $\sim$25%). If we correct for such a bias, the neutrino mass limits become tighter, with the maximal mass dropping to 4 eV. To be conservative, we have not included this effect in our final quoted results.

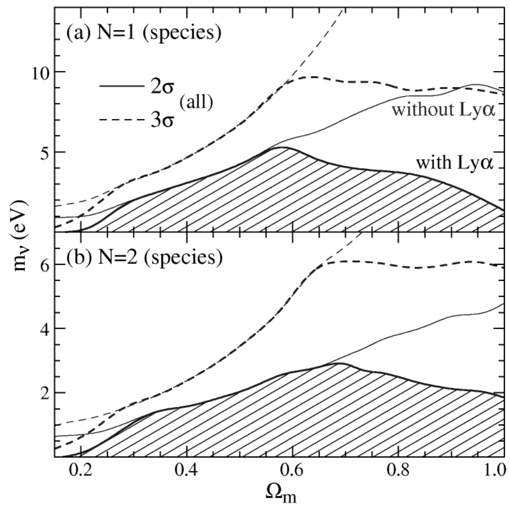

FIG. 2. Constraints on the neutrino mass (a) single massive species (b) two (degenerate) mass species with and without the Lyα constraint.

In applying our constraints, we assume that the Universe is flat ($\Omega_m + \Omega_\Lambda = 1$) and that gravity waves do not contribute to the COBE normalization. However, assuming an open universe or gravity waves from power law inflation does not change the m_ν limits significantly since the tilt can be used to offset small changes in normalization. The simplest inflationary models can also predict a variation of the spectral index with scale. This is a small effect compared to the neutrino power suppression, so that we do not include it. Future CMB observations should address this point definitively.

Future prospects.—Additional Lyα forest data from quasar surveys such as the Sloan Digital Sky Survey (SDSS [26]) have the potential to increase the precision of these mass constraints substantially. Furthermore, we expect that the next generation of CMB satellites will not only verify the existence of the underlying framework for structure formation, which we currently assume, but also provide limits on the neutrino mass itself.

How precise must these Lyα measurements be to improve on projected CMB limits on the neutrino mass? To answer this question, we employ Fisher information matrix techniques to approximate the joint covariance matrix. For the Lyα forest power spectrum measurement, the Fisher matrix is given by $\mathbf{F}_{ij} = (\Delta P/P)^{-2}(\partial \ln P/\partial p_i)(\partial \ln P/\partial p_j)$. We add this to the CMB Fisher matrix projected for the MAP and Planck satellites including polarization information [27]. The variance of the optimal unbiased estimator of p_i marginalized over the other parameters is $(F^{-1})_{ii}$.

A fractional error of $\Delta P/P = 0.1$ (1σ) on the Lyα power spectrum would improve the MAP upper limit from 1.1 to 0.54 eV and the Planck limit from 0.51 to 0.29 eV both at 2σ. Note that these represent limits in a wider 10 parameter space including spatial curvature and gravity waves. These improvements would exceed those that can be achieved by the SDSS galaxy survey [27].

Is 10% precision in power achievable from Lyα forest measurements? The constraint in this paper relies on data from $\sim$10 full quasar spectra [6]. The SDSS quasar survey [26] will yield spectra of roughly similar quality (resolution 2.5 Å compared to $\sim$1.5 Å for Ref. [6]) for $\sim 10^5$ quasars. The mean distance between sight lines in the SDSS will be somewhat larger than the scale on which clustering was measured in Ref. [6], so that the decrease in statistical errors will not be too far off the factor $\sim(10^5/10)^{1/2} \sim 100$ which would be expected if the sight lines were independent. A 1σ statistical error of $\Delta P/P < 1\%$ should therefore be possible in the future, so that systematic errors will become dominant. Studying larger hydrodynamic simulations should enable us to understand these systematic effects and refine our analysis techniques. The vast size of the SDSS data set will also enable us to pin down nongravitational contributions to Lyα forest clustering, for example, by analyzing the evolution of the forest with redshift. The signature of massive neutrinos, if they are present, should therefore be obvious, even if m_ν is a fraction of an eV.

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[1] Y. Fukuda *et al.,* Phys. Lett. B **433**, 9 (1998); **436**, 33 (1998); Phys. Rev. Lett. **81**, 1562 (1998).
[2] C. Athanassopoulos *et al.,* Phys. Rev. Lett. **75**, 2650 (1995); Phys. Rev. C **54**, 2685 (1996).
[3] J. N. Bahcall, Astrophys. J. **467**, 475 (1996); V. Barger and K. Whisnant, e-print hep-ph/9903262, 1999.
[4] A sterile species may be required. See K. S. Babu, R. K. Schaefer, and Q. Shafi, Phys. Rev. D **53**, 606 (1996); G. L. Fogli, E. Lisi, D. Montanino, and G. Scioscia, Phys. Rev. D **56**, 4365 (1997).
[5] C. R. Lynds, Astrophys. J. **164**, L73 (1971); W. L. W. Sargent, P. J. Young, A. Boksenberg, and D. Tytler, Astrophys. J. Suppl. Ser. **42**, 41 (1980); M. Rauch, Annu. Rev. Astron. Astrophys. **36**, 267 (1998).
[6] R. A. C. Croft, D. H. Weinberg, M. Pettini, L. Hernquist, and N. Katz, astro-ph/9809401 [Astrophys. J. (to be published)].
[7] W. Hu, N. Sugiyama, and J. Silk, Nature (London) **386**, 37 (1997).
[8] M. White, Astrophys. J. **506**, 495 (1998).
[9] R. Cen, J. Miralda-Escudé, J. P. Ostriker, and M. Rauch, Astrophys. J. **437**, L9 (1994); Y. Zhang, P. Anninos, and M. L. Norman, *ibid.* **453**, L57 (1995); L. Hernquist, N. Katz, D. H. Weinberg, and J. Miralda-Escudé, *ibid.* **457**, L5 (1996).
[10] H. G. Bi, Astrophys. J. **405**, 479 (1993); L. Hui and N. Gnedin, Mon. Not. R. Astron. Soc. **292**, 27 (1997).
[11] R. A. C. Croft, D. H. Weinberg, N. Katz, and L. Hernquist, Astrophys. J. **488**, 532 (1997).
[12] R. A. C. Croft, D. H. Weinberg, N. Katz, and L. Hernquist, Astrophys. J. **495**, 44 (1998).
[13] R. Davé, L. Hernquist, N. Katz, and D. Weinberg, Astrophys. J. **511**, 521 (1999).
[14] R. Davé, J. Dubinski, and L. Hernquist, New Astron. **2**, 277 (1997).
[15] E. F. Bunn and M. White, Astrophys. J. **480**, 6 (1997).
[16] A. Klypin and J. Holtzmann, e-print astro-ph/9712217, 1997.
[17] N. Gnedin and L. Hui, Mon. Not. R. Astron. Soc. **296**, 44 (1998).
[18] P. McDonald and J. Miralda-Escudé, astro-ph/9807137, 1999; L. Hui, A. Stebbins, and S. Burles, Astrophys. J. **511**, L5 (1999).
[19] W. Hu, D. J. Eisenstein, and M. Tegmark, Phys. Rev. Lett. **80**, 5255 (1998).
[20] B. F. Madore *et al.,* astro-ph/9812157, 1999.
[21] P. T. P. Viana and A. R. Liddle, e-print astro-ph/9803244.
[22] J. A. Peacock and S. J. Dodds, Mon. Not. R. Astron. Soc. **267**, 1020 (1994).
[23] S. Burles, K. M. Nollett, J. N. Truran, and M. S. Turner, Phys. Rev. Lett. **82**, 4176 (1999).
[24] E. Carretta, R. G. Gratton, G. Clementini, and F. Fusi Pecci, e-print astro-ph/9902086, 1999.
[25] D. J. Eisenstein and W. Hu, Astrophys. J. **511**, 5 (1998).
[26] SDSS: http://www.astro.princeton.edu/BBOOK.
[27] D. J. Eisenstein, W. Hu, and M. Tegmark, astro-ph/9807130, 1999 [Astrophys. J. (to be published)].