

MEASURING THE CURVATURE OF THE UNIVERSE[†]

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We discuss how the curvature of the universe can be robustly measured employing only the gross features of the CMB anisotropy spectrum. Though the position of the first peak is not robust, uncertainties in the model for structure formation can be removed by using the spacing of the acoustic peaks and the location of the damping tail. Combined these provide important consistency tests that can be used to discriminate against a truly exotic model.

If we knew the model of structure formation exactly, we could determine many cosmological parameters, including the curvature of the universe, to several percent accuracy from features in the CMB anisotropy power spectrum (for a review see [1] and references therein). The question arises: how is our ability to measure them degraded as we relax our assumptions about the underlying model. Once we understand which features are model independent (and why), we can go on to study the process of structure formation from those which are model *dependent*. For concreteness, we will focus here on one step of this program [2]: measuring the spatial curvature of the universe, i.e. $\Omega_{\text{tot}} = \Omega_0 + \Omega_\Lambda$. The complementary approach of first verifying the model and then measuring the cosmological parameters is taken in a companion piece [3].

Let us assume that we understand the “big picture”, i.e. that gravitational instability enhances initially small fluctuations, and that the CMB is coupled to the baryon-electron plasma before recombination. Can we build a measurement of the curvature from such minimal assumptions? Are there sufficient cross checks such that we can have confidence in the measurement? The answers to these questions lie in the acoustic signature of the small angle CMB anisotropy spectrum.

Our “big picture” leaves several questions unanswered:

1. What is the fundamental nature of the fluctuations?

- do curvature perturbations exist outside the horizon as in the inflationary model or are the perturbations initially isocurvature as in a defect model (see also [2,3]).

2. What is the matter content of the universe?

- does the baryon-photon ratio ($\Omega_b h^2$) follow the big bang nucleosynthesis (BBN) prediction?

- what is the matter-radiation ratio ($\Omega_0 h^2$ or more generally, the equality epoch z_{eq})?

3. Does the thermal history of the universe follow the prediction of standard recombination at $z_* \sim 10^3$?

These questions and their consequence for the curvature measurement can be addressed by examining the gross properties of the CMB spectrum taken as a whole. The fine details, so useful for making precision measurements in a fixed model, are too model-dependent to serve us here.

Under our minimal assumptions, we have two striking features (1) the acoustic peaks: their positions, position ratios, spacings and relative heights; and (2) the damping tail: its position, position relative to the peaks and shape. The acoustic peaks probe the sound horizon at last scattering; the damping tail probes the photon diffusion scale at that epoch. Both reflect the curvature of the universe in the projection from physical scale at last scattering to angular scale on the sky. We shall show that these features have complementary strengths and weaknesses in guarding against model uncertainties. Combined they can be proof against any one of a host of exotic possibilities.

To set the stage for this discussion, let us briefly review the angular size distance test for curvature in the universe as it relates to the CMB spectrum. A feature in the temperature fluctuations on the last scattering surface, corresponding to wavenumber k_{feature} , is viewed as an anisotropy on the sky at the multipole moment of a spherical harmonic decomposition $\ell_{\text{feature}} = k_{\text{feature}} D$, where D is the comoving angular size distance and is dependent strongly on the curvature of the universe:

$$D = |K|^{-1/2} \sinh[|K|^{1/2}(\eta_0 - \eta_*)], \quad (1)$$

where $\eta_0 - \eta_*$ is the conformal distance to the last scattering surface and the curvature $K = -H_0^2(1 - \Omega_{\text{tot}})$ (for $K > 0$ replace $\sinh \rightarrow \sin$). Here the Hubble constant is $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Since the projection depends sensitively on $\Omega_K = 1 - \Omega_{\text{tot}}$, any feature in the CMB at last scattering may serve in the angular size distance test for curvature in the universe (see Fig. 1). Let us now

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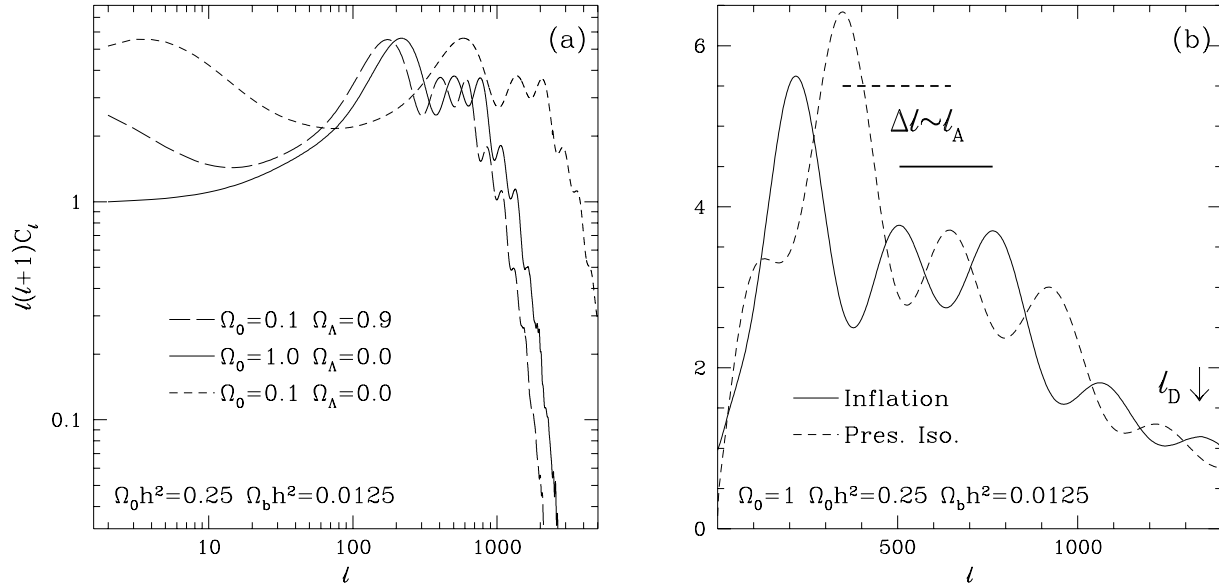


FIG. 1. (a) Angular size distance. For an inflationary model, features at last scattering such as the peaks and damping tail are fixed in real space by $\Omega_0 h^2$ and $\Omega_b h^2$, providing in the anisotropy power spectrum $\ell(\ell+1)C_\ell$ standard rulers for the angular size distance test. The cosmological constant Ω_Λ yields a minor effect compared with the curvature $1 - \Omega_0 - \Omega_\Lambda$. (b) In a broad class of models, the peak spacing $\Delta\ell$ and damping tail location ℓ_D depend only on the background parameters and provide rulers that are robust to model changes. The inflationary model is here compared with the pressure scaling model [7].

turn to the two acoustic features, the peak spacings and damping tail location, that best suit this purpose.

Before recombination, the photons and baryons are tightly coupled into a single fluid by Compton scattering. Acoustic oscillations are stimulated as the gravitational compression or rarefaction of the fluid is halted and turned around by photon pressure as the Jeans length (or sound horizon) passes the wavelength. Because gravity is impotent under the Jeans length, typically its effects subsequently die away leaving the fluid to oscillate at its natural frequency thereafter.

As discussed further in [2,3], the nature of the fluctuations basically determines whether the photon-baryon fluid is undergoing a compression or rarefaction at Jeans crossing which affects the acoustic phase (see Fig. 1b). For either case, the spacing of the peaks reflects the natural frequency of the oscillator. More specifically, in the radiation-dominated era the oscillator equation for the effective temperature fluctuation T of the CMB becomes $T'' + c_s^2 T = 2\Psi''$, where Ψ is the Newtonian potential, c_s is the sound speed, and primes are derivatives with respect to $k\eta$, where $\eta = \int dt/a$ is the conformal time. From the Poisson equation, $|\Psi| \sim (k\eta)^{-2} \delta\rho/\rho$. Since in the radiation-dominated era, the density fluctuation $\delta\rho/\rho = \mathcal{O}(T)$ typically, Ψ is usually negligible well inside the horizon. More generally, if Ψ'' is small or slowly-varying then the solution is an oscillation at the frequency $\omega = kc_s$, possibly with a zero point offset. On the

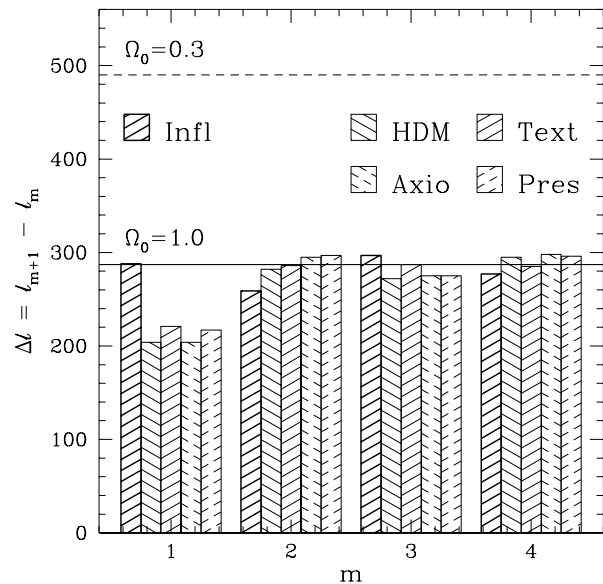


FIG. 2. The peak spacing, especially between higher peaks, is mainly dependent on the sound horizon at last scattering (projected on the sky) yielding a robust feature for the angular size distance test. Here five $\Omega_0 = 1$ models (see text; $\Omega_0 h^2 = 0.25$, $\Omega_b h^2 = 0.0125$ and $\Omega_\Lambda = 0$) are compared with the simple prediction $\Delta\ell = \ell_A$ (solid line).

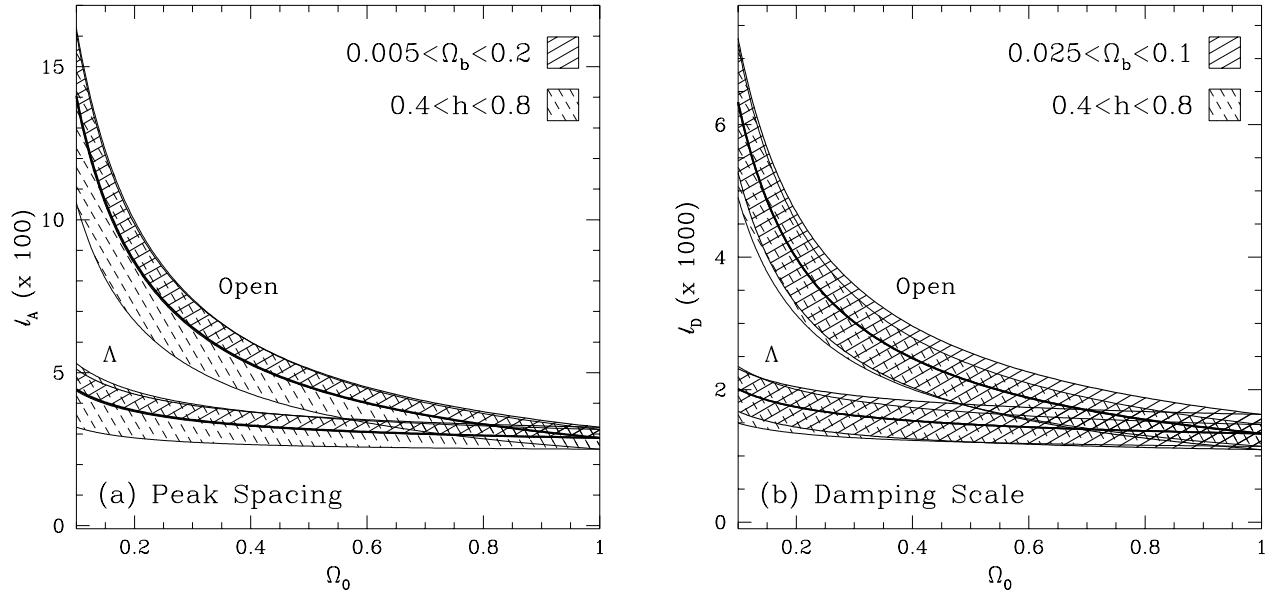


FIG. 3. (a) Peak spacing as a function of Ω_0 in a flat cosmological constant ($\Omega_K = 0$) and open model ($\Omega_\Lambda = 0$). Uncertainties in $\Omega_b h^2$ and h have little effect on the ability to distinguish an open from a flat model. (b) Damping tail as a function of Ω_0 . The damping tail is more dependent on $\Omega_b h^2$ than the peak spacing but still provides interesting constraints.

last scattering surface, the acoustic peaks will be spaced by

$$k_{m+1} - k_m = k_A = \pi/r_s, \quad \Delta\ell = \ell_A = k_A D, \quad (2)$$

where $r_s = \int c_s d\eta$ is the sound horizon at last scattering. To summarize: while the phase and first peak bear the mark of the model-dependent driving force, the spacing of the higher peaks reflects the model-independent natural frequency, set by the sound horizon at last scattering.

If the sound horizon at last scattering is a known quantity, then the peak spacing provides a sensitive angular size distance test of the curvature that is relatively robust to the nature of the fluctuations. In Fig. 2, we show that the peak spacings for the inflationary, texture [5], hot dark matter [4], axionic [6], and pressure scaling [7] isocurvature models are to good approximation related by Eq. (2) to the sound horizon scale.

There are two possible drawbacks to this method of measuring the curvature. The first is that the sound horizon at last scattering depends on the baryon content $\Omega_b h^2$, the matter-radiation ratio $\Omega_0 h^2$ and the thermal history. In Fig. 3a, we show that an uncertain baryon content does not pose an obstacle nor do reasonable values of the Hubble constant $0.4 < h < 0.8$. We shall return to comment on the thermal history below. The second drawback is that for a precise measurement, gravitational forcing effects must be negligible so that the peak spacing reflects the natural frequency of the oscillator. In some models, this may not occur until the higher peaks where damping and secondary effects may make the signal dif-

ficult to observe (compare the 1st-2nd peak spacing with the higher ones in Fig. 2).

The location of the damping tail in the CMB spectrum provides yet another angular size distance test of the curvature [2,8]. The damping is a function of the duration of recombination, or the thickness of the last scattering surface. As the universe recombines, the coupling between the photons and the baryons decreases and the distance that photons can travel before scattering increases. Acoustic oscillations are destroyed as the photons random walk through the electron-baryon fluid. The random walk distance, approximately the geometric mean of the horizon and Compton mean free path, sets the scale of this feature in the CMB.

The benefit of this test is that it is entirely independent of the nature of the fluctuations (see Fig. 1b). As long as the baryon fluctuations are linear, the random walk scale depends only on the background baryon density, ionization fraction, and expansion rate, not on the fluctuations themselves. The main drawback is that it is difficult to measure accurately. The signature of diffusion damping is a sharp exponential cutoff in ℓ at the diffusion scale (see Fig. 1a). Although this exponential shape is essentially unique, secondary effects such as gravitational redshifts between last scattering and the present (ISW effect) can quickly overwhelm the signal making it difficult to measure. How much these factors will degrade the measurement of the curvature will vary from model to model. In inflationary models, both this effect and various other secondary anisotropies are small enough that

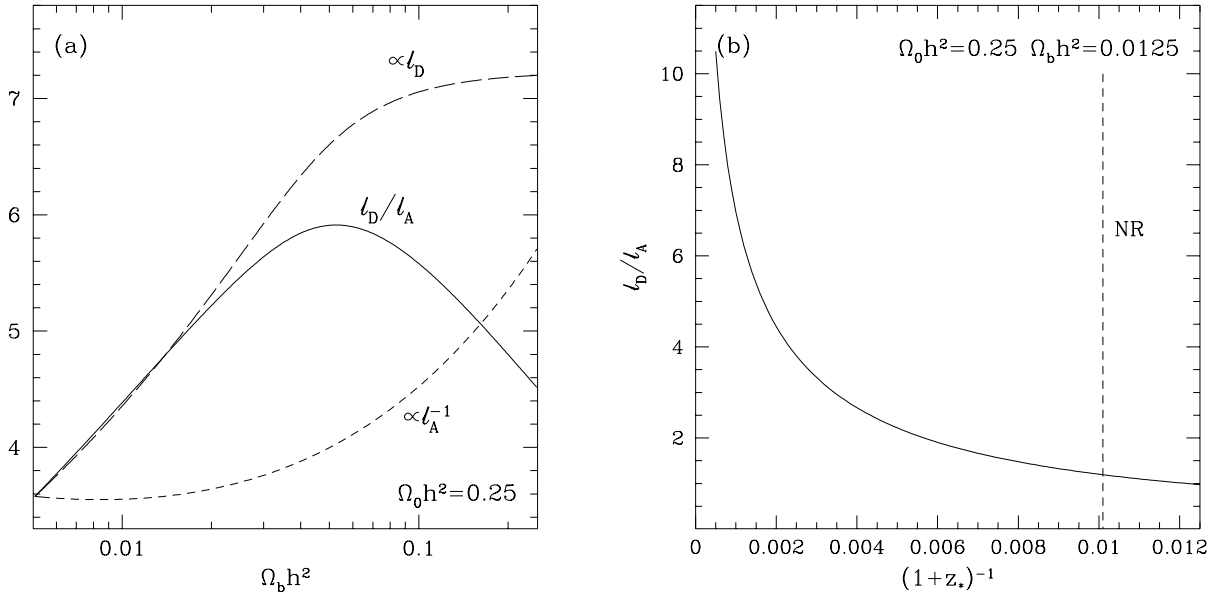


FIG. 4. Discriminating exotic cases. (a) If the baryon content is far from the BBN value, it will be reflected in the ratio of the damping scale to peak spacing ℓ_D/ℓ_A , which gives roughly the number of observable peaks. (b) Likewise, ℓ_D/ℓ_A will also detect any delay in recombination. For simplicity, we have here assumed that the universe recombinates instantaneously at z_* . A realistic recombination scenario will lower ℓ_D/ℓ_A . If recombination occurs late, the model becomes identical to the no-recombination (NR) scenario in its prediction for the CMB.

ℓ_D should be measurable [8].

Another concern is that the damping scale is quite sensitive to the background baryon content, expansion rate and thermal history. In Fig. 3b, we show that for reasonable values of $\Omega_b h^2$ and h this will not prevent us from distinguishing between $\Omega_0 \approx 0.3$ and 1.0 models.

Interestingly the benefits and drawbacks of the peak spacing and damping tail tests are complementary. The peak spacing is easily measured and relatively robust to changes in the other background parameters, but not fully immune to radical behavior in the model for the gravitational fluctuations. The damping tail is immune to such effects but is more difficult to measure and suffers more from uncertainties in the other background parameters.

Ideally, we would like to measure both quantities. By combining these two tests, we have an important consistency check on the underlying assumptions of the model and a discriminator against truly exotic models. For example, the baryon content $\Omega_b h^2$ could be far from the BBN value or recombination could be delayed by early energy injection from decaying particles or non-linear

structure formation.* Since these exotic possibilities affect the two scales differently, a useful discriminator is the ratio of the two scales ℓ_D/ℓ_A (see Fig. 4). If this ratio is anomalous, it is a clear indication that one of the fundamental assumptions is invalid. If one knew from external information which assumption that is (e.g. baryon content, radiation content, thermal history, etc.) then accurate measurements of the curvature could again be made.

The full acoustic signature in the CMB provides additional consistency checks. The Compton drag of the baryons on the photons tends to enhance the fluctuations inside gravitational potential wells. For the acoustic oscillations, this implies that compressional phases will be enhanced over rarefaction phases leading to an alternating series of relative peak heights (see Fig. 1b). If the baryon content is far lower than the BBN value, then this effect is too weak to modulate the peak heights. If it is far higher then it is so strong that the rarefaction phases will not appear as peaks in the anisotropy at all [2]. This could also occur if recombination is delayed such

*Late reionization does not interfere with these tests since it mainly suppresses power uniformly on small scales without changing the peak spacing or damping tail location.

that the ratio of baryon to photon densities is higher at last scattering. Since the modulation of the peaks is a unique signature of baryons at last scattering, its detection would provide an important consistency check on the assumptions underlying the curvature measurement.

In summary, the acoustic signature in the CMB anisotropy spectrum provides a sufficient number of features such that a curvature measurement which is essentially robust to the nature of the fluctuations, the other background parameters, and thermal history may be constructed. The full battery of tests will require complete information on the acoustic signal – from the first peak all the way through to the damping tail.

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