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TENSOR ANISOTROPIES IN AN OPEN UNIVERSE

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ABSTRACT

We calculate the anisotropies in the cosmic microwave background induced by long-wavelength primordial gravitational waves in a universe with negative spatial curvature, such as are produced in the "open inflation" scenario. The impact of these results on the *COBE* normalization of open models is discussed.

Subject headings: cosmic microwave background — cosmology: theory

There is considerable observational prejudice suggesting that we live in a universe with negative spatial curvature, an open universe. Recently, Bucher, Goldhaber, & Turok (1995) have devised an open inflationary cosmogony. This has allowed one, for the first time, to calculate the spectrum of primordial fluctuations in an open model. The scalar field modes, which give rise to density perturbations, come in three types and have been extensively discussed in the literature (for a recent review, see Cohn 1996). As with all inflationary models, a nearly scale-invariant spectrum of gravitational waves (tensor modes) is also produced, and the spectrum of such modes has recently been computed (Tanaka & Sasaki 1997; Bucher & Cohn 1997). If the energy density during inflation is high enough, these modes induce a measurable cosmic microwave background (CMB) anisotropy (Abbott & Wise 1984). In this Letter we calculate these CMB anisotropies and their implications for the *COBE* normalization of open models.

The gravitational waves represent tensor perturbations to the metric

$$ds^2 = a^2(\eta)\,[-\,d\eta^2 + (\gamma_{ij} + h_{ij})\,dx^i\,dx^j], \qquad (1)$$

where $a(\eta)$ is the scale factor, $d\eta = dt/a(t)$ is the conformal time, and γ_{ij} is the three-metric for a space of constant (negative) curvature $K = -H_0^2(1 - \Omega_0)$. The perturbations decompose as $h_{ij} = 2hQ_{ij}$, where the harmonic modes are the transverse-traceless tensor eigenfunctions of the Laplacian $\nabla^2 Q_{ij} = -k^2 Q_{ij}$ (Kodama & Sasaki 1984; Abbott & Schaefer 1986). The photon temperature distribution function Θ can likewise be expanded in mode functions $\Theta(\eta, \boldsymbol{x}, \hat{n}) = \sum_k \sum_l \Theta_1(\boldsymbol{k}) G_l(\boldsymbol{x}, \boldsymbol{k}, \hat{n})$, which form a complete basis constructed out of covariant derivatives of Q_{ij}.

The Einstein equations reduce to a single relation that expresses the evolution of the amplitude of the tensor metric perturbation in the presence of tensor anisotropic stress in the matter $p\pi$ (Kodama & Sasaki 1984; Abbott & Schaefer 1986),

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + (k^2 + 2K)h = 8\pi G a^2 p\pi. \qquad (2)$$

The quadrupolar variations in the metric induced by $\dot{h}$ leave a corresponding signature through the photon quadrupole ($l = 2$), which acts as the source to the tensor Boltzmann hierarchy (Hu & White 1997),

$$\dot{\Theta}_2 = -k\frac{\sqrt{5}}{7}\kappa_3\Theta_3 - \dot{h} - \frac{9}{10}\dot{\tau}\Theta_2,$$
$$\dot{\Theta}_l = k\left[\frac{\sqrt{l^2 - 4}}{(2l - 1)}\kappa_l\Theta_{l-1} - \frac{\sqrt{(l + 1)^2 - 4}}{(2l + 3)}\kappa_{l+1}\Theta_{l+1}\right] - \dot{\tau}\Theta_l, \qquad (3)$$

where $\dot{\tau} = an_e\sigma_T$ is the differential optical depth to Thomson scattering, and the geodesic deviation factors are $\kappa_l^2 = [1 - (l^2 - 3)K/k^2]$. In the above, we have neglected the coupling of the temperature anisotropy to the CMB polarization since this has a small effect on the temperature anisotropy spectrum calculated (see Hu & White 1997 and its generalization to open universes Hu et al. 1997 for more details). The power spectrum of temperature anisotropies today is defined as

$$(2l + 1)^2 C_l^{(T)} = \frac{2}{\pi}\int_0^\infty \frac{dq}{q}\, q^3 |\Theta_l(\eta_0, q)|^2, \qquad (4)$$

where $q^2 = k^2 + 3K$ ranges from 0 to ∞ since the subcurvature modes are complete for $k \geq \sqrt{-3K}$ (Abbott & Schaeffer 1986). There are no supercurvature modes for the gravity wave background (Tanaka & Sasaki 1997).

The initial gravitational wave power spectrum may be parameterized as

$$q^3 P_h(q) \equiv q^3 |h(\eta = 0, q)|^2 \propto \frac{(q^2 + 4)}{(q^2 + 1)} f(q). \qquad (5)$$

The bubble models predict $f(q) \sim q$ for $q \ll 1$ and $f(q) \simeq 1$ for $q \gtrsim 2$ (Tanaka & Sasaki 1997; Bucher & Cohn 1997). The *minimal* anisotropies are produced when this turnover occurs at the largest allowed q, or $f(q) = \tanh(\pi q/2)$ (see Bucher & Cohn 1997, eq. [6.5]), which we display in Figure 1 (*dashed lines*). Decreasing the turnover scale increases the low-l anisotropy, as shown in the $f(q) = \tanh(\pi q)$ example of Figure 1 (*dotted lines*).

The exact form of the function $f(q)$ is sensitive to the

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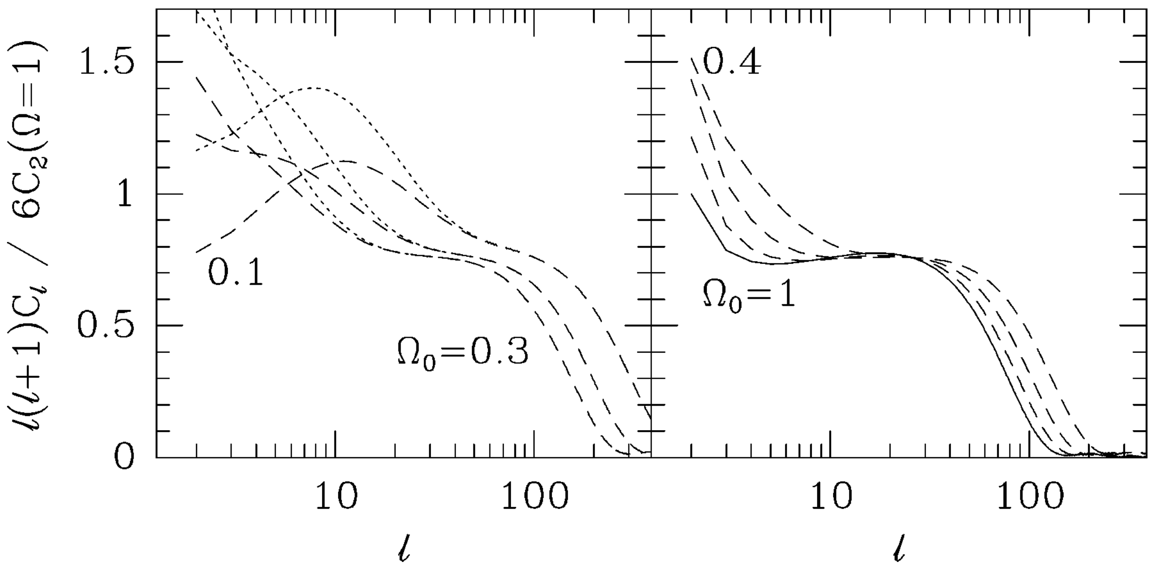

FIG. 1.—The CMB anisotropy spectrum induced by a spectrum of gravitational waves with $n_T = 0$. The solid line is $\Omega_0 = 1$ with $h = 0.75$. Dashed lines show the *minimal* anisotropy for (*left*) $\Omega_0 = 0.1$, 0.2, and 0.3, and (*right*) 0.4, 0.6, and 0.8 relative to $\Omega_0 = 1$. The dotted lines (*left*) illustrate how low-l anisotropies are enhanced as the bubble parameters are altered from their minimal values.

parameters of the bubble. However, as illustrated in Figure 1, variations are confined to the low multipoles for reasonable values of $\Omega_0 \gtrsim 0.3$, as was the case for the scalar modes (Yamamoto & Bunn 1996). Small changes in bubble parameters will thus be lost in cosmic variance. Since a calculation of nonscale-invariant spectra has not been performed for tensors, we make an *Ansatz* that the spectral tilt is a pure power law in k, viz. $(k/H_0)^{n_T}$ times the above result.

In flat space, as $k \to 0$ the power in the Boltzmann hierarchy of equation (3) remains in the *quadrupole*. Thus, we expect that tensor contributions in a flat universe will have an enhanced quadrupole due to long-wavelength modes (as seen in Fig. 2). However, as $k \to 0$, equation (2) requires that $\dot{h} \to 0$, so the source of the anisotropy dies off to low k, ensuring a finite quadrupole. If $K < 0$, then even as $q \to 0$, the metric perturbation damps when $\eta \gtrsim |K|^{-1/2}$, i.e., when the curvature scale crosses the horizon. This provides a source to the anisotropy to arbitrarily low q, and indeed the C_l from a scale-invariant spectrum, $q^3P(q)$ = const, would diverge logarithmically. A similar divergence would occur for the scalar monopole because of the decay of the gravitational potential (see Fig. 2 and Sugiyama & Silk 1994), but the monopole is not observable. The open inflation models regulate this divergence with the finite energy difference before and after the tunneling event that defines the bubble (Tanaka & Sasaki 1997; Bucher & Cohn 1997) and translates into the turnover in $f(q)$ discussed above. If one goes to sufficiently low $\Omega_0 \lesssim 0.1$, the effect of the curvature cutoff $k(q = 0) = \sqrt{-3K}$ can also be seen as a low multipole suppression in the spectrum. However, for Ω_0 of interest for structure formation the curvature cutoff is absent, and the large contribution from low-q is the dominant effect, leading to anisotropy spectra that decrease strongly with l on *COBE* scales.

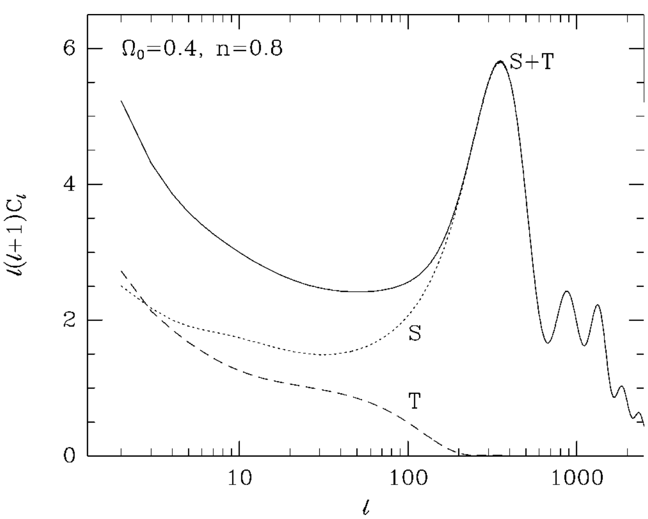

FIG. 2.—Relative tensor and scalar anisotropy contributions for a model with $\Omega_0 = 0.4$ and $n = 0.8$ assuming $n_S - 1 = n_T$, $\Omega_B h^2 = 0.02$, and $h = 0.6$. This model has been chosen for illustration to have $T/S \simeq 1$ and would produce insufficient large-scale structure, e.g., $\sigma_8 \simeq 0.25$, as well as provide a poor fit to the *COBE* data.

Since gravitational waves provide anisotropies but no density fluctuations, they lower the normalization of the matter power spectrum. Open models already have quite a low normalization (e.g., White & Silk 1996; White & Scott 1996), so we seek the *minimal* anisotropies induced by gravity waves. These minimal anisotropies are also, in fact, close to what most models would predict, with reasonable inflationary potentials. We express the *COBE* 4 yr normalization (Bennett et al. 1996) in terms of the value of the density perturbations per logarithmic interval in k evaluated at horizon crossing. Writing $\Delta^2(k) = k^3P(k)/(2\pi^2)$, we define $\delta_H \equiv \Delta(k = H_0)$. If we hold the "shape" of the matter power spectrum fixed, then the small-scale power (e.g., σ_8) is proportional to δ_H. For minimal models with $\tilde{n} \equiv n - 1 = n_T$ the *COBE* 4 yr data give

$$10^5\delta_H = 1.95\Omega_0^{-0.35-0.19\ln\Omega_0 + 0.15\tilde{n}} \exp(1.02\tilde{n} + 1.70\tilde{n}^2). \quad (6)$$

The fitting function works to 3% over the range $0.2 < \Omega_0 \leq 1$ and $0.7 < n < 1$, whereas the *COBE* 1 σ error is 10%. Compared with the equivalent expression *without* gravity waves (Bunn & White 1997, eq. [31]), we isolate the additional suppression as $\Omega_0^{0.32\tilde{n}}$ exp $(2.02\tilde{n} + 1.84\tilde{n}^2)$. This suppression exacerbates the problem that low-Ω_0 models have with the present-day abundance of rich clusters (White & Silk 1996).

In conclusion, we have presented the first calculation of the anisotropy in the CMB from a spectrum of long-wavelength gravitational waves in an open universe. The spectrum exhibits a peak in large-angle power that is dependent on the modifications to the initial power spectrum near the curvature scale from the bubble wall. For very low Ω_0 a curvature cutoff is seen in the spectrum, analogous to the case of scalar modes. Since gravitational waves provide anisotropies but no density fluctuations, they lower the normalization of the matter power spectrum. We have calculated this normalization from the *COBE* 4 yr data and expressed our result in terms of a fitting function (eq. [6]). The lower normalization of the tilted models with gravitational waves exacerbates the difficultly such models have in fitting the present-day abundance of rich clusters.

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REFERENCES

Abbott, L. F., & Schaefer, R. K. 1986, ApJ, 308, 546
Abbott, L. F., & Wise, M. 1984, Nucl. Phys. B, 244, 541
Bennett, C. L., et al. 1996, ApJ, 454, L1
Bucher, M., & Cohn, J. D. 1997, Phys. Rev. D, 55, 7461
Bucher, M., Goldhaber, A. S., & Turok, N. 1995, Phys. Rev. D, 52, 3314
Bunn, E. F., & White, M. 1997, ApJ, 480, 6
Cohn, J. D. 1996, in Proc. 31st Rencontres de Moriond, Microwave Background Anisotropies, ed. F. Bouchet, in press
Hu, W., Seljak, U., White, M., & Zaldarriaga, M. 1997, in preparation
Hu, W., & White, M. 1997, Phys. Rev. D, 56, 596
Kodama, H., & Sasaki, M. 1984, Prog. Theor. Phys., 78 1
Sugiyama, N., & Silk, J. 1994, Phys. Rev. Lett., 73, 509
Tanaka, M., & Sasaki, M. 1997, Prog. Theor. Phys., 97, 243
White, M., & Scott, D. 1996, Comments Astrophys., 18, 289
White, M., & Silk, J. 1996, Phys. Rev. Lett., 77, 4704
Yamamoto, K., & Bunn, E. F. 1996, ApJ, 464, 8