The Principal Components of

Falsifying Cosmological Paradigms

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The Standard Cosmological Model

- Standard ΛCDM cosmological model is an exceedingly successful phenomenological model
- Rests on three pillars

  Inflation: sources all structure

  Cold Dark Matter: causes growth from gravitational instability

  Cosmological Constant: drives acceleration of expansion

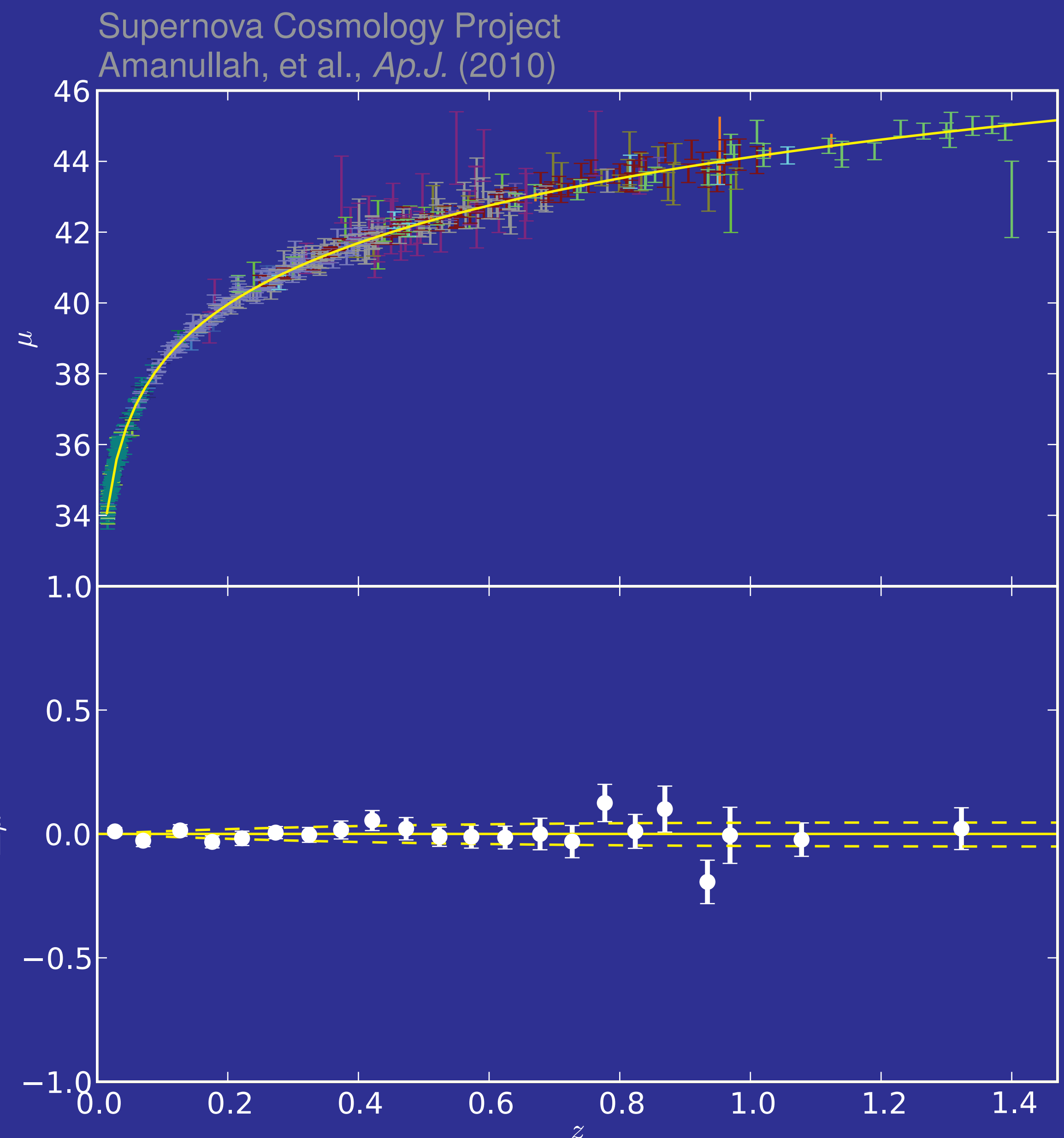
  that are poorly understood from fundamental physics
- ΛCDM and its generalizations to dark energy and slow-roll inflationary models is highly predictive and hence highly falsifiable
- Parameterization of the paradigm encompasses free functions ($w(z)$, $V(\phi)$, $x_e(z)$)
- Principal components form an observationally complete basis

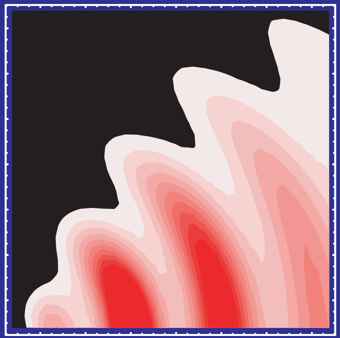
Dark Energy $w(z)$

Falsifying ΛCDM

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

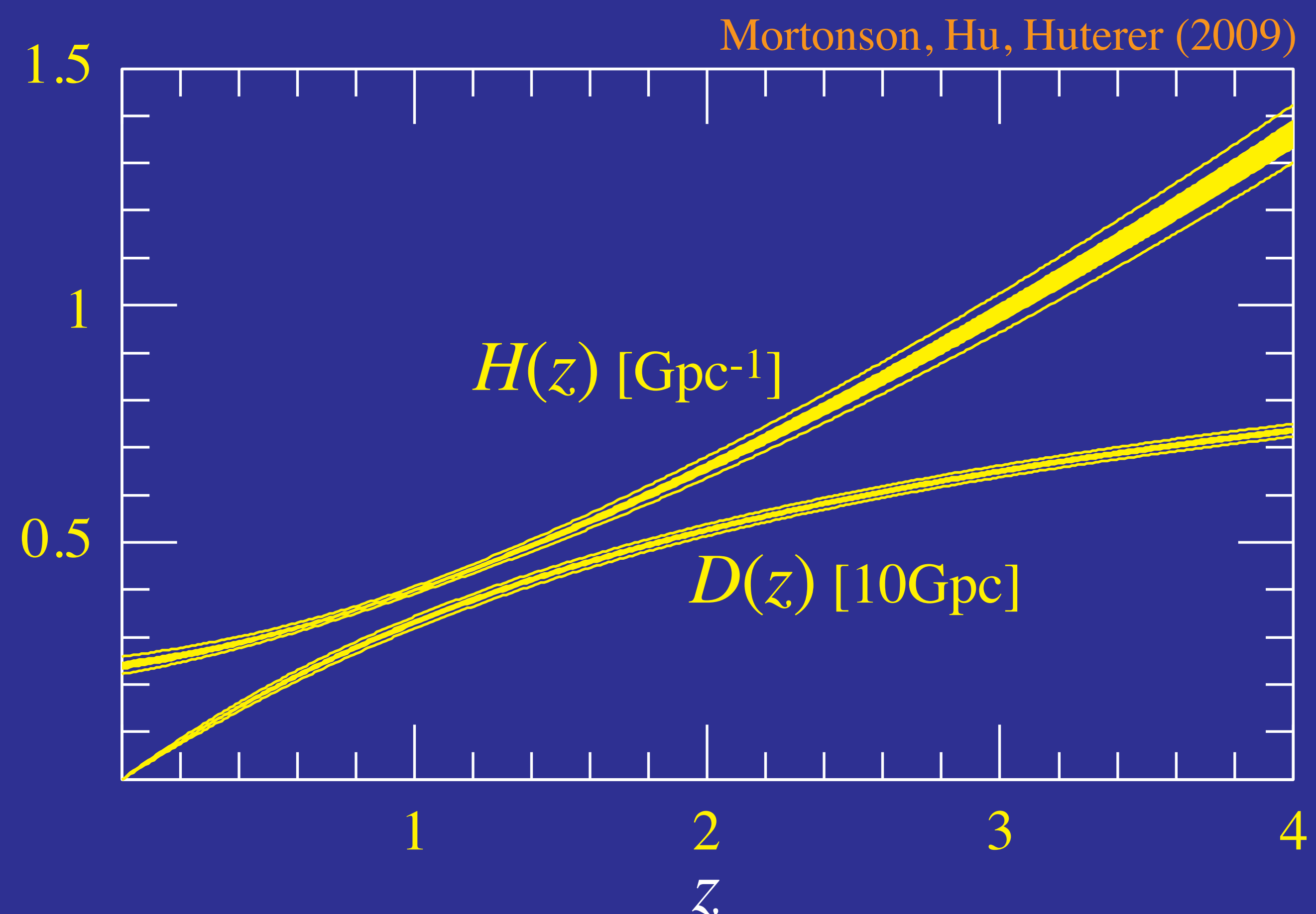




Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Flat ΛCDM

- CMB predicts expansion history and distance redshift relation at all redshifts to few percent precision
- Any violation falsifies flat ΛCDM
(violation of flatness falsifies standard inflation)



Is H_0 Interesting?

- WMAP infers that in a flat Λ cosmology H_0=71±2.5
- Key project measures H_0=72±8; SHOES 74.2±3.6
- Are local H_0 measurements still interesting?

- YES!!!

- CMB best measures only high-z quantites:
  distance to recombination
  energy densities and hence expansion rate at high z

- CMB observables then predict H_0 for a given hypothesis about the dark energy (e.g. flat Λ)
- Consistency with measured value is strong evidence for dark energy and in the future can reveal properties such as its equation of state *if* H_0 *can be measured to percent precision*

Fixing the Past; Changing the Future

Fixed Deceleration Epoch

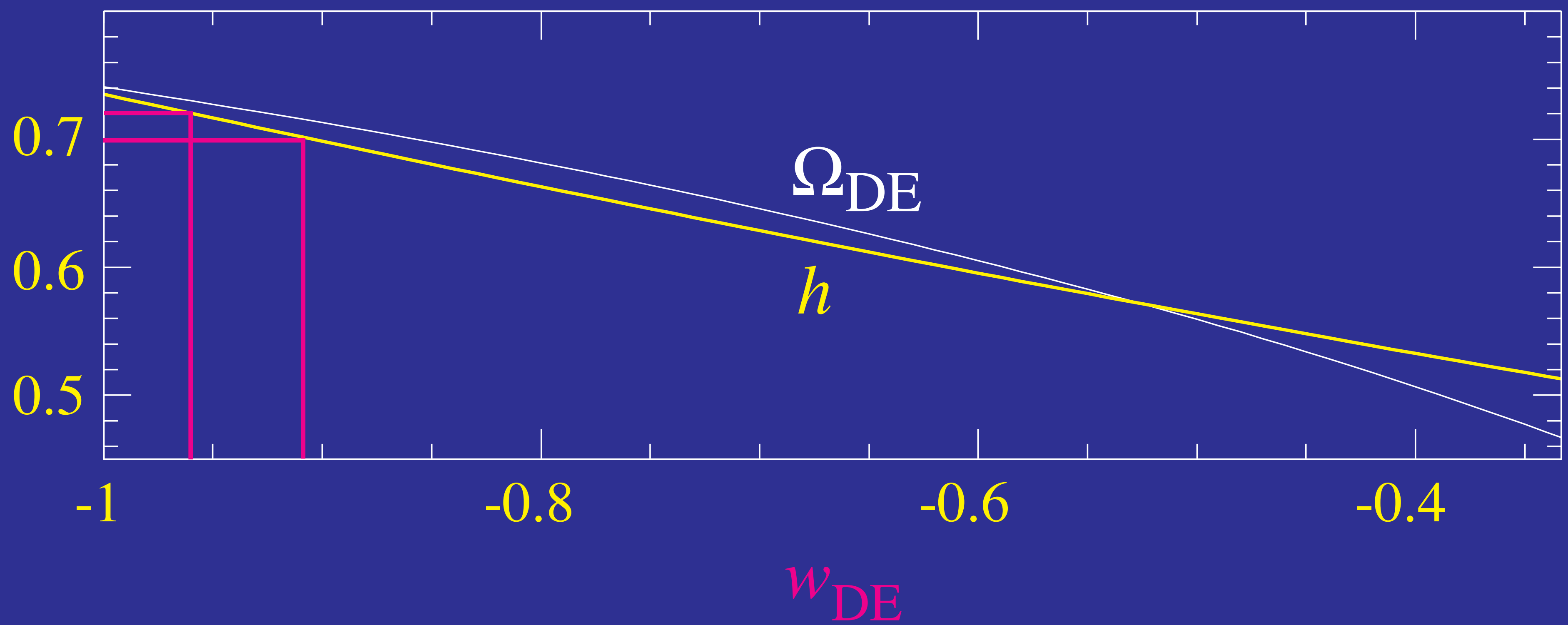
- CMB determination of matter density controls all determinations in the deceleration (matter dominated) epoch
- WMAP7: $\Omega_m h^2 = 0.133 \pm 0.006 \rightarrow 4.5\%$
- Distance to recombination D_* determined to $\frac{1}{4}4.5\% \approx 1\%$
- Expansion rate during any redshift in the deceleration epoch determined to 4.5%
- Distance to any redshift in the deceleration epoch determined as

$$D(z) = D_* - \int_z^{z_*} \frac{dz}{H(z)}$$

- Volumes determined by a combination $dV = D_A^2 d\Omega dz / H(z)$
- Structure also determined by growth of fluctuations from z_*
- $\Omega_m h^2$ can be determined to $\sim 1\%$ from Planck.

H_0 = Dark Energy

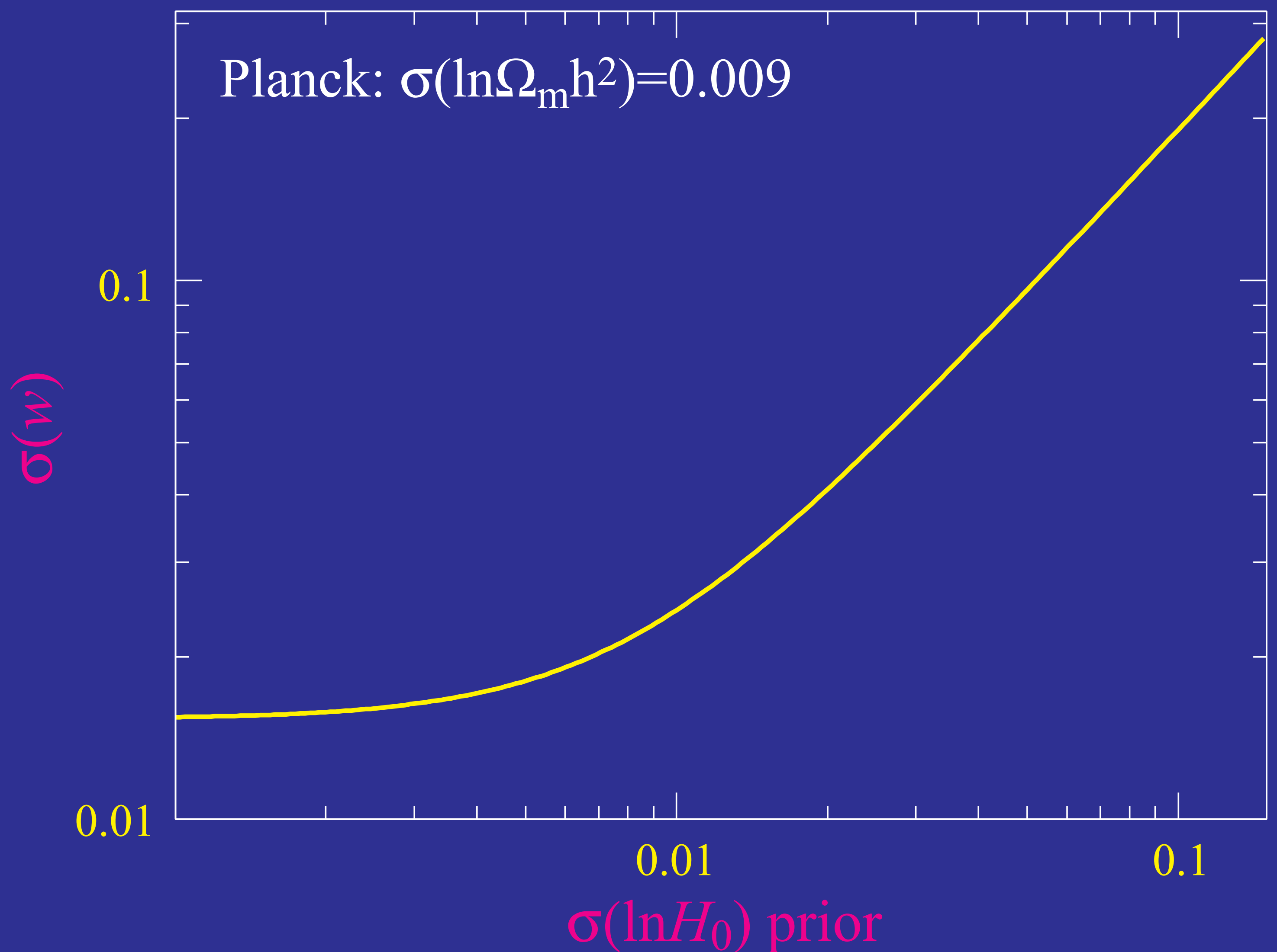
- Flat constant w dark energy model
- Determination of Hubble constant gives w to comparable precision



- For evolving w, equal precision on average or pivot w, equally useful for testing a cosmological constant

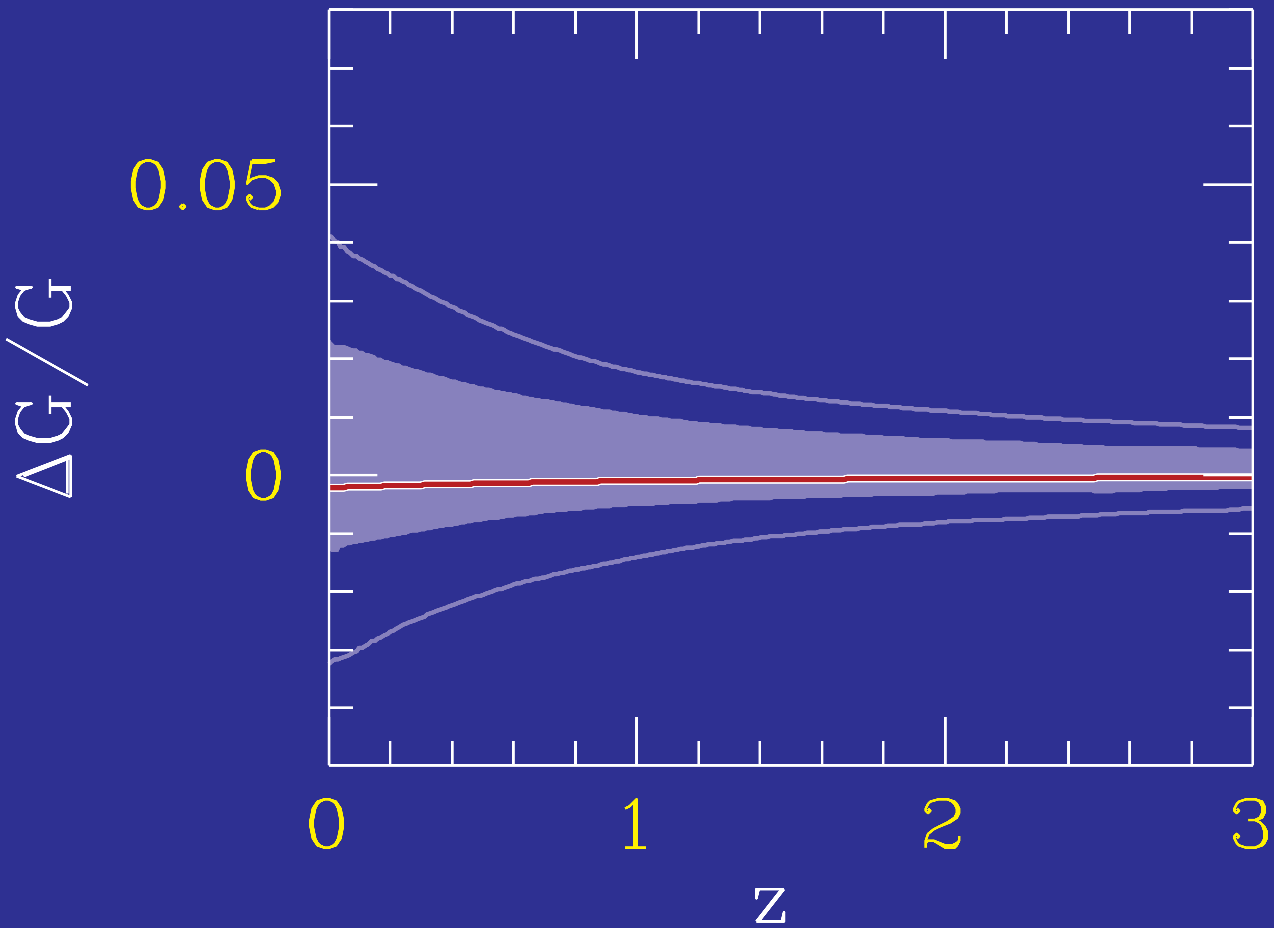
Forecasts for CMB+H_0

- To complement CMB observations with $\Omega_m h^2$ to 1%, an H_0 of ~1% enables constant w measurement to ~2% in a flat universe



Falsifying ΛCDM

- Λ slows growth of structure in highly predictive way



Cosmological Constant

Beyond ΛCDM

Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Why PCs

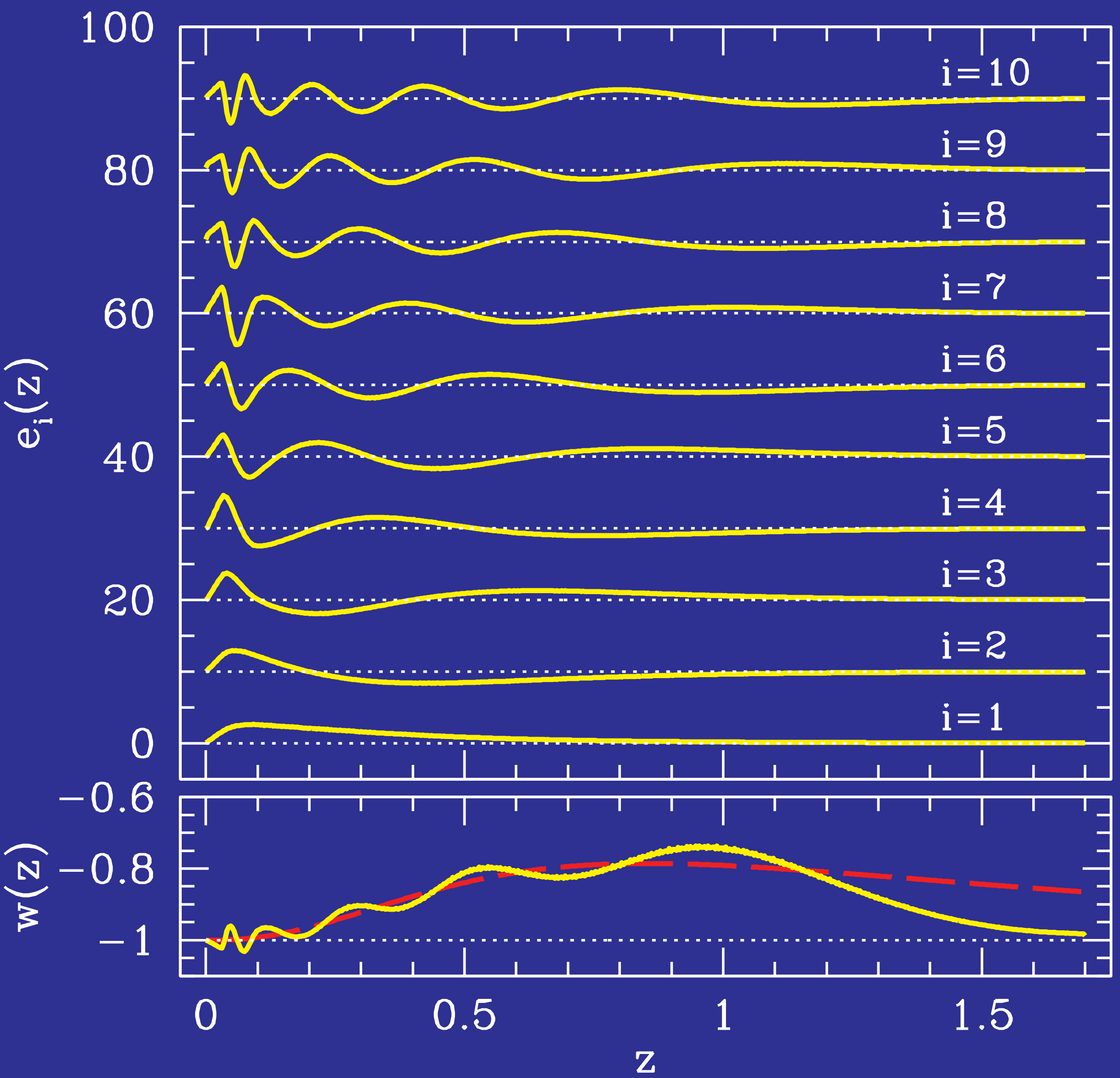
- Principal components are the eigenbasis of the projected or actual covariance matrix for a discrete representation of $f(x_i)$
- Rank ordered in observability and decorrelated linear combination

Advantages:

- Define according to Fisher projected covariance matrix – no a posteriori bias in looking for features
- Efficient – can keep only observable modes and never requires MCMC over large correlated discrete space
- Complete – can include as many modes as required to make basis observationally complete
- Paradigm testing – rapidly explore all possible observational outcome of a given paradigm
- Falsifiable predictions for other observables not yet measured

Equation of State PCs

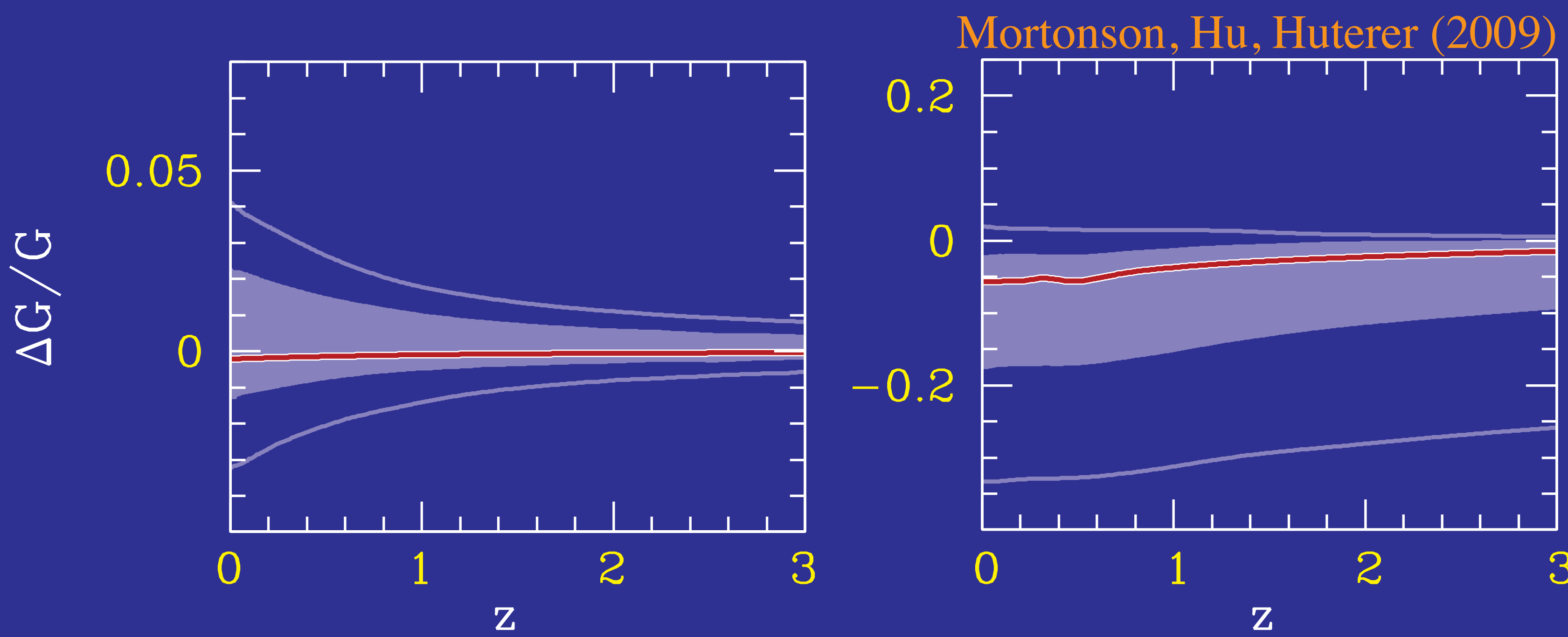
- 10 PCs defined for StageIV (SNAP+Planck) define an observationally complete basis out to z=1.7



Mortonson, Huterer, Hu (2010)

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way



Cosmological Constant

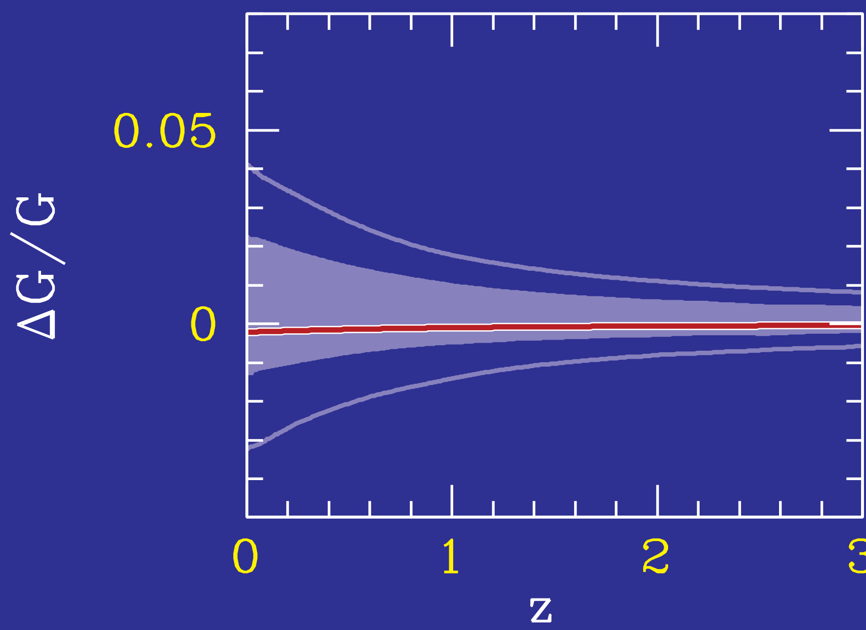
Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

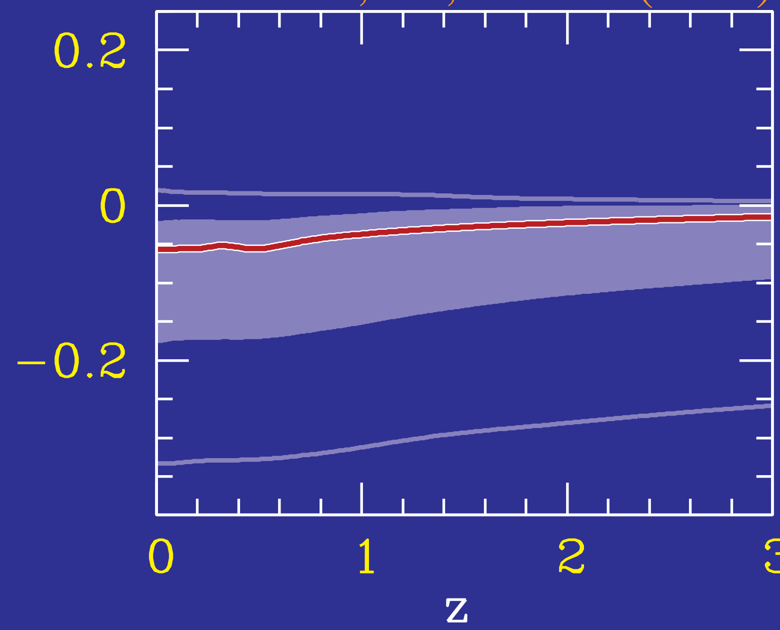
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

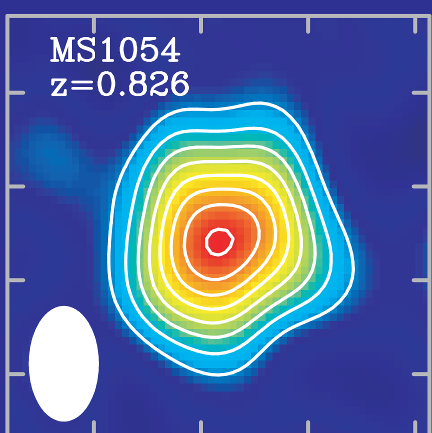
Mortonson, Hu, Huterer (2009)

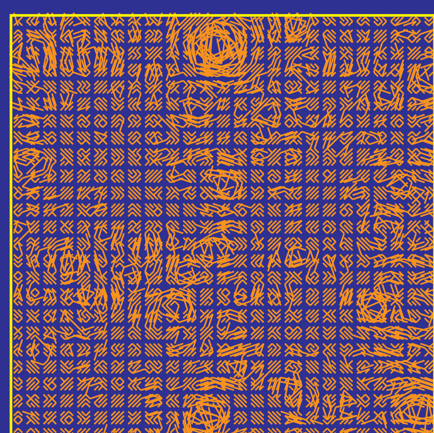


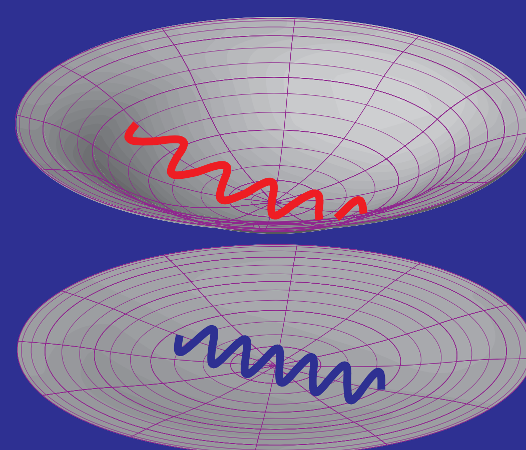
Cosmological Constant

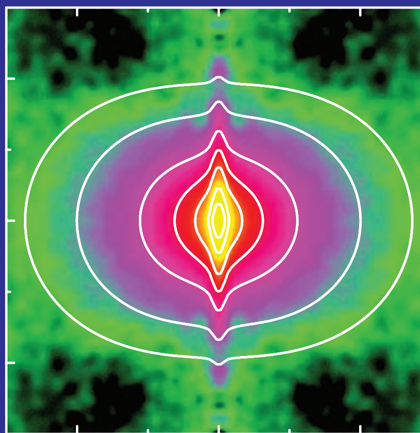


Quintessence



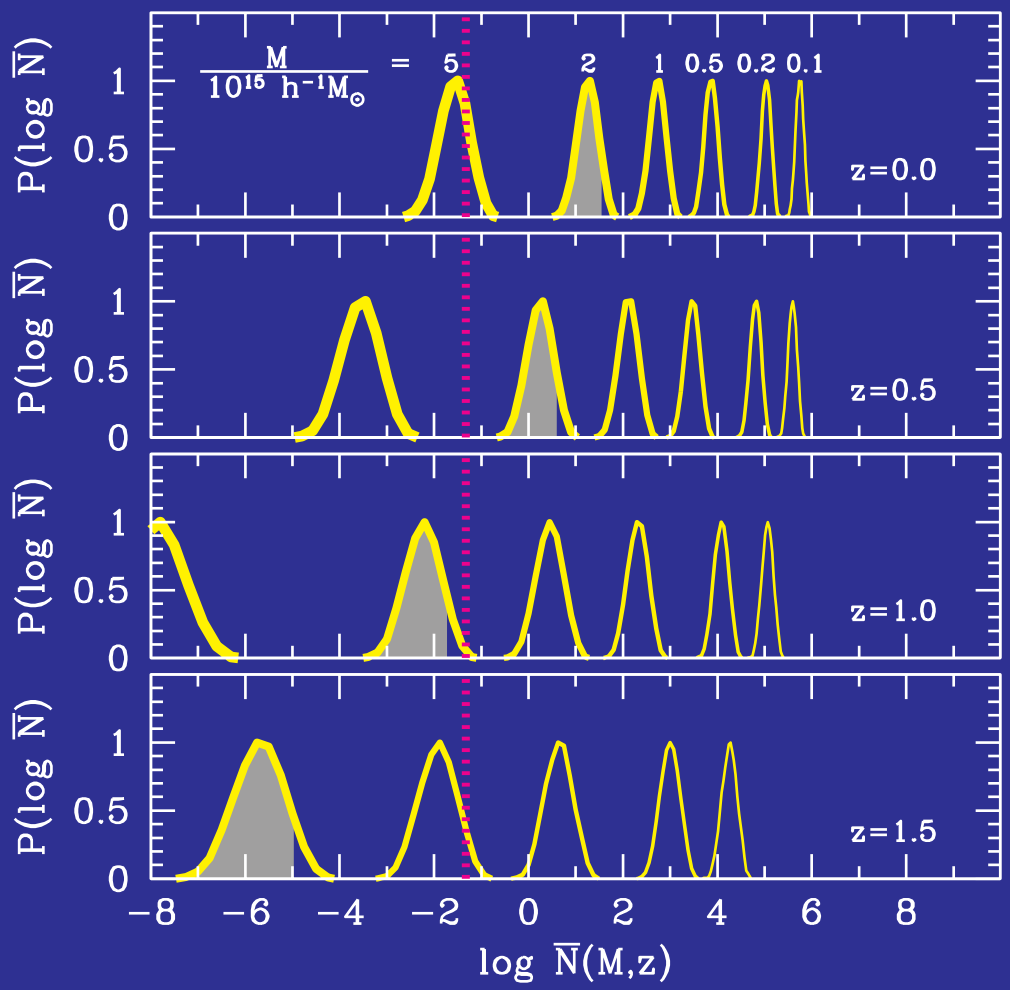






Elephantine Predictions

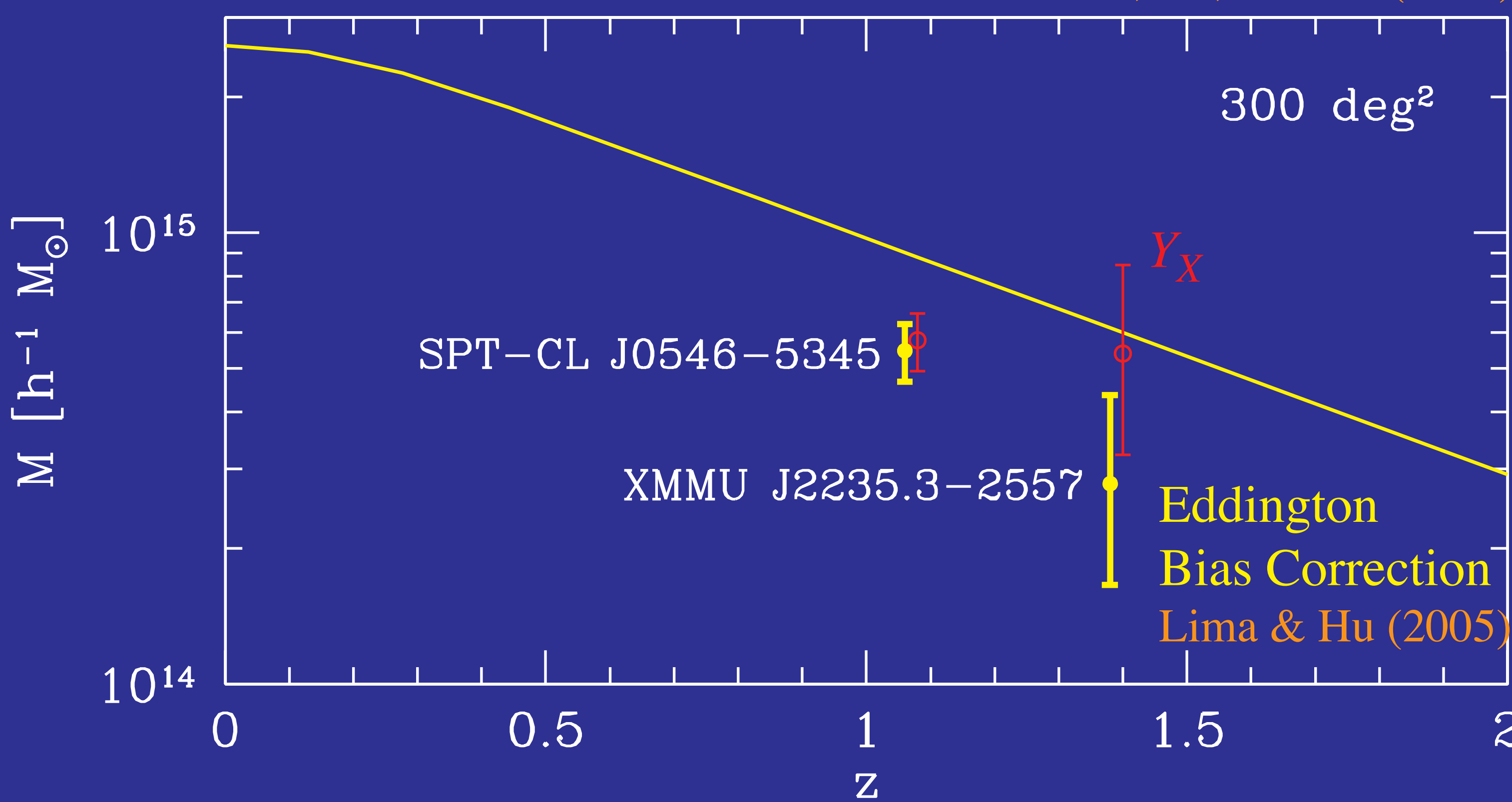
- Geometric constraints on the cosmological parameters of ΛCDM
- Convert to distributions for the predicted average number of clusters above a given mass and redshift



ΛCDM Falsified?

- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- No currently known high mass, high redshift cluster violates this bound

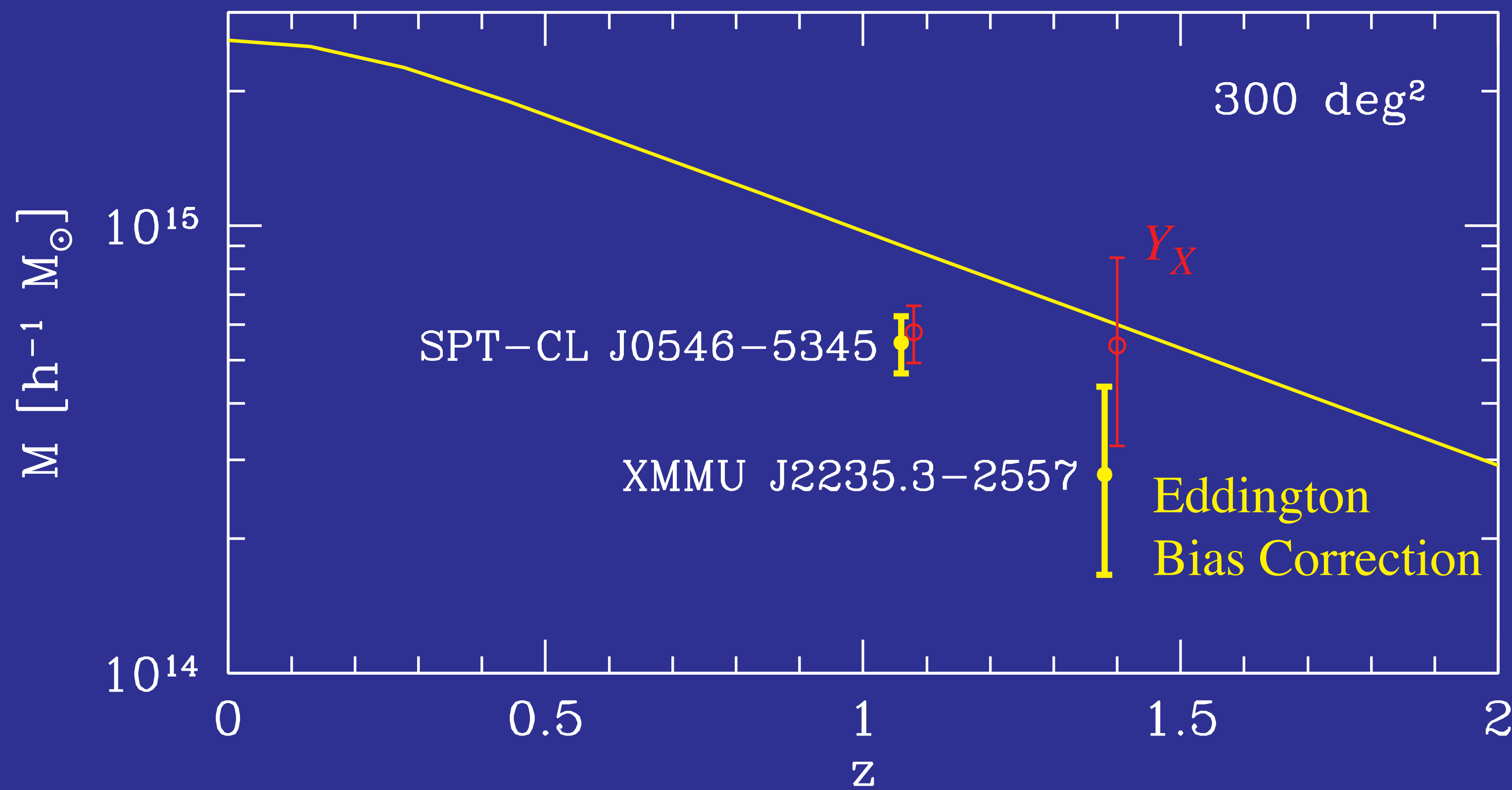
Mortonson, Hu, Huterer (2010)



ΛCDM Falsified?

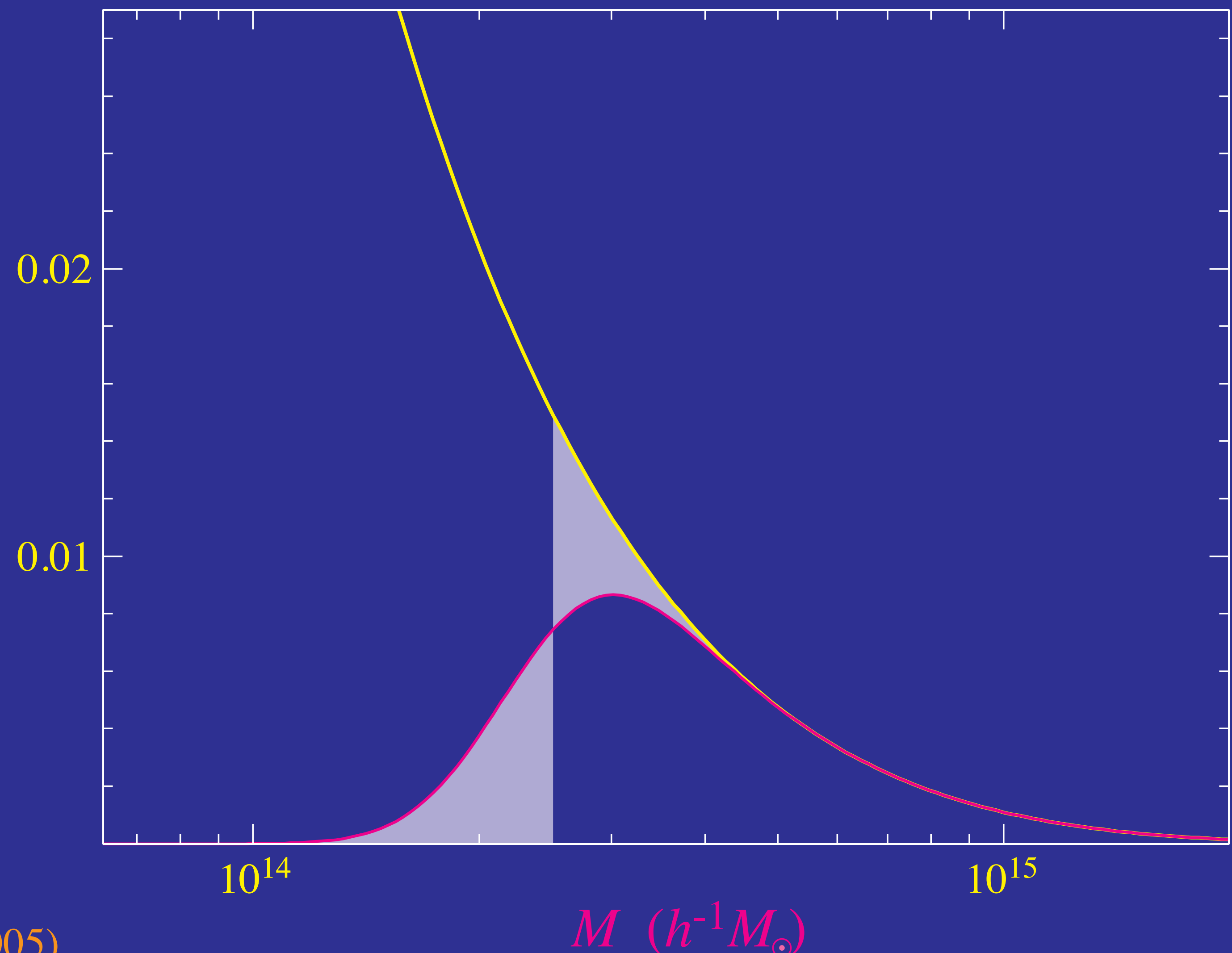
- 95% of ΛCDM parameter space predicts less than 1 cluster in 95% of samples of the survey area above the $M(z)$ curve
- Convenient fitting formulae for future elephants:
  http://background.uchicago.edu/abundance

Mortonson, Hu, Huterer (2010)



Number Bias

- For $>M_{\rm obs}$, scatter and steep mass function gives excess over $>M$
- Equate the number $>M_{\rm obs}$ to $>M_{\rm eff}$
- Not the same as best estimate of true mass given model!



Lima & Hu (2005)

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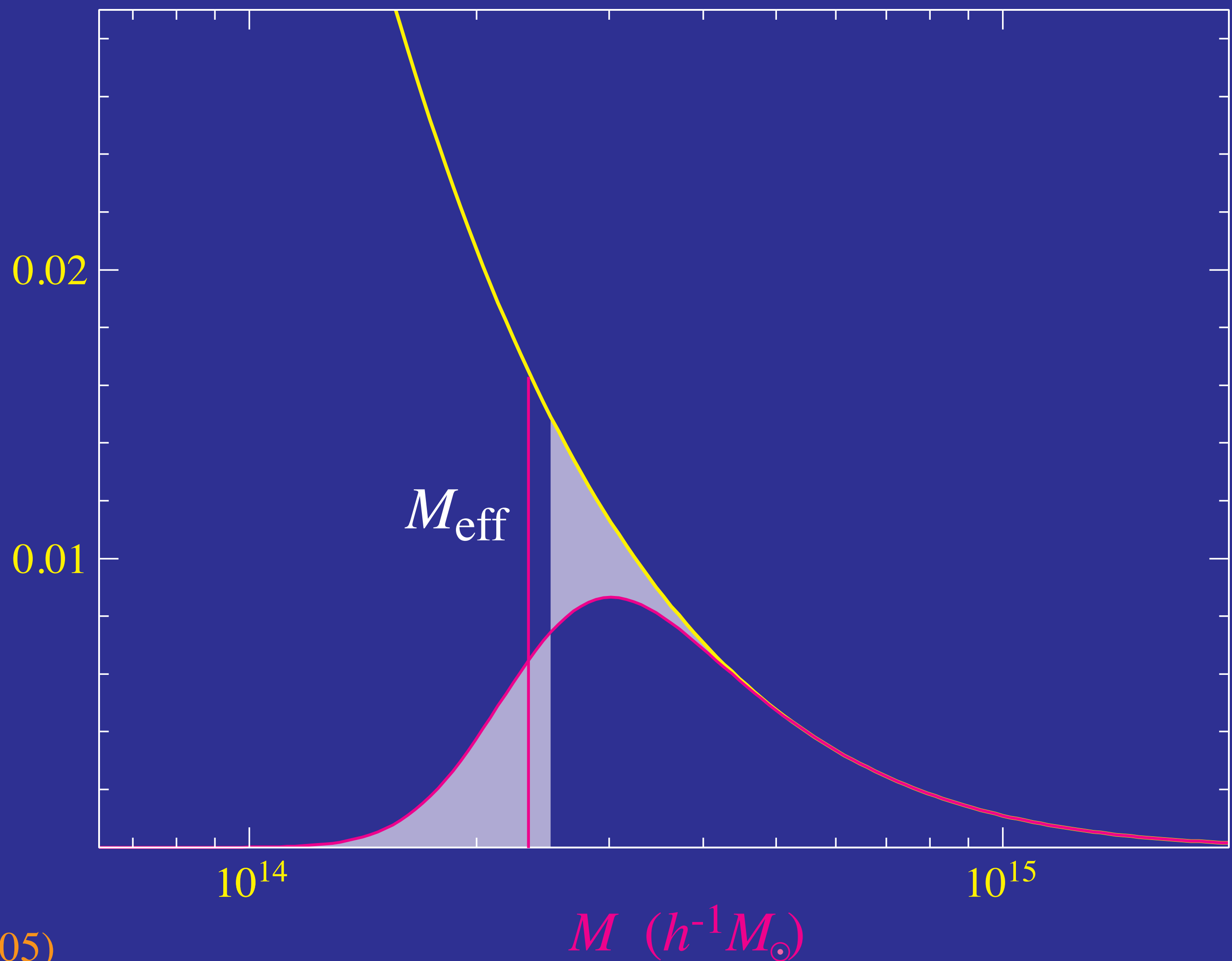
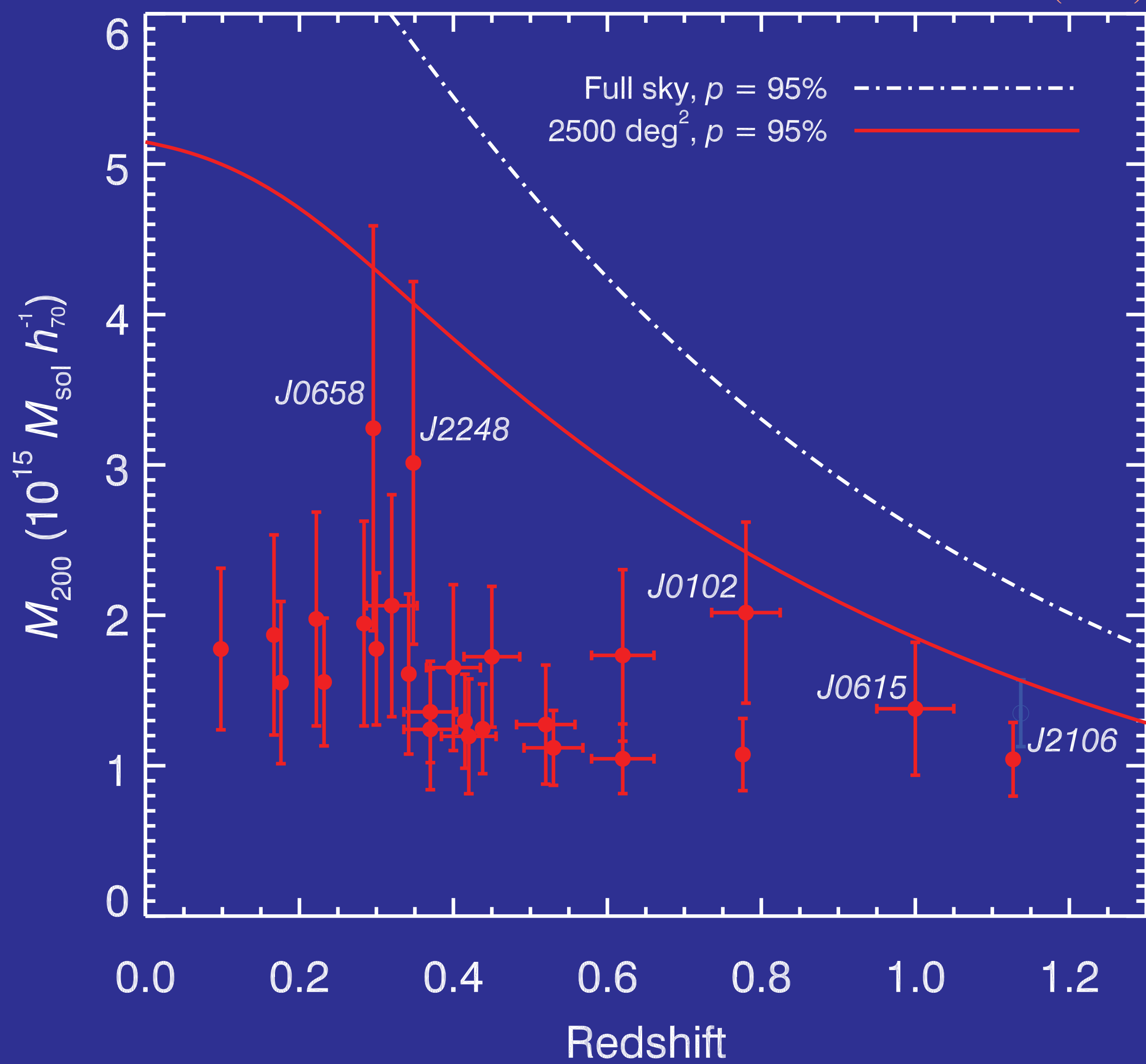


Lima & Hu (2005)

Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

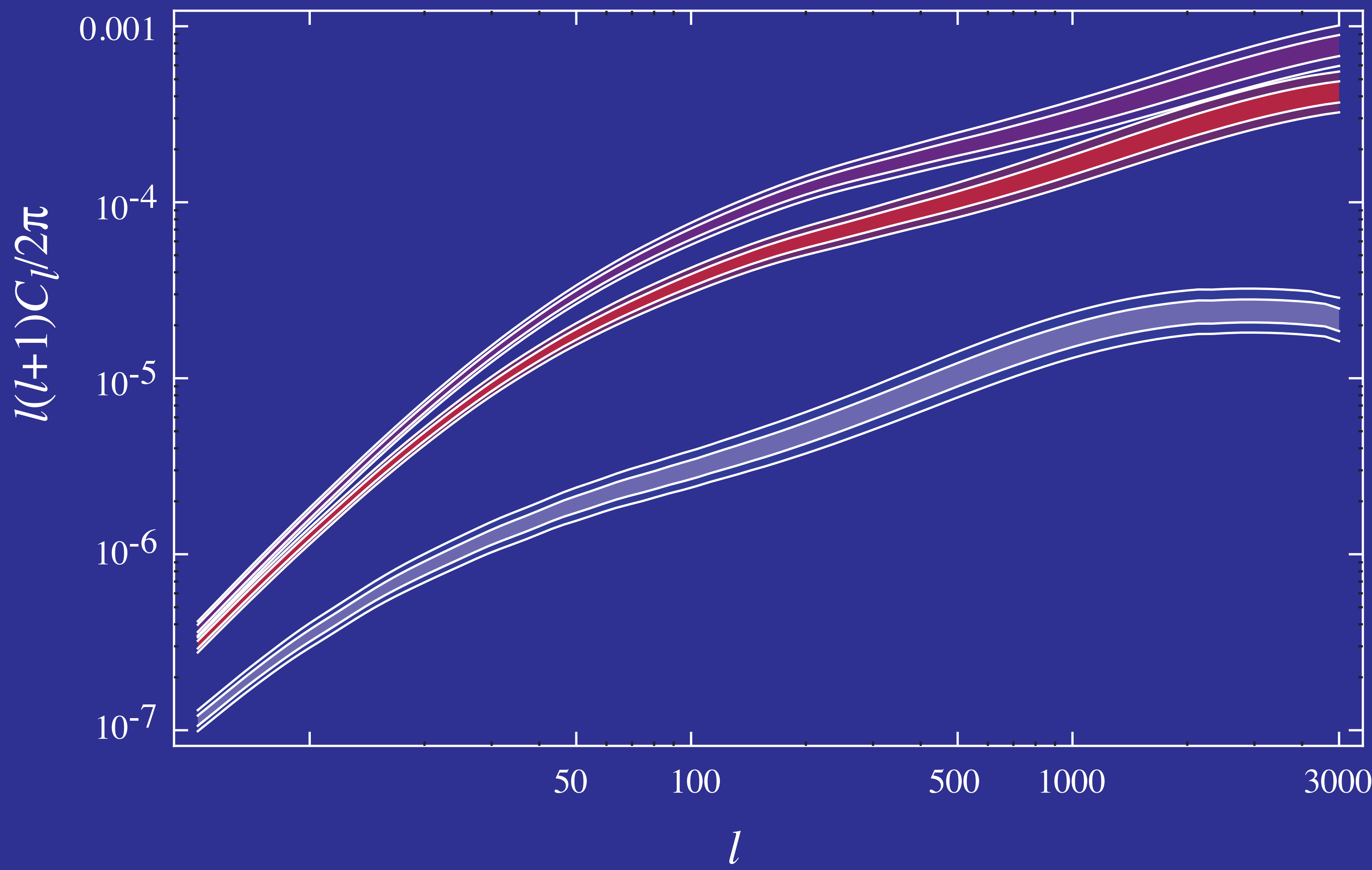
Williamson et al (2011)



Predictions for Cosmic Shear

- ΛCDM statistical prediction for cosmic shear and sources at z=0.5,2,3.5

Vanderveld et al (2011)



Inflaton Potential $V(\phi)$

Slow Roll Inflation

- Standard paradigm: quantum fluctuations of a single canonical scalar field slowly rolling in a smooth potential
- Predictions:

  Scalar fluctuations

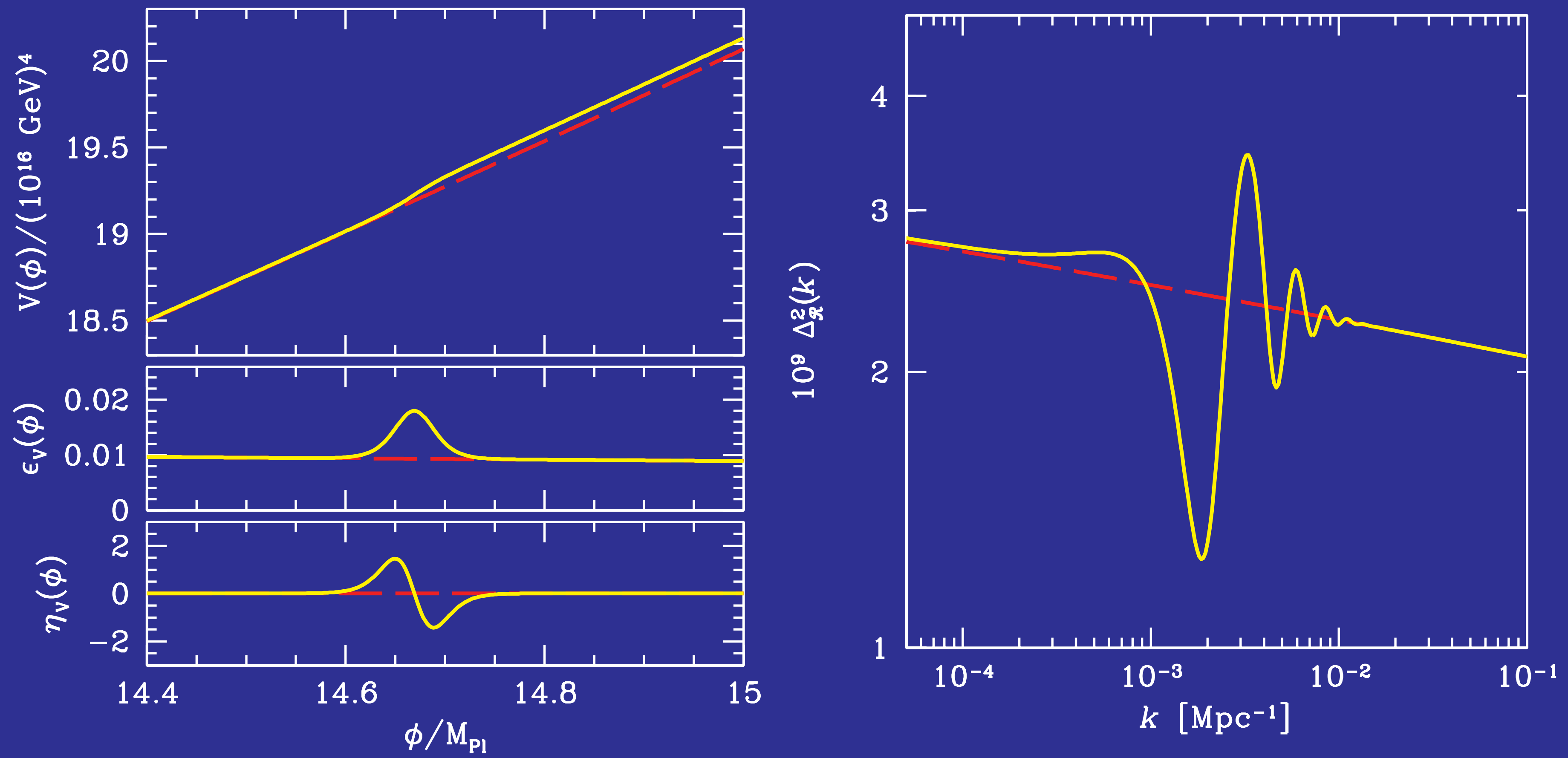
  - scale free
  - adiabatic
  - highly Gaussian fluctuations

  Tensor (gravitational wave) fluctuations:

  - scale free with power law related to T/S
  - amplitude related to energy scale

Features in Potential

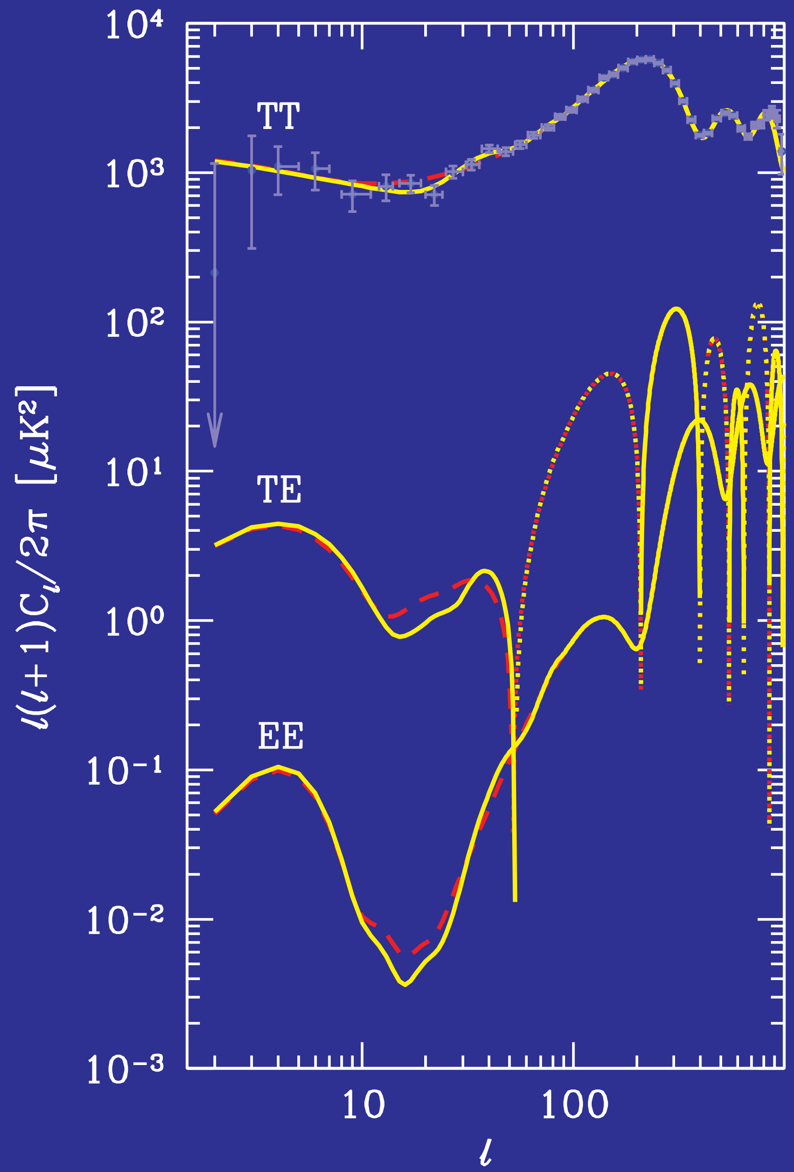
- Rolling of inflaton across a sharp feature causes ringing



Mortonson, Dvorkin, Peiris, Hu (2008) [Covi et al 2006; Hamann et al 2007]

Features in Potential

- Possible expanation of glitches
- Predicts matching glitches in polarization
- Falsifiable independent of ionization history through PC analysis
- Planck 2.5-3σ
- Cosmic variance 5-8σ



Mortonson, Dvorkin, Peiris, Hu (2008)

Inflaton Fluctuations

- Single field inflaton fluctuations obey the linearized Klein-Gordon equation for $u = a\delta\phi$

$$\ddot{u} + \left[k^2 - \frac{\ddot{z}}{z}\right] u = 0$$

where

$$z(\eta) = \dot{\phi}/H$$

- Oscillatory response to rapid slow down or speed up of roll $\dot{\phi}$ due to features in the potential
- Single function $z(\eta)$ controls curvature fluctuations but
  - direct PC or other functional constraints cumbersome
  - link to $V(\phi)$ obscured

Generalized Slow Roll

- Green function approach allowing slow roll parameters to be strongly time varying (Stewart 2002)
- Generalized for large features by promoting second order to non-linear in controlled fashion (Dvorkin & Hu 2009)
- Functional constraints on the source function of deviations from scale invariance

$$G'(\ln \eta) = \frac{2}{3}\left[\frac{f''}{f} - 3\frac{f'}{f} - \left(\frac{f'}{f}\right)^2\right], \qquad f = 2\pi\eta z(\eta)$$

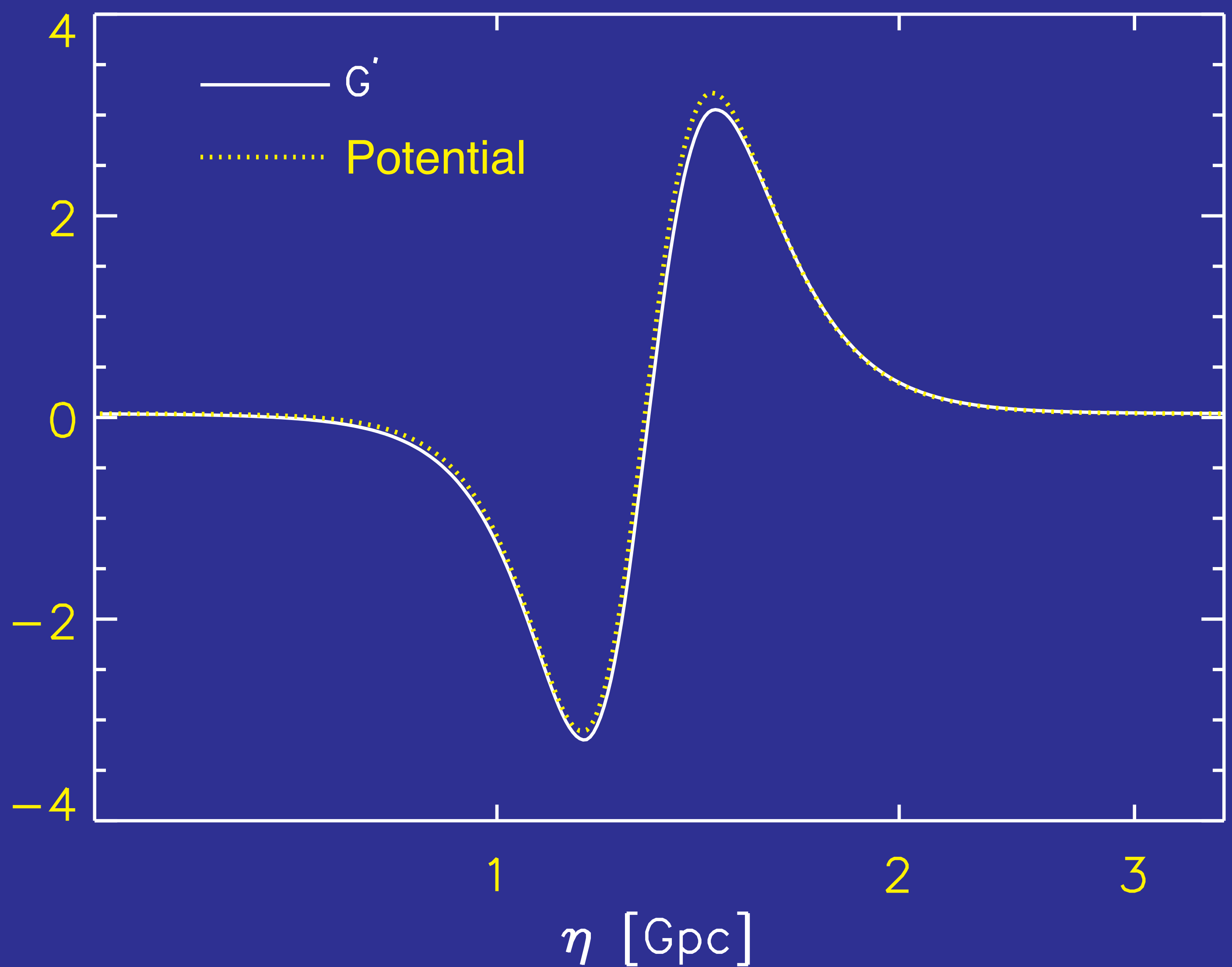
- As long as large features are crossed on order an e-fold or less

$$G' \approx 3\left(\frac{V'}{V}\right)^2 - 2\frac{V''}{V}$$

same combination that enters into tilt n_s in slow roll

GSR and the Potential

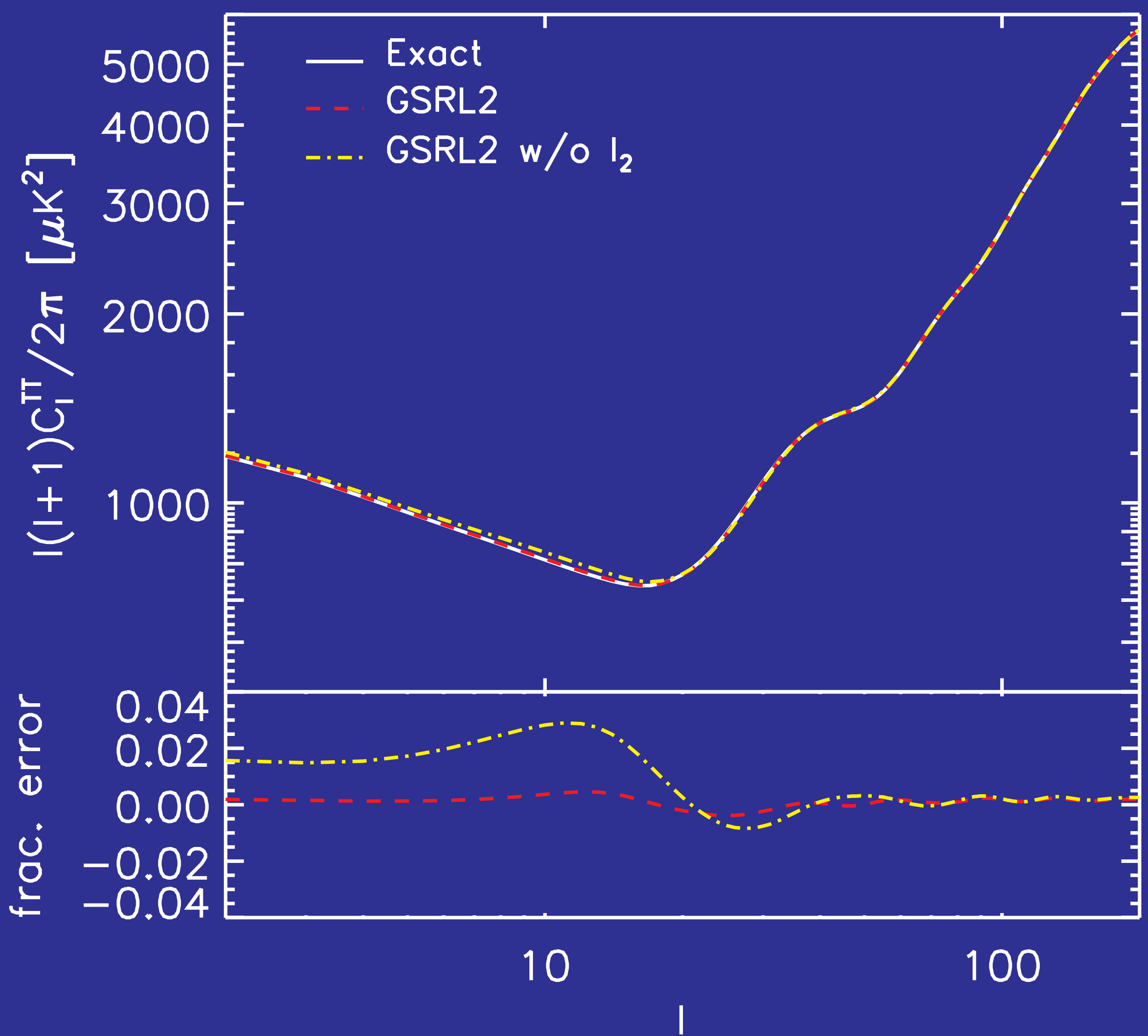
- GSR source function *G'* vs potential combination $3(V'/V)^2 - 2V''/V$



Dvorkin & Hu (2009)

GSR Accuracy

- ~2% for order unity features (can be improved to <0.5% with iteration)



Dvorkin & Hu (2009)

Generalized Slow Roll

- Heuristically, a non-linear mapping or transfer function

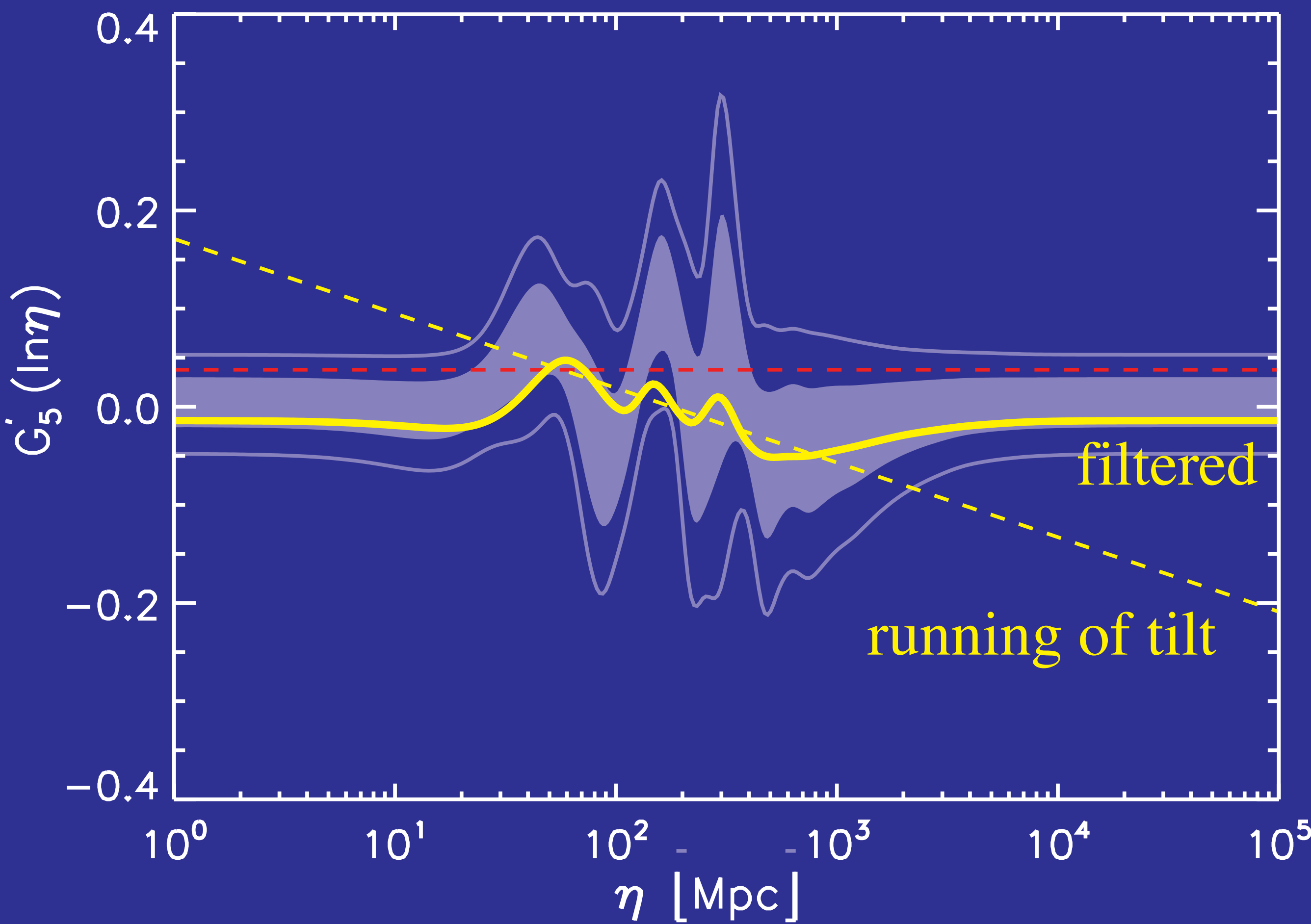
$$\Delta^2_{\mathcal{R}}(k) = A_s T[G'(\ln \eta)]$$

- Allows only initial curvature spectra that are compatible with single field inflation
- Disallowed behavior falsifies single field inflation
- PC decomposition of G' allows efficient computation - precompute responses and combine non-linearly
- Changes in initial power spectrum do not require recomputing radiation transfer in CMB – fast parameters in CAMB
- Bottleneck is WMAP likelihood evaluation. Fast OMP parallelized code ($\sim 5N_{\rm core}$ speedup)

http://background.uchicago.edu/wmap_fast

Functional Constraints on Source

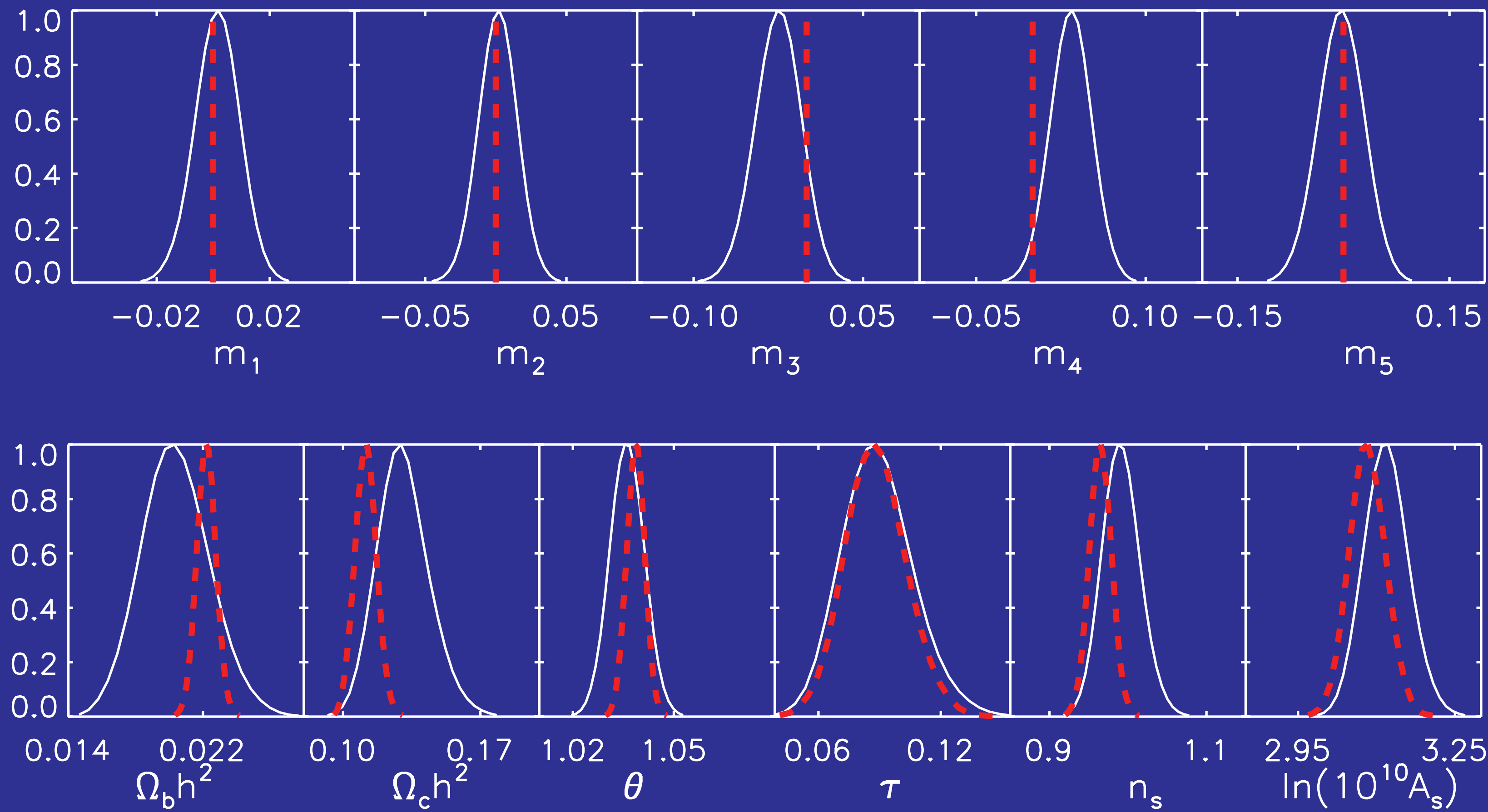
- 5 nearly Gaussian independent constraints on deviations from scale invariance for model testing
- Not a reconstruction due to truncation



Dvorkin & Hu (2010)

WMAP Constraints on 5PCs

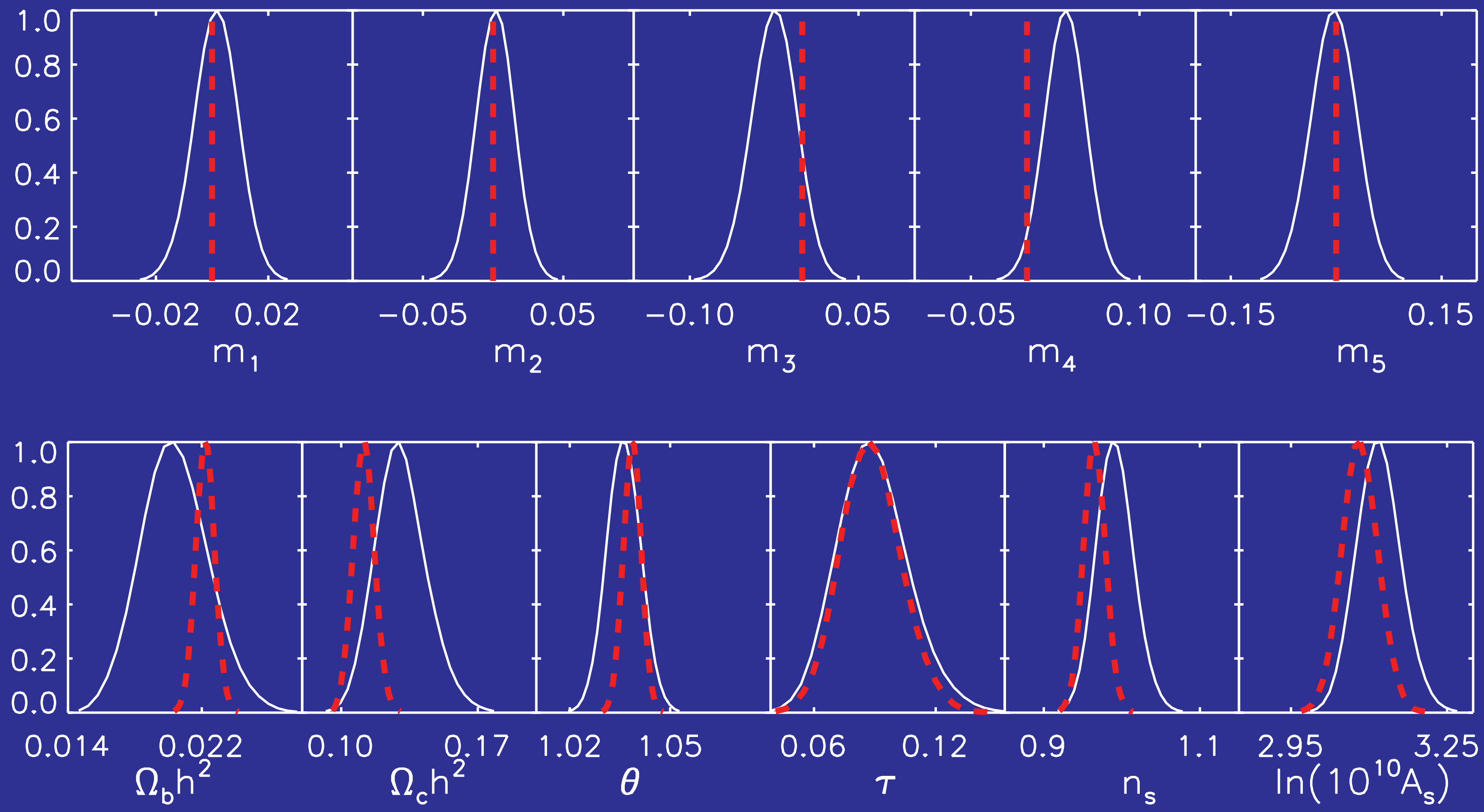
- 1 out of 5 shows a 95% preference for non-zero values though only if CDM density is high



Dvorkin & Hu (2010)

WMAP Constraints on 5PCs

- Interestingly 4th component carries most of the information about running of tilt
- But outside of the PC range data does not prefer a constant running of that size - local preference around few 100Mpc

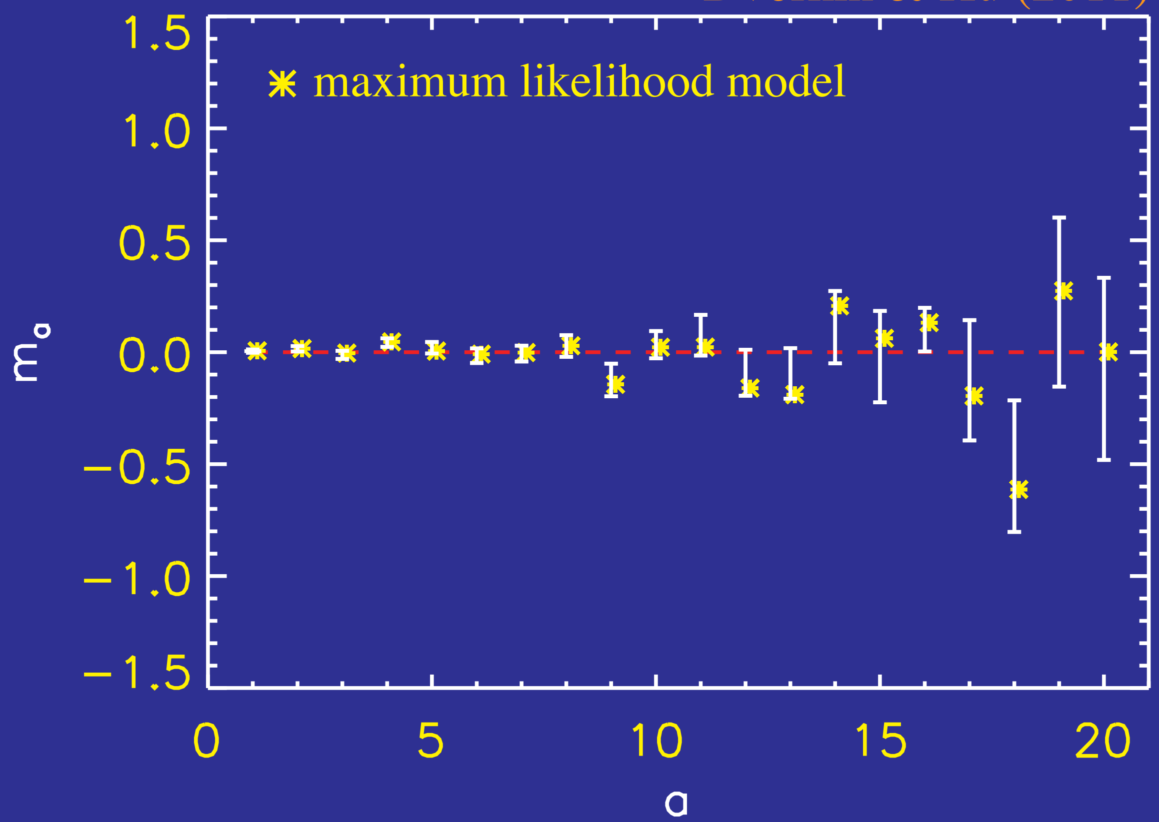


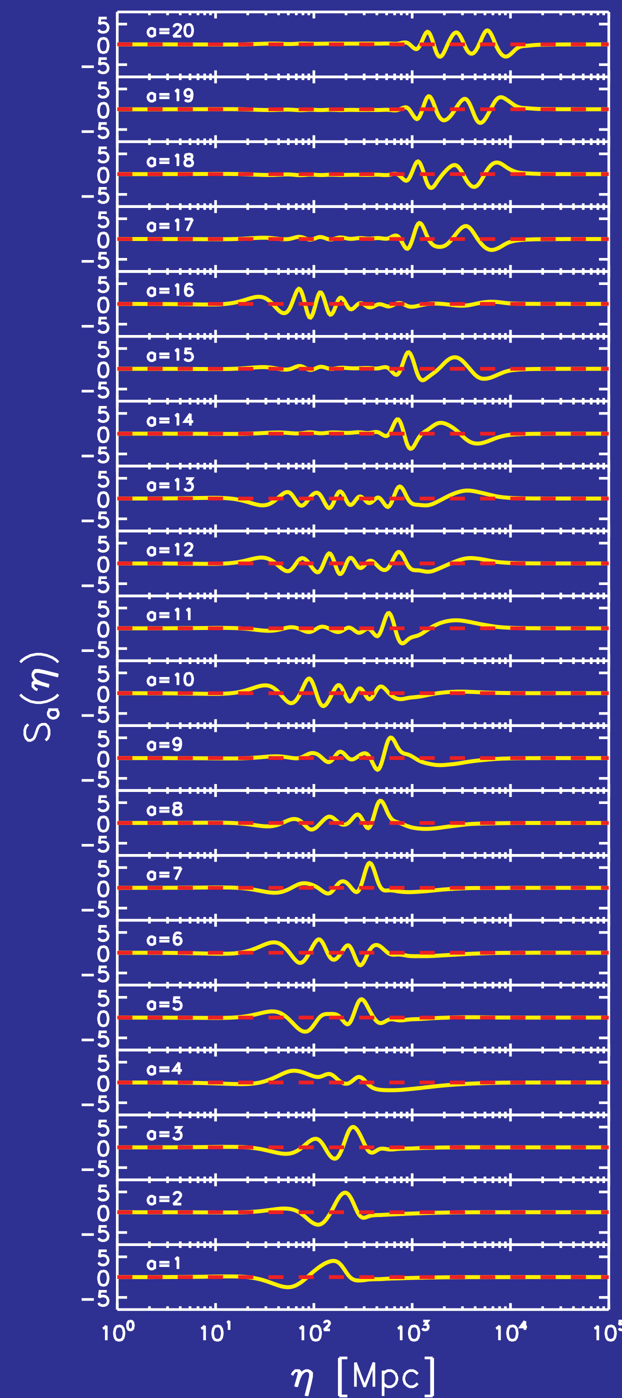
Dvorkin & Hu (2010)

Complete Basis

- Higher order PCs out to 20 carry information on weakly constrained modes
- Horizon scale features, WMAP beam scale features
- Maximum likelihood $2\Delta\ln L=17$

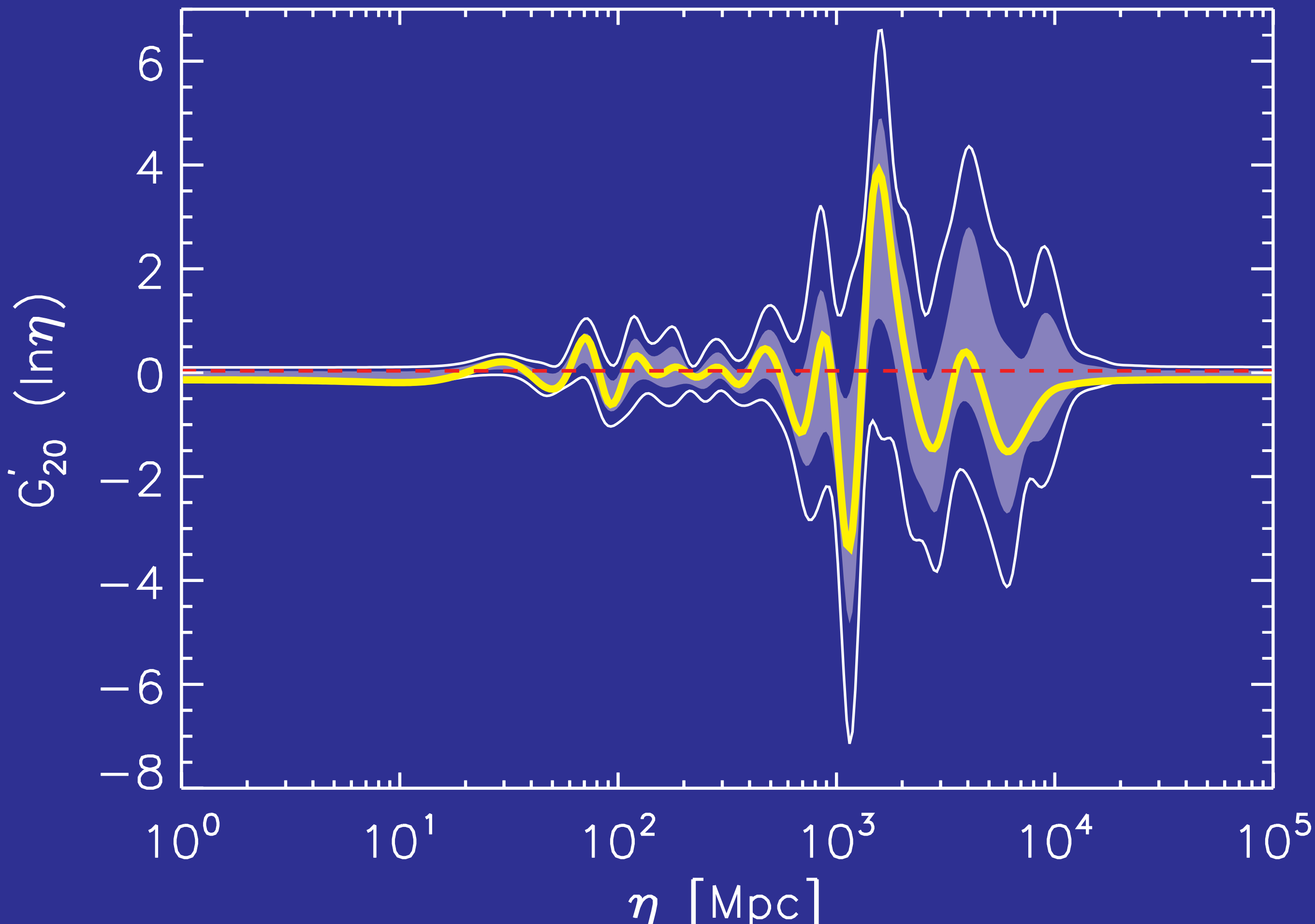
Dvorkin & Hu (2011)





Functional Constraints on Source

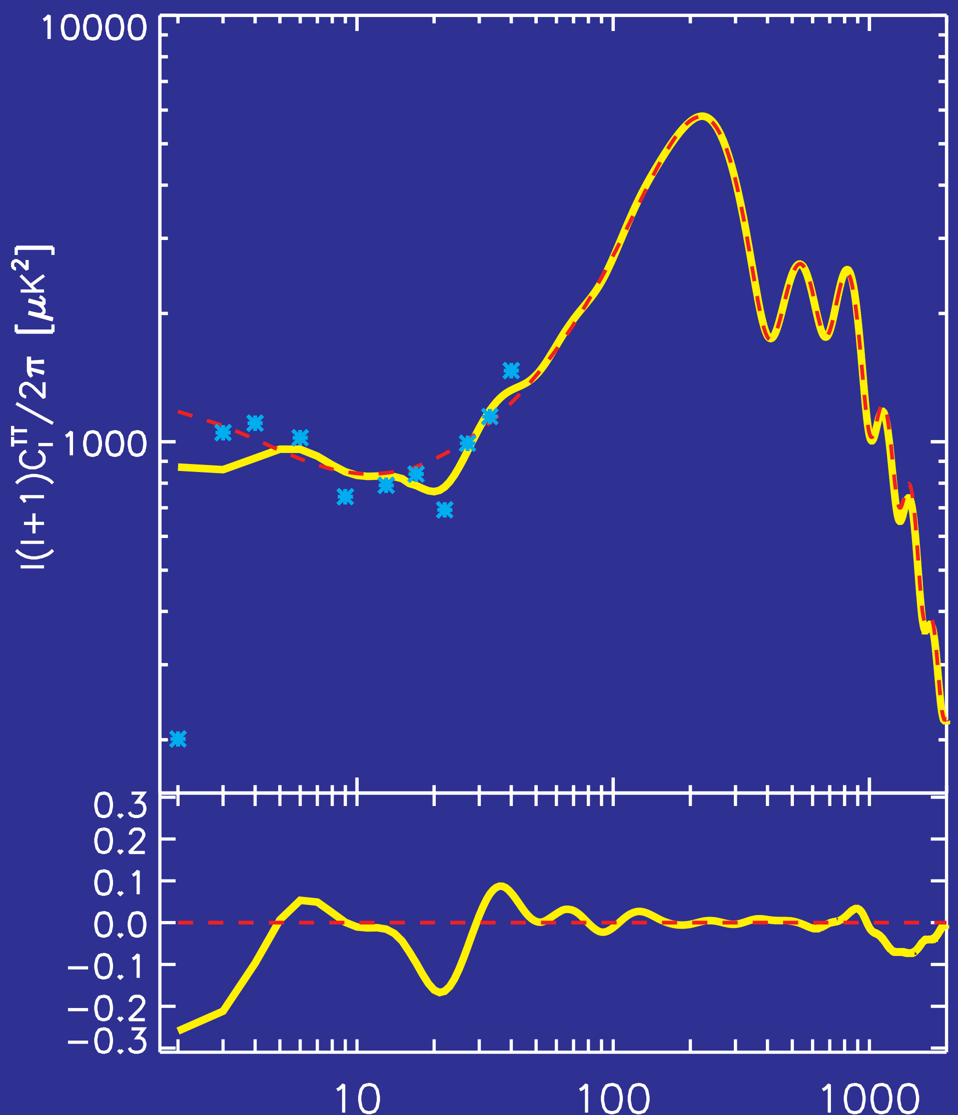
- 20 PC filter on source function
- Consistent with no deviations from scale free conditions; most significant deviations at 1000 Mpc



Dvorkin & Hu (2011)

Posterior Power

- Posterior probability distribution of temperature power spectrum given single field inflation (GSR)



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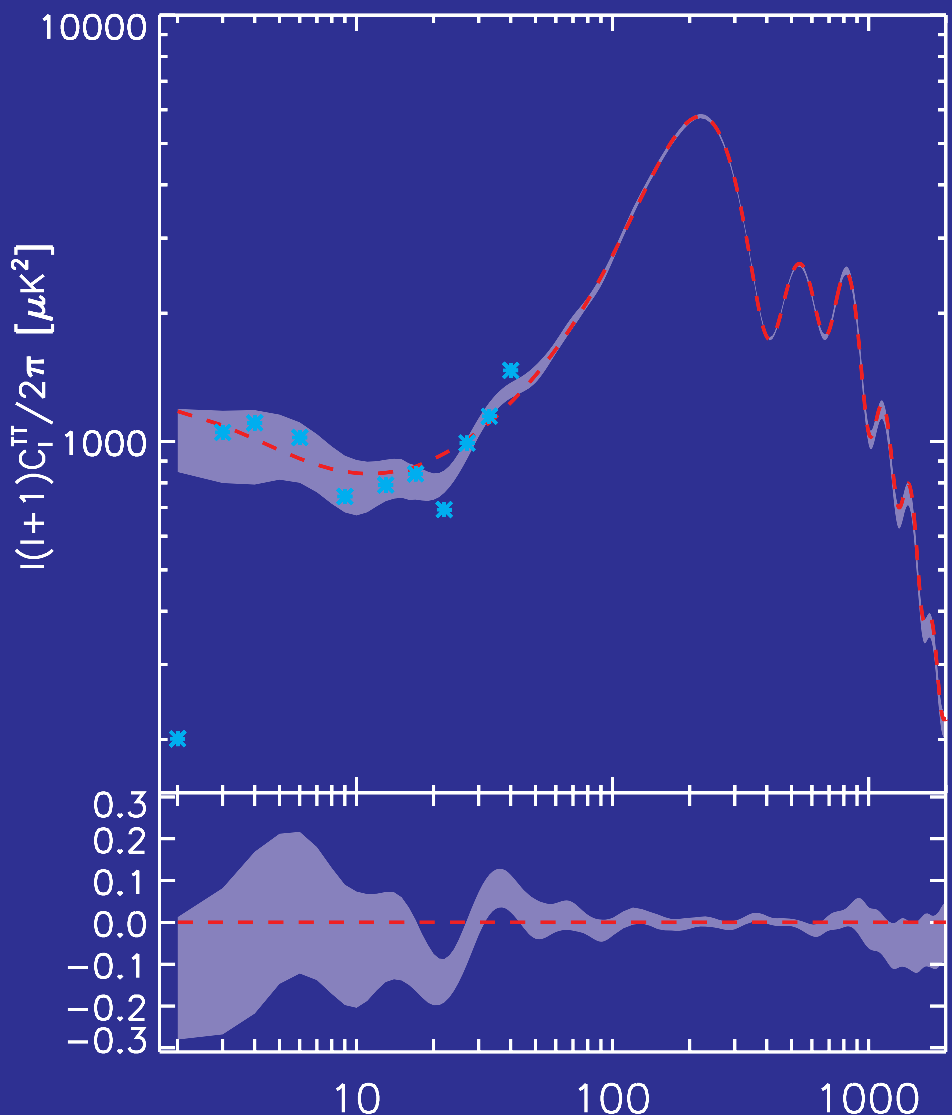

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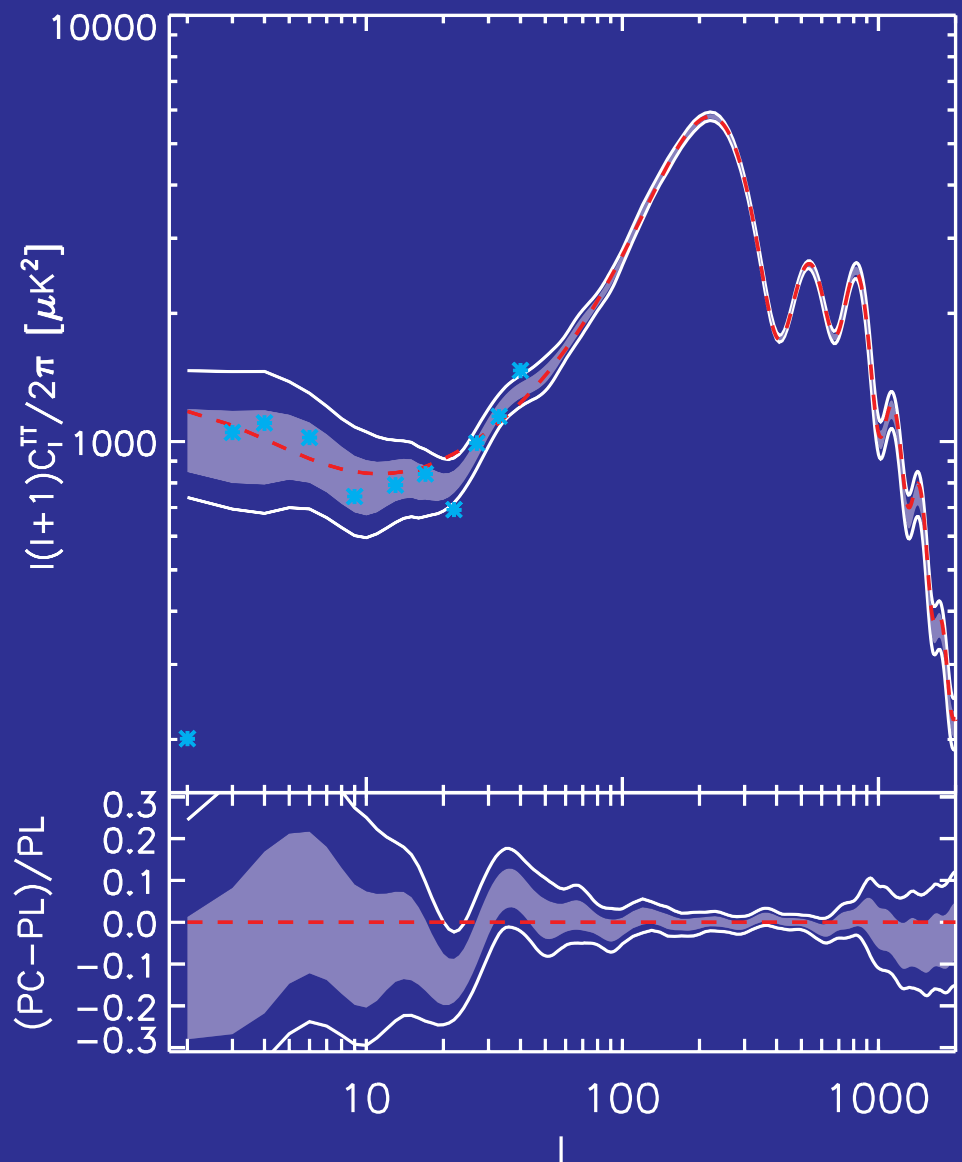
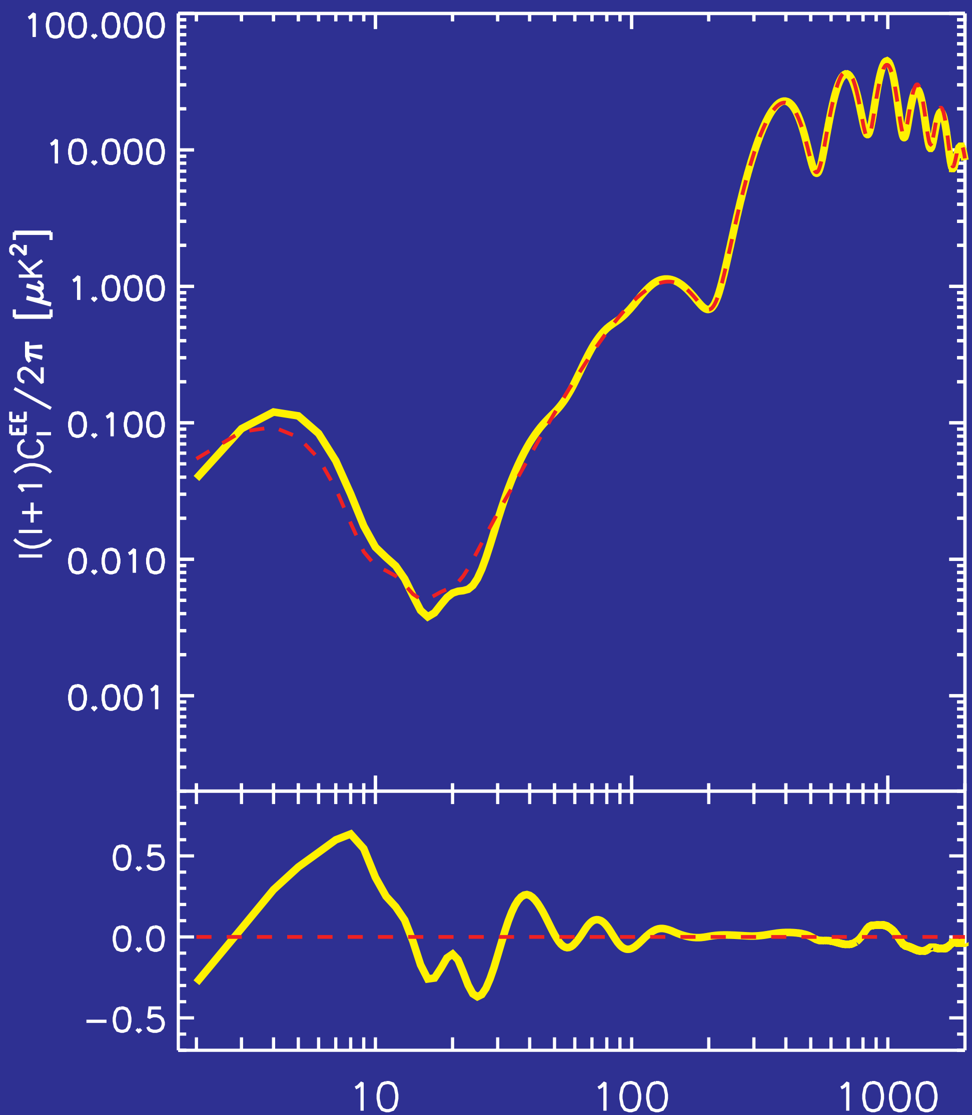


Dvorkin & Hu (2011)

Predicted Polarization

- If features are due to single field inflation (GSR) there must be corresponding ones in polarization



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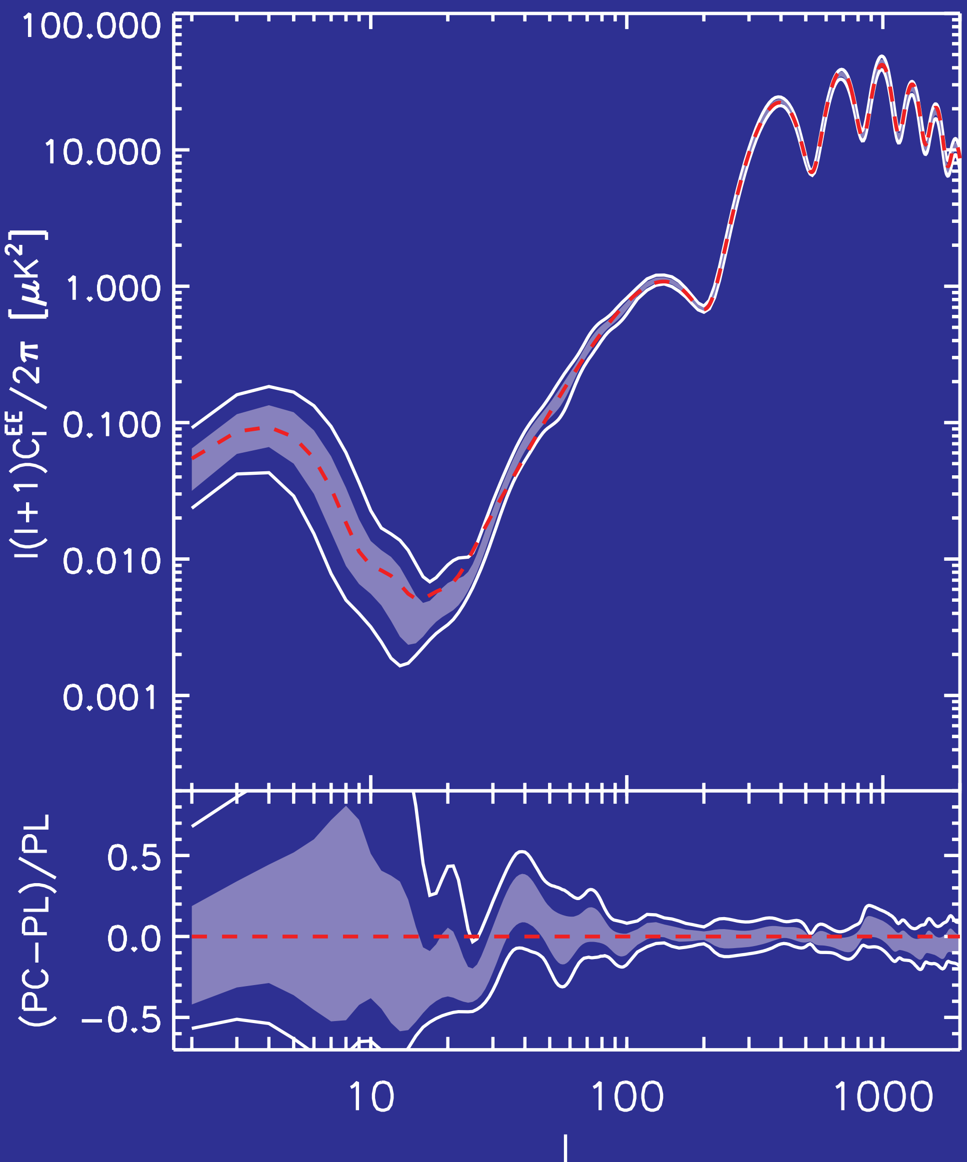
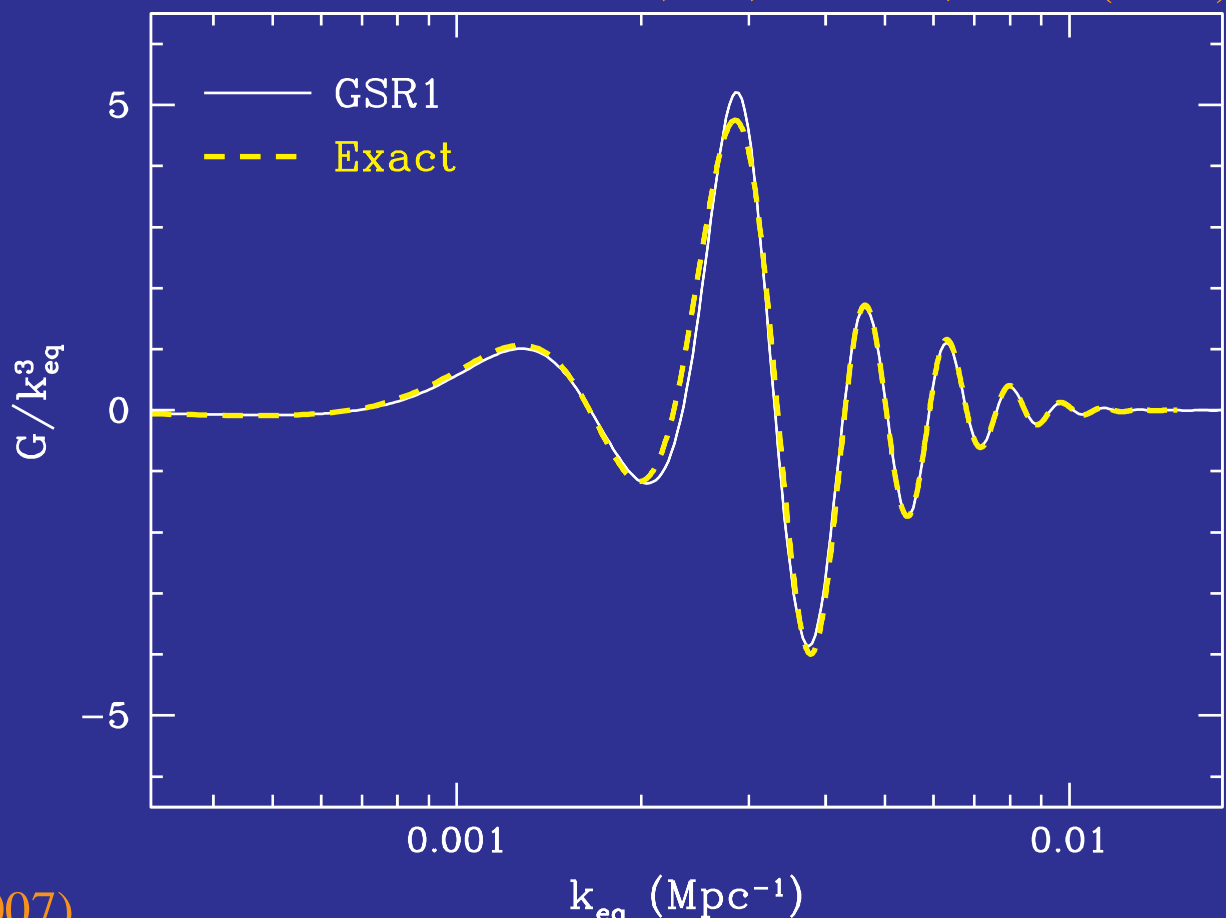


Dvorkin & Hu (2011)

Bispectrum Features

- Predicts features in the bispectrum
- Efficiently calculated through generalized slow-roll
- Bispectrum features related to the *l*~20-40 glitch are large but confined to too small a range to be observed

Adshead, Hu, Dvorkin, Peiris (2011)

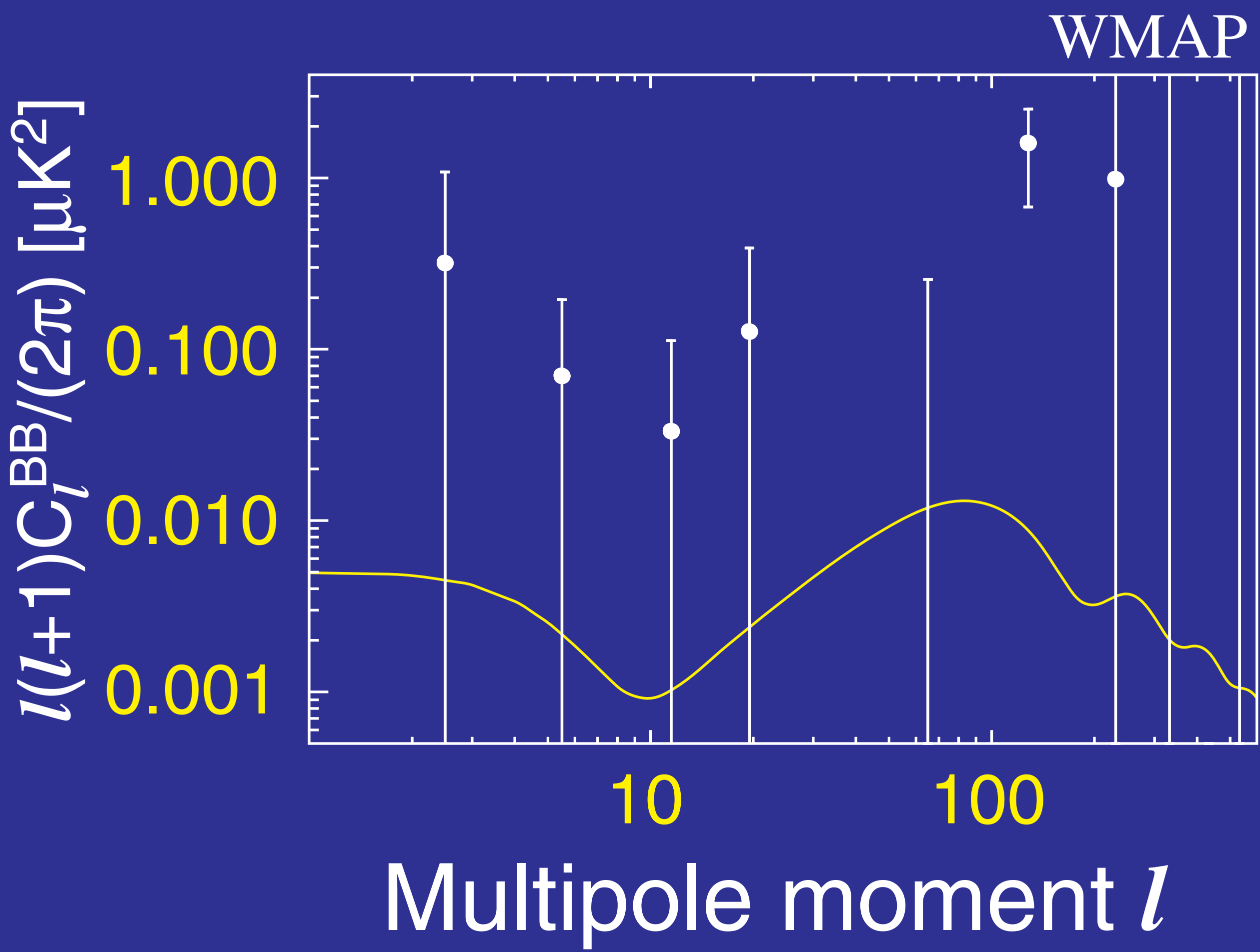


equilateral configuration

Chen et al (2007)

Tensor Slope

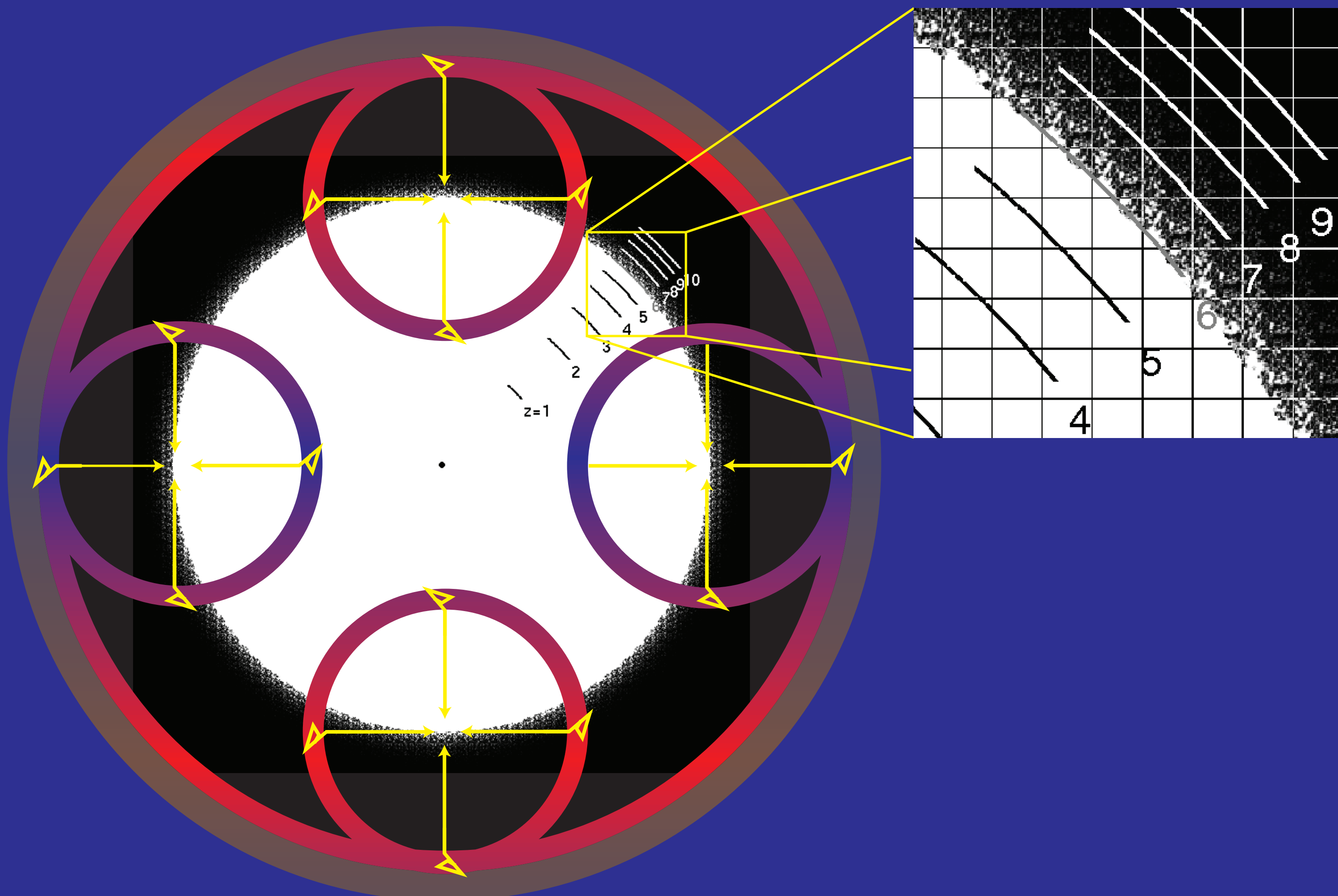
- If degree scale tensors are observed, reionization enables test of slow roll infation through consistency between n_T-r



Ionization History $x_e(z)$

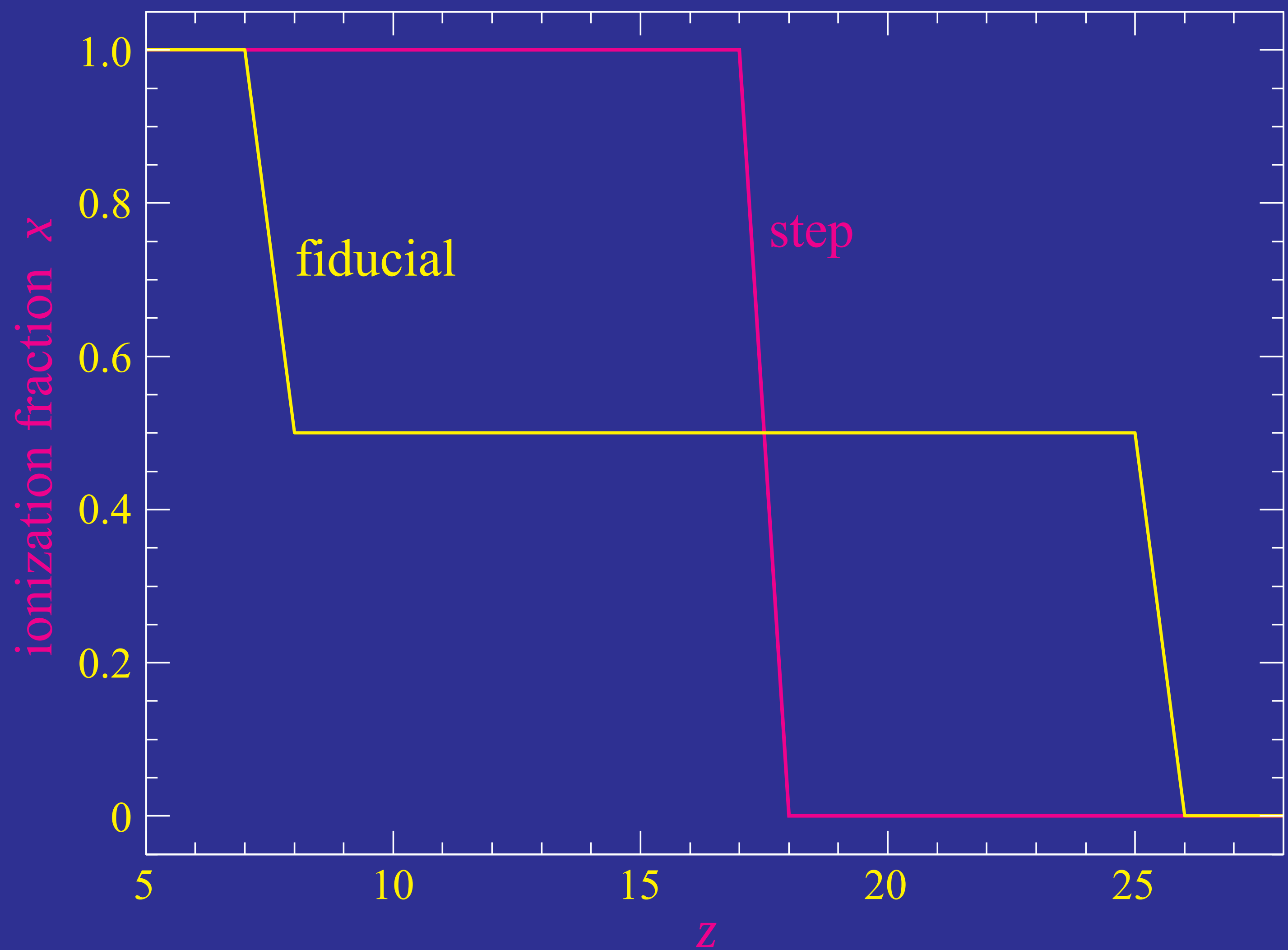
Polarization & Reionization

- Rescattering of anisotropic radiation during reionization leads to large scale polarization
- Sensitive to the average ionization fraction



Ionization History

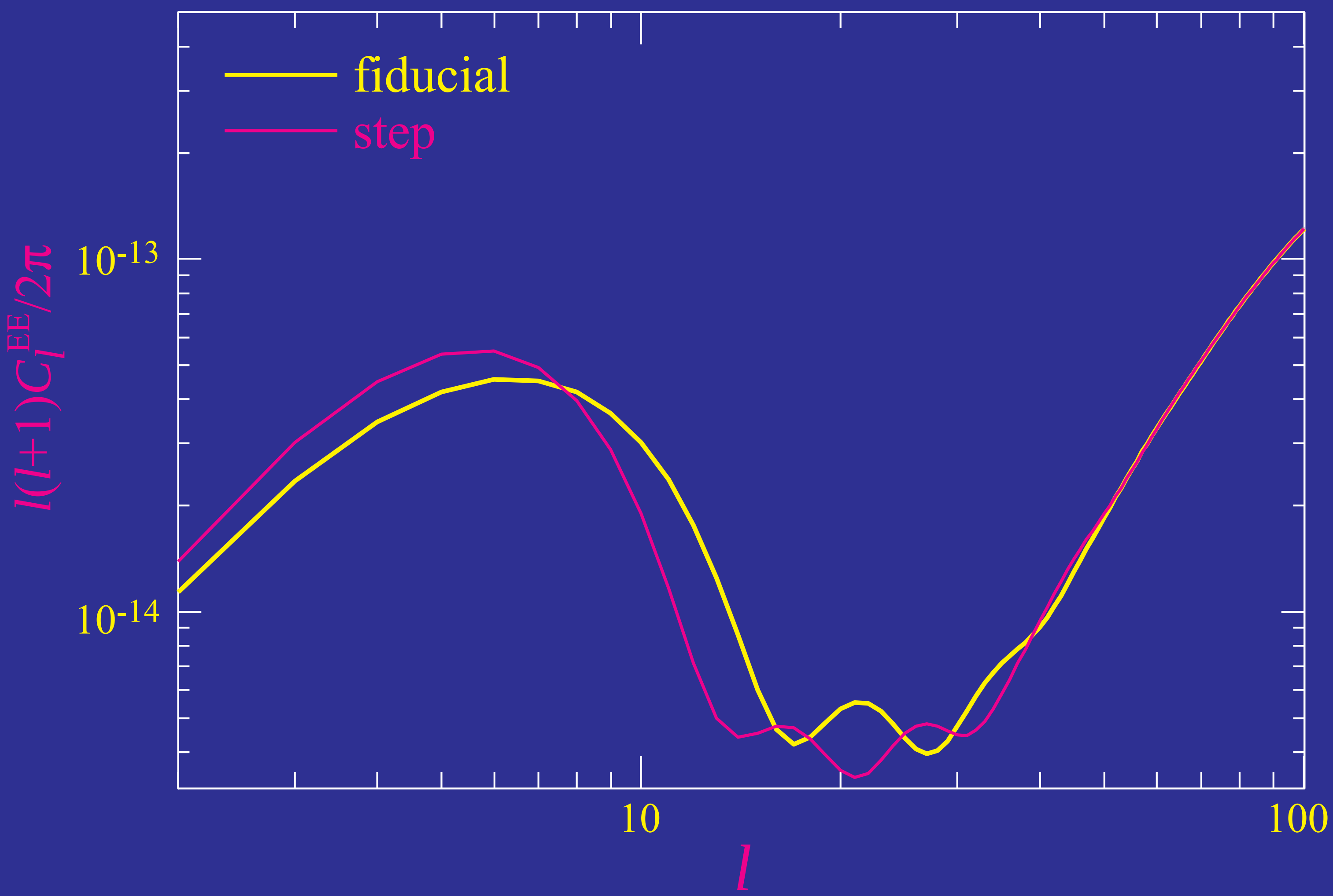
- Two models with same optical depth τ but different ionization history



Kaplinghat et al. (2002); Hu & Holder (2003)

Distinguishable History

- Same optical depth, but different coherence - horizon scale during scattering epoch

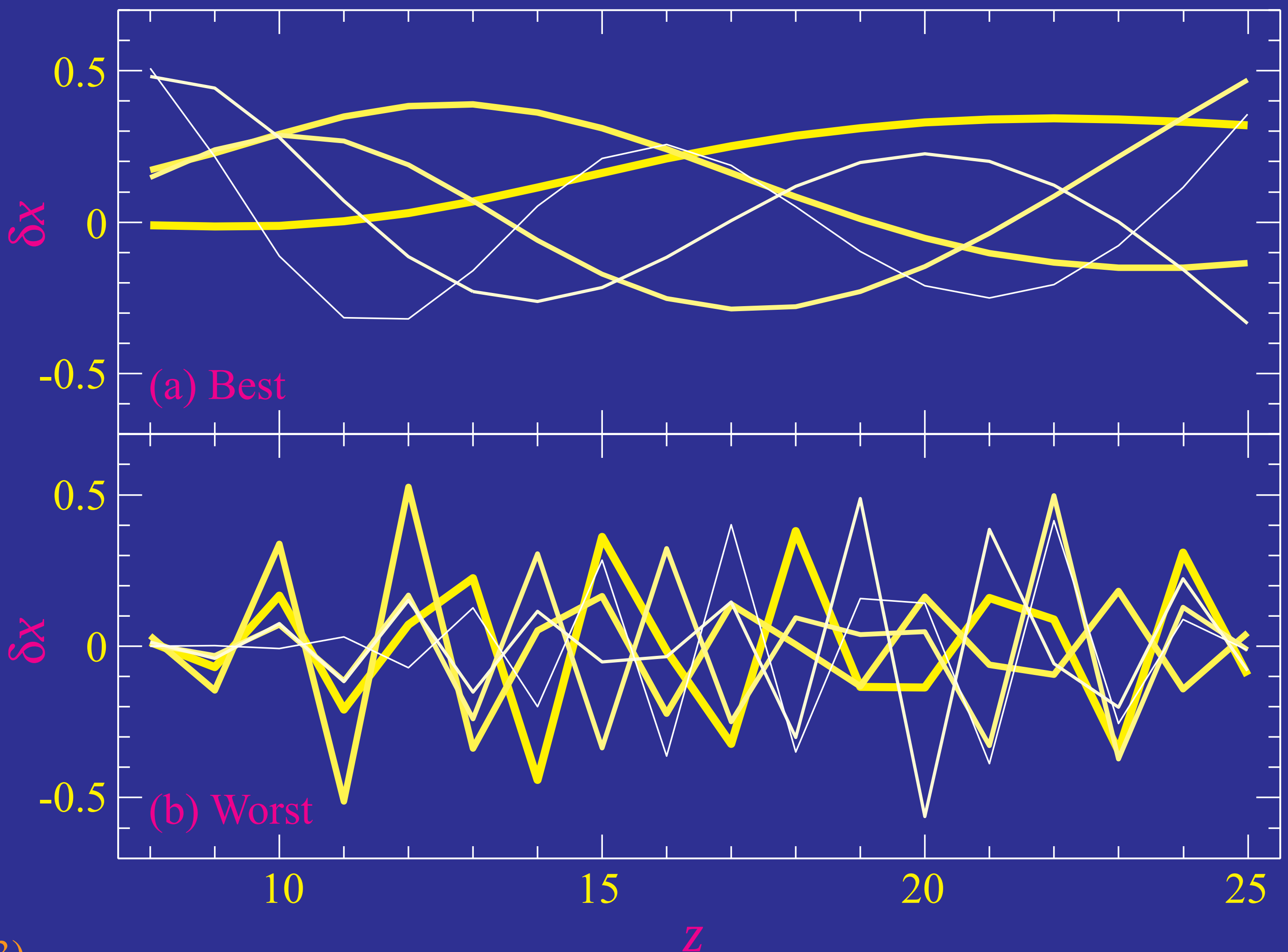


Kaplinghat et al. (2002); Hu & Holder (2003)

Principal Components

- Eigenvectors of the Fisher Matrix

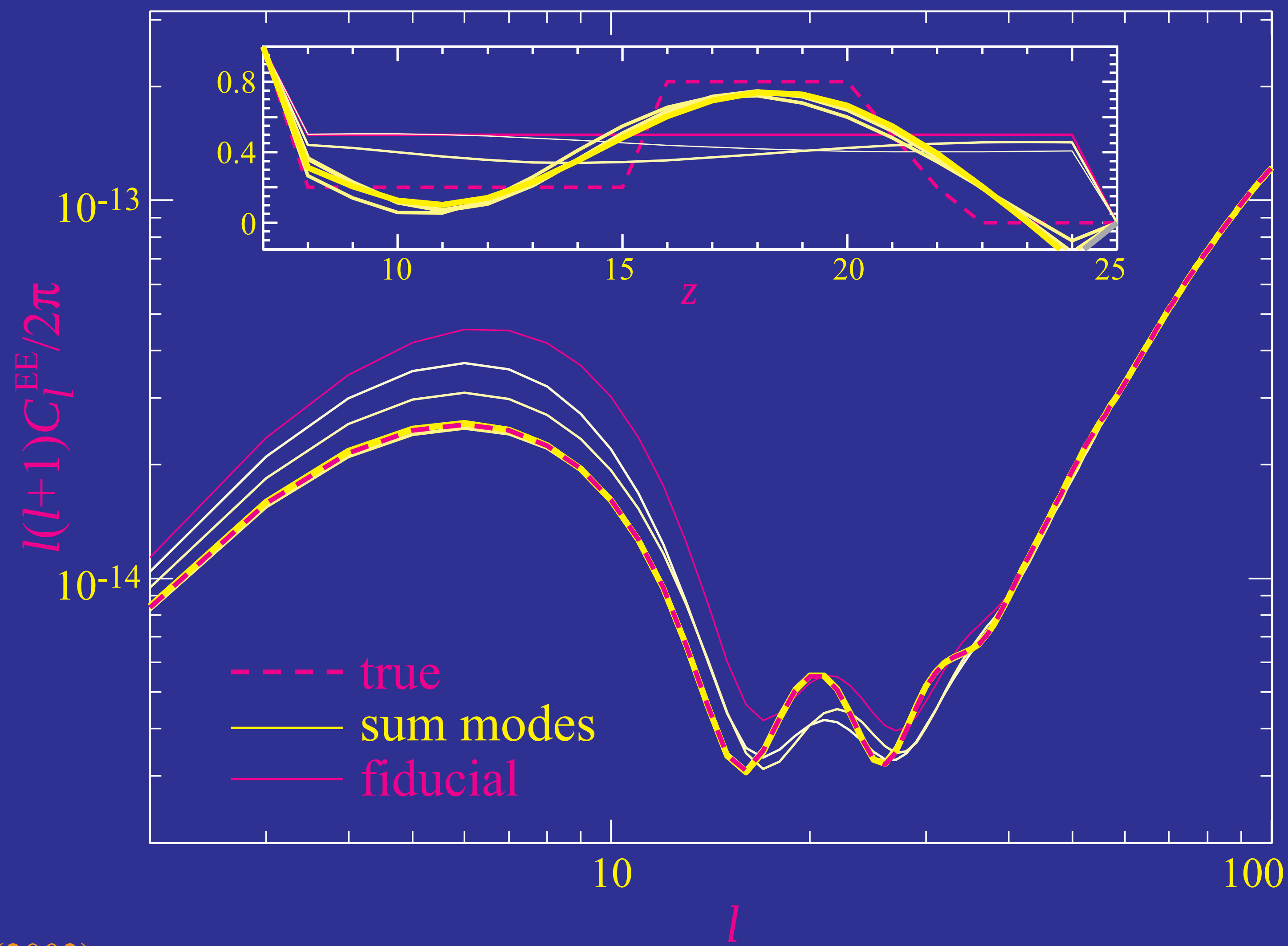
$$F_{ij} \equiv \sum_\ell (\ell + 1/2) T_{\ell i} T_{\ell j} = \sum_\mu S_{i\mu} \sigma_\mu^{-2} S_{j\mu}$$



Hu & Holder (2003)

Representation in Modes

- Reproduces the power spectrum with sum over >3 modes
  more generally 5 modes suffices: e.g. total τ=0.1375 vs 0.1377

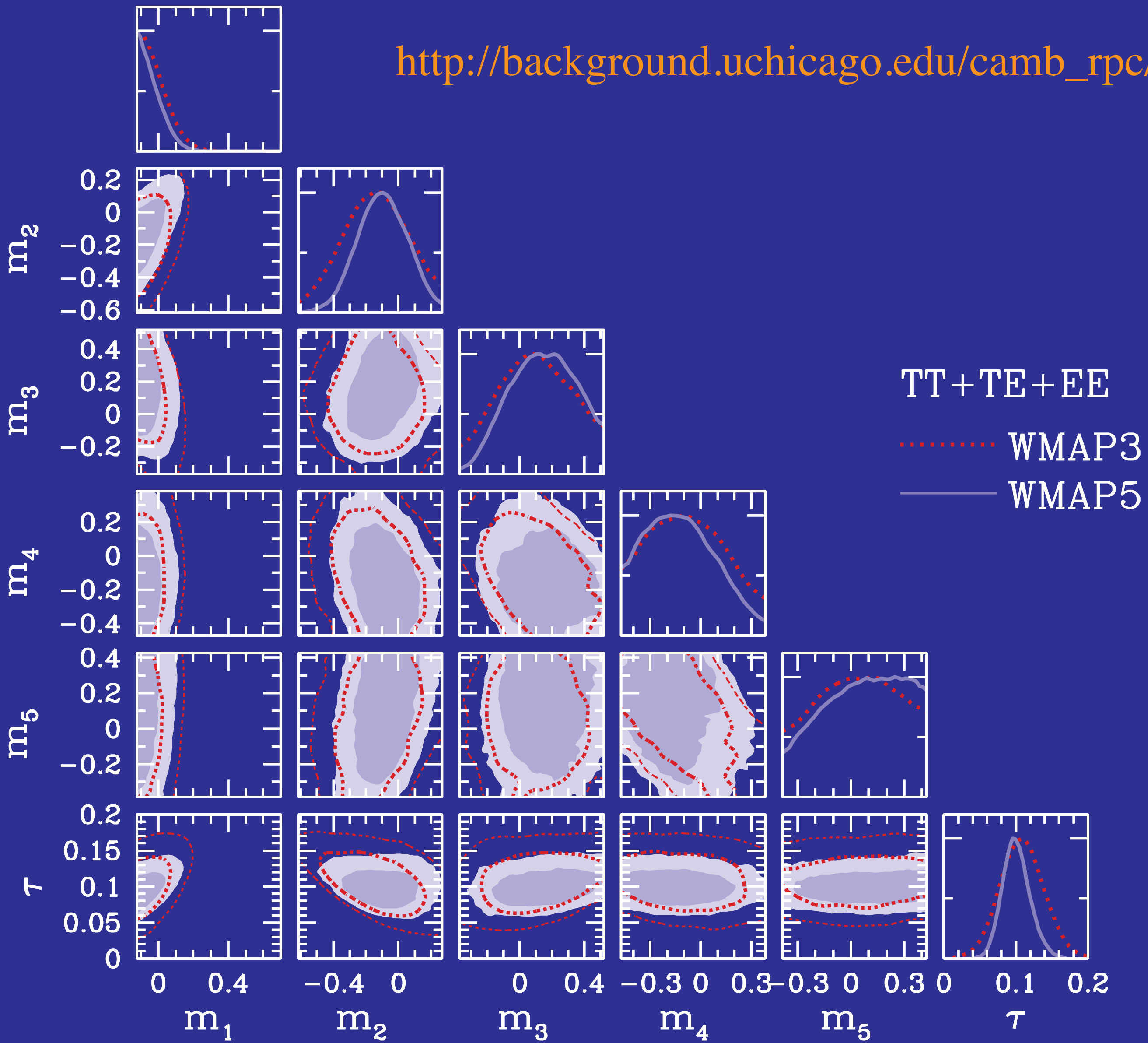


Hu & Holder (2003)

WMAP5 Ionization PCs

- Only first two modes constrained, τ=0.101±0.017

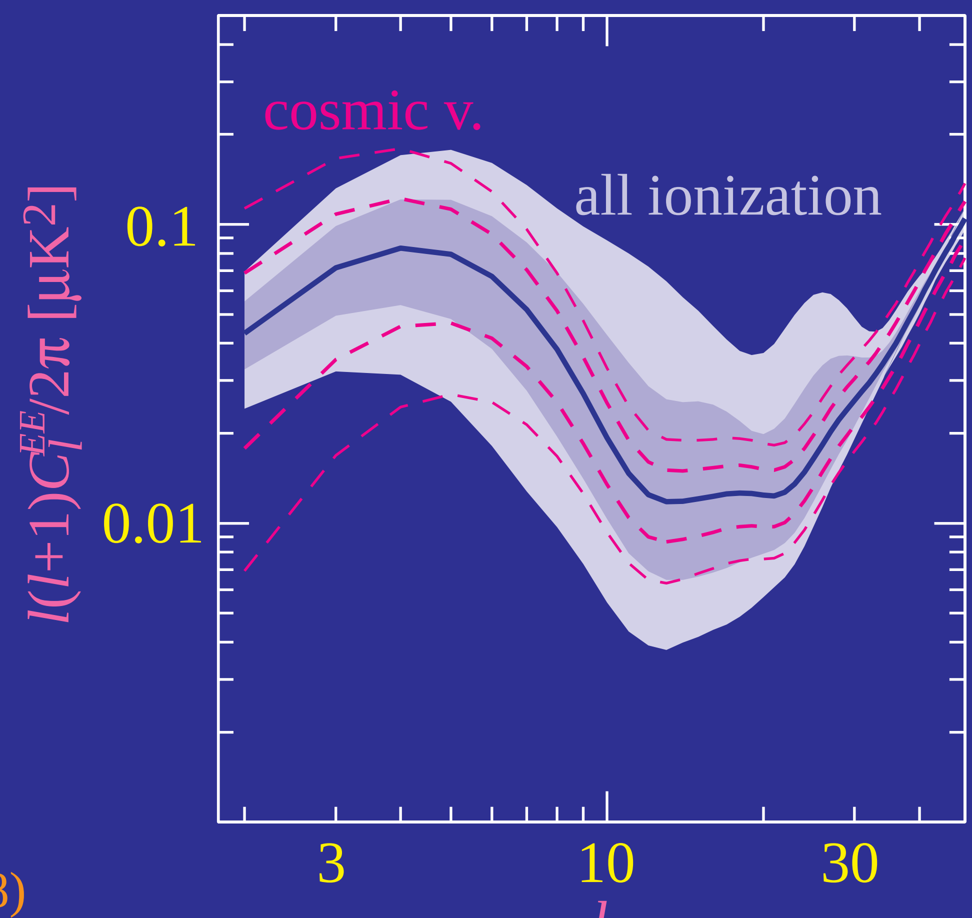
http://background.uchicago.edu/camb_rpc/



Mortonson & Hu (2008)

Model-Independent Reionization

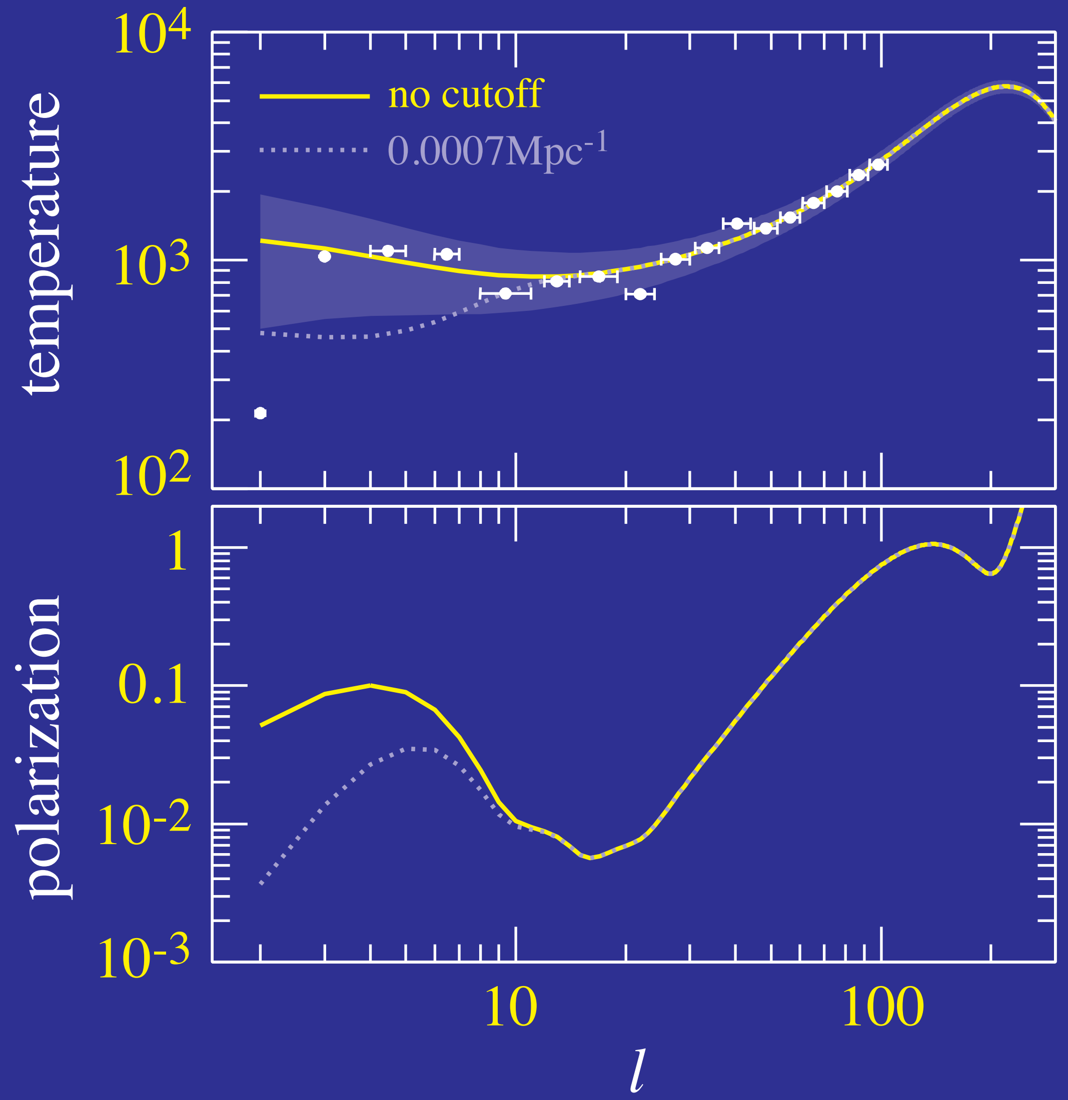
- All possible ionization histories at $z<30$
- Detections at $20<l<30$ required to further constrain general ionization which widens the τ-n_s degeneracy allowing $n_s=1$
- Quadrupole & octopole predicted to better than cosmic variance test ΛCDM for anomalies



Mortonson & Hu (2008)

Horizon-Scale Power

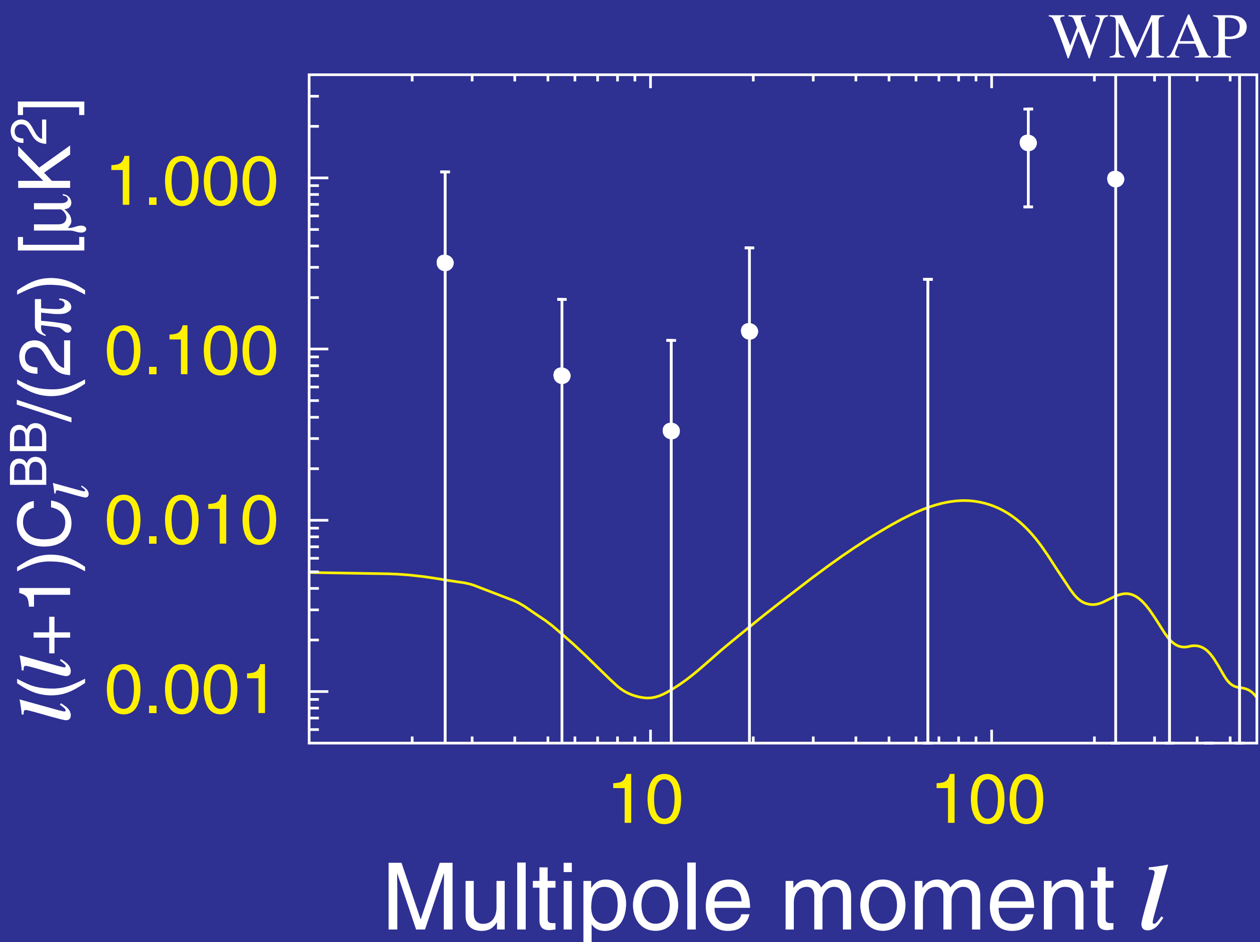
- Polarization is a robust indicator of horizon scale power and disfavors suppression as explanation of low quadrupole independently of ionization or acceleration model



Mortonson & Hu (2009)

Tensor Slope

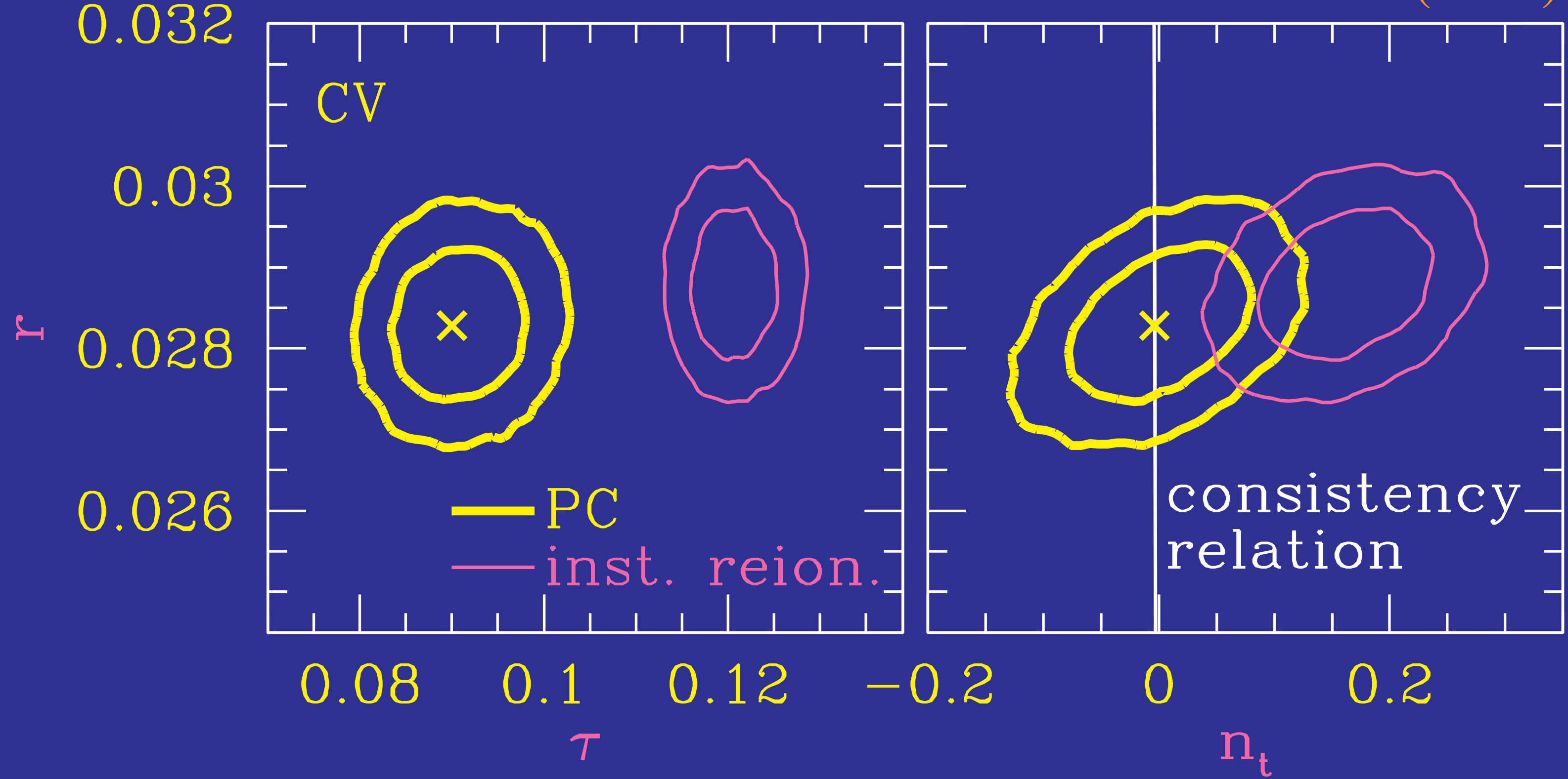
- If degree scale tensors are observed, reionization enables test of slow roll infation through consistency between n_T-r



Consistency Relation & Reionization

- By assuming the wrong ionization history can falsely rule out consistency relation
- Principal components eliminate possible biases

Mortonson & Hu (2007)



Summary

- Standard inflationary ΛCDM is highly predictive and falsifiable
- Distance-redshift relation at all redshifts, including $z = 0$ and H_0 fixed at the few percent level largely from CMB
- ΛCDM places firm upper bound on growth of structure for all quintessence models (smooth dark energy with $w \geq -1$) e.g. for high-z cluster abundance falsification
- Deviations from slow roll constrained $<$ few % around first peak
- Deviations at larger scale allowed and marginally favored yielding testable predictions in the polarization
- Polarization can falsify the whole single field inflationary paradigm independently of presence of features
- Tensor consistency relation testable with reionization B-modes even if reionization is complex