Cosmic Acceleration

Dark Energy
v.
Modified Gravity

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Outline

- Dark Energy vs Modified Gravity
- Three Regimes of Modified Gravity
- Worked (Toy) Models: $f(R)$ and DGP Braneworld
- Collaborators

Wenjuan Fang	Dragan Huterer
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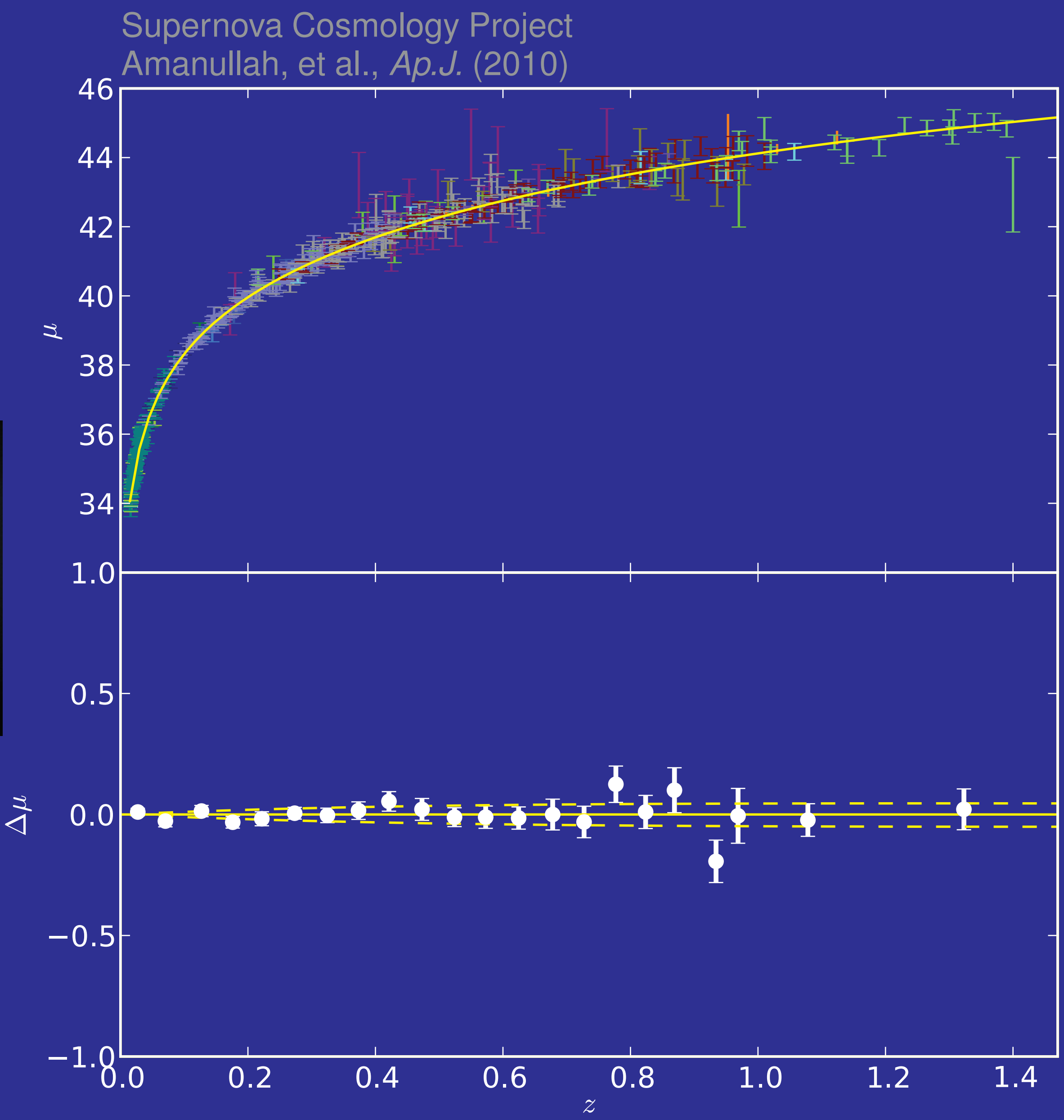
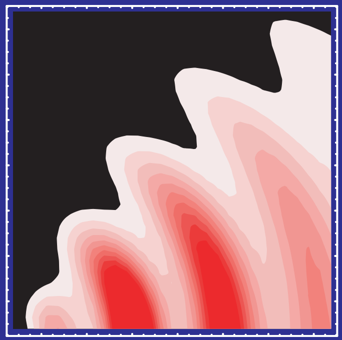
Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable)
Candle
Supernovae
Luminosity v Flux

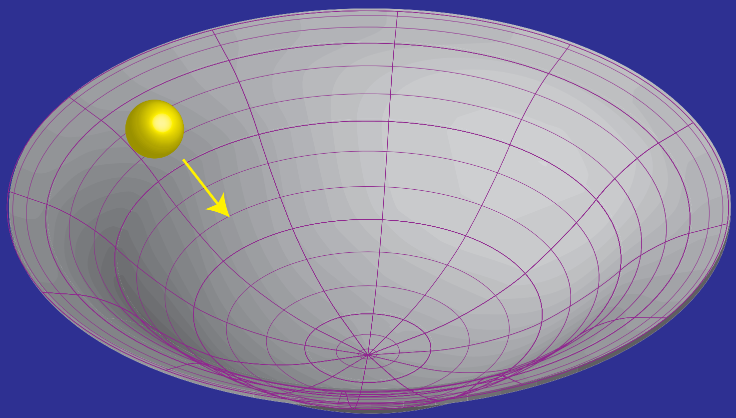




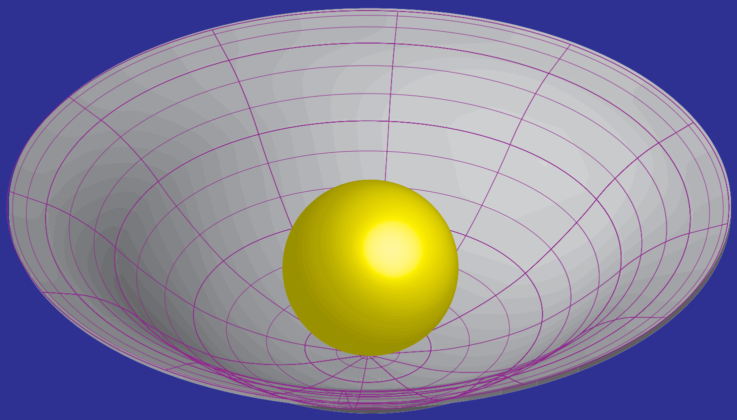
Standard Ruler
Sound Horizon
v CMB, BAO angular
and redshift separation

Mercury or Pluto?

- General relativity says Gravity = Geometry



- And Geometry = Matter-Energy



- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?

Modified Gravity = Dark Energy?

- Solar system tests of gravity are informed by our knowledge of the local stress energy content
- With no other constraint on the stress energy of dark energy other than conservation, modified gravity is formally equivalent to dark energy

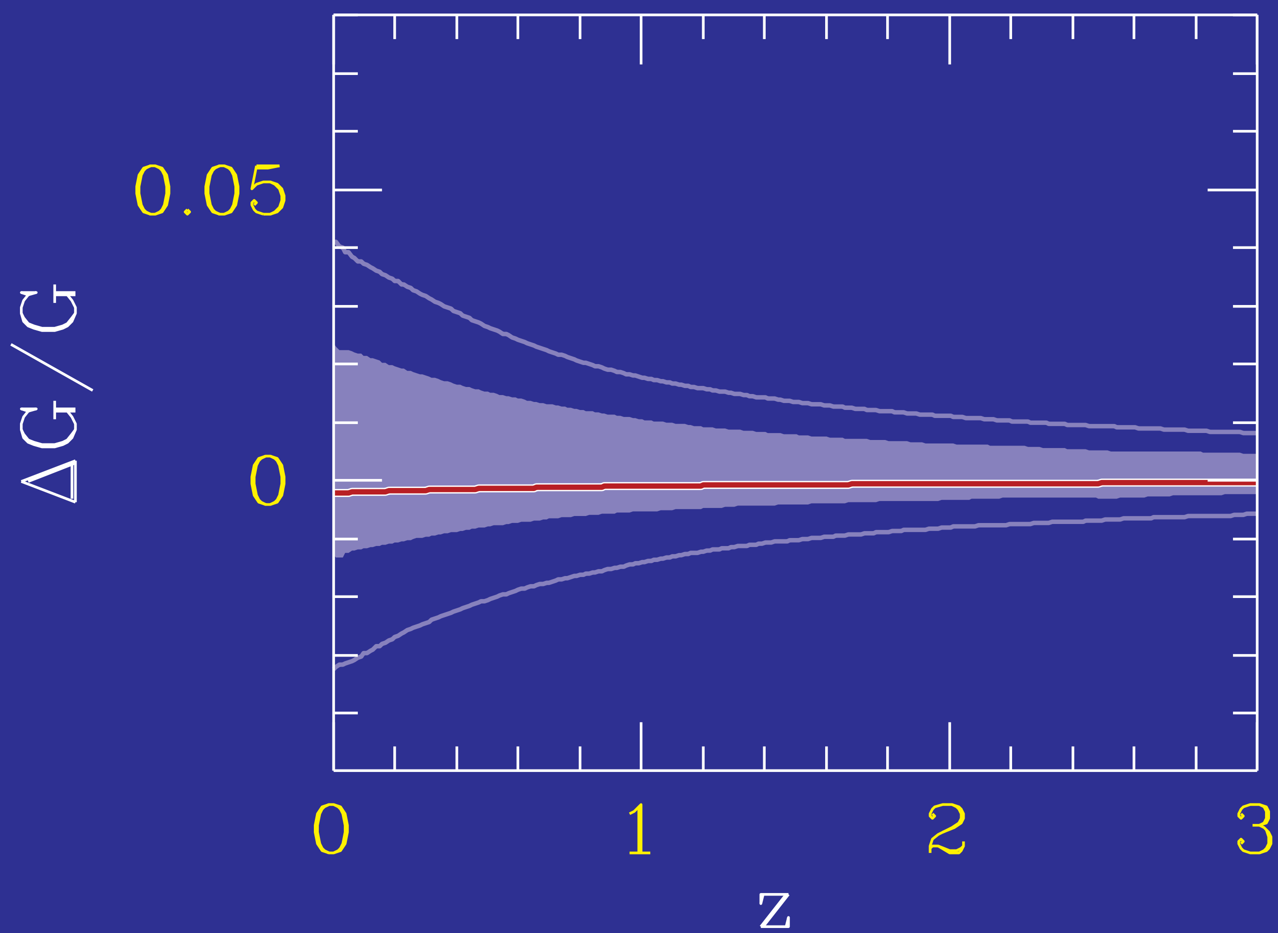
$$
\begin{aligned}
F(g_{\mu\nu}) + G_{\mu\nu} &= 8\pi G T^{\rm M}_{\mu\nu} \qquad\qquad \boxed{-F(g_{\mu\nu}) = 8\pi G T^{\rm DE}_{\mu\nu}} \\
G_{\mu\nu} &= 8\pi G [T^{\rm M}_{\mu\nu} + T^{\rm DE}_{\mu\nu}]
\end{aligned}
$$

and the Bianchi identity guarantees $\nabla^\mu T^{\rm DE}_{\mu\nu} = 0$

- Distinguishing between dark energy and modified gravity requires closure relations that relate components of stress energy tensor
- For matter components, closure relations take the form of equations of state relating density, pressure and anisotropic stress

Falsifying ΛCDM

- Λ slows growth of structure in highly predictive way



Cosmological Constant

Modified Gravity $\neq$ "Smooth DE"

- Scalar field dark energy has $\delta p = \delta\rho$ (in constant field gauge) – relativistic sound speed, no anisotropic stress
- Jeans stability implies that its energy density is spatially smooth compared with the matter below the sound horizon

$$\begin{aligned} ds^2 &= -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2 \\ \nabla^2(\Phi-\Psi) &\propto \text{matter density fluctuation} \end{aligned}$$

- Anisotropic stress changes the amount of space curvature per unit dynamical mass

$$\nabla^2(\Phi+\Psi) \propto \text{anisotropic stress}$$

but its absence in a smooth dark energy model makes $g = (\Phi+\Psi)/(\Phi-\Psi) = 0$ for non-relativistic matter

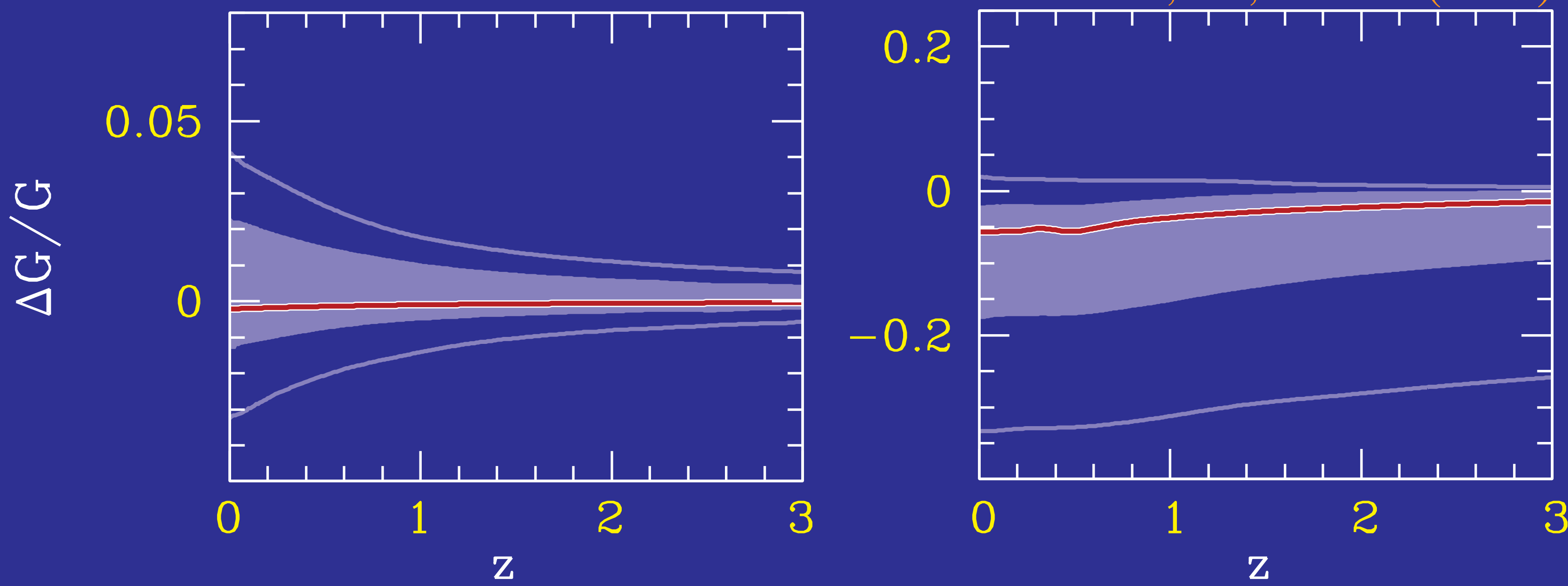
Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate
- Hence geometric measurements of the expansion rate predict the growth of structure
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm
  - Cluster Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)

Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)



Cosmological Constant

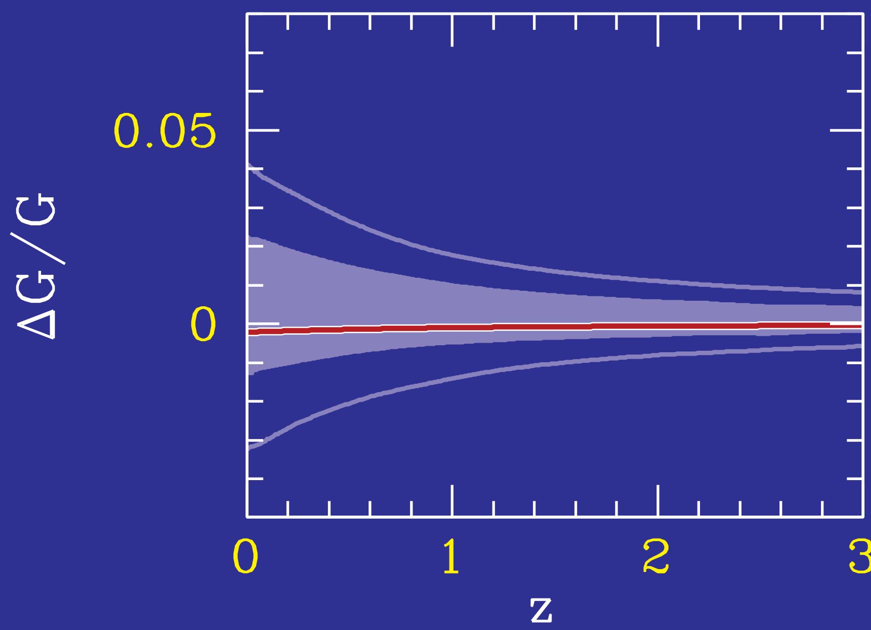
Quintessence

- Deviation significantly >2% rules out Λ with or without curvature
- Excess >2% rules out quintessence with or without curvature and early dark energy [as does >2% excess in H_0]

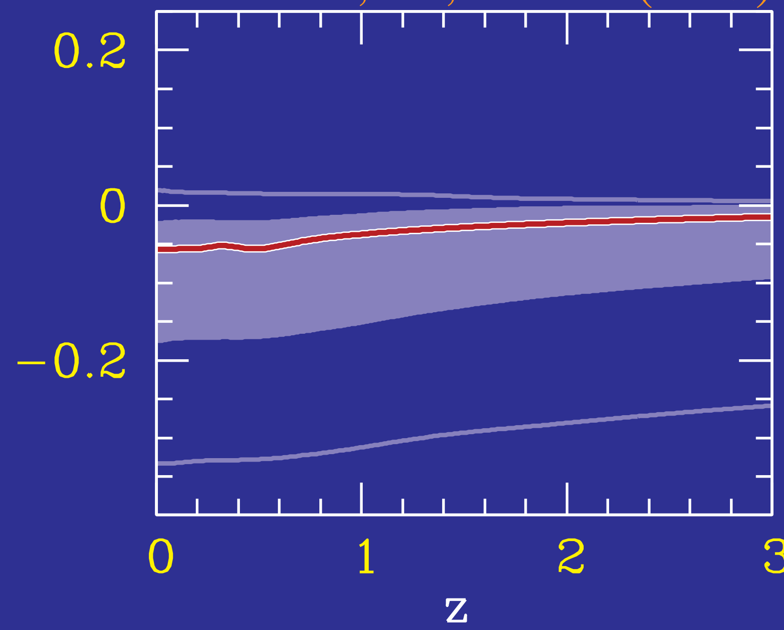
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

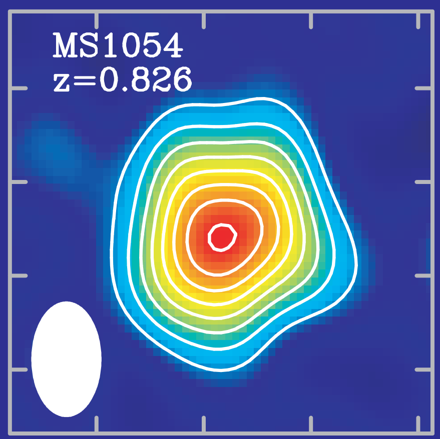
Mortonson, Hu, Huterer (2009)

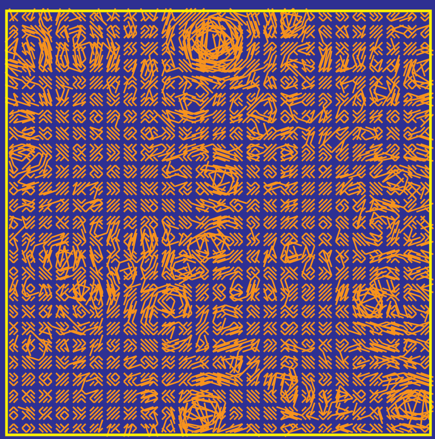


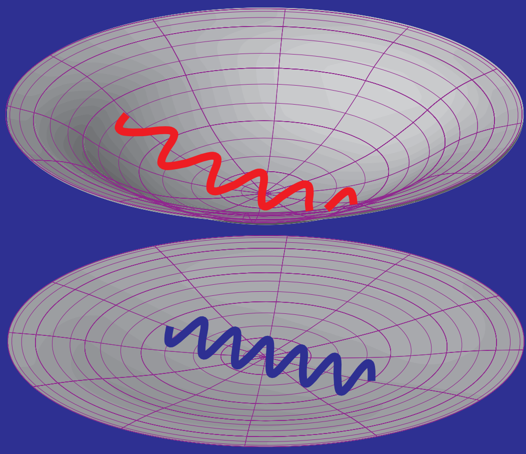
Cosmological Constant

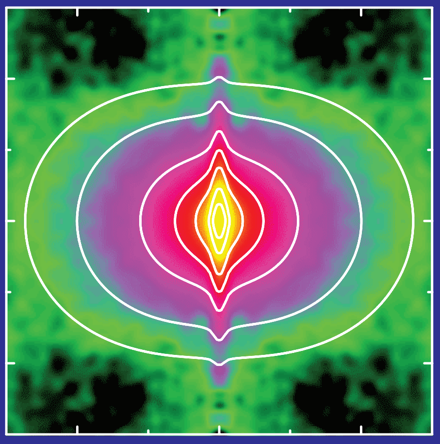


Quintessence





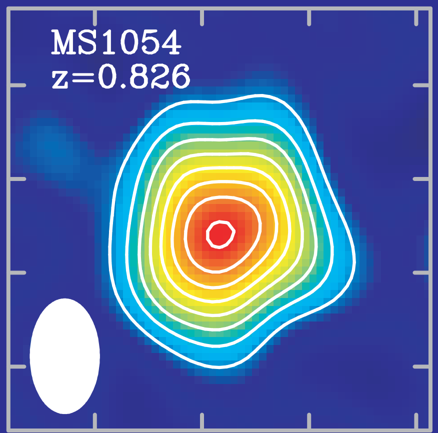


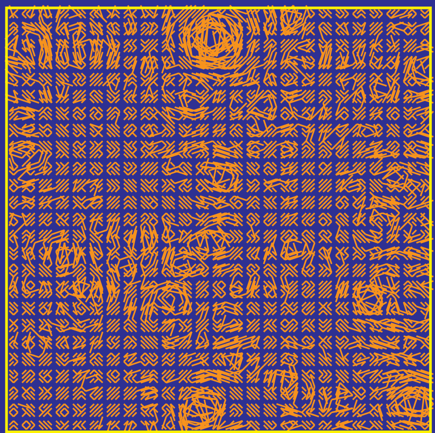


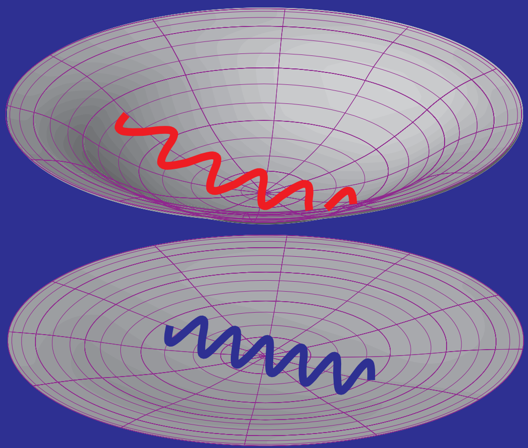
Quintessence Falsified?

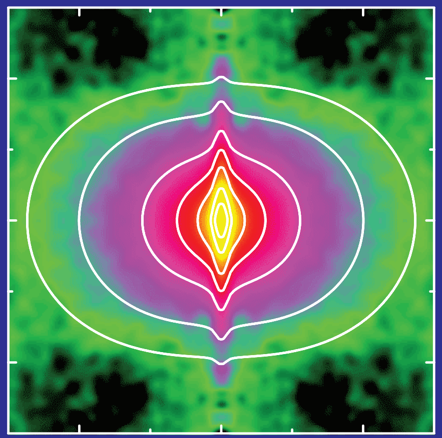
- No excess numbers of massive z>1 X-ray or SZ clusters with Gaussian initial conditions (Jee et al 2009, Brodwin et al 2010)
- No excess power in gravitational lensing at high z relative to low z (Bean 0909.3853)

- But would such violations favor modified gravity?
- Given astrophysical systematics, expect purported 2σ violations of smooth dark energy predictions will be common in coming years!





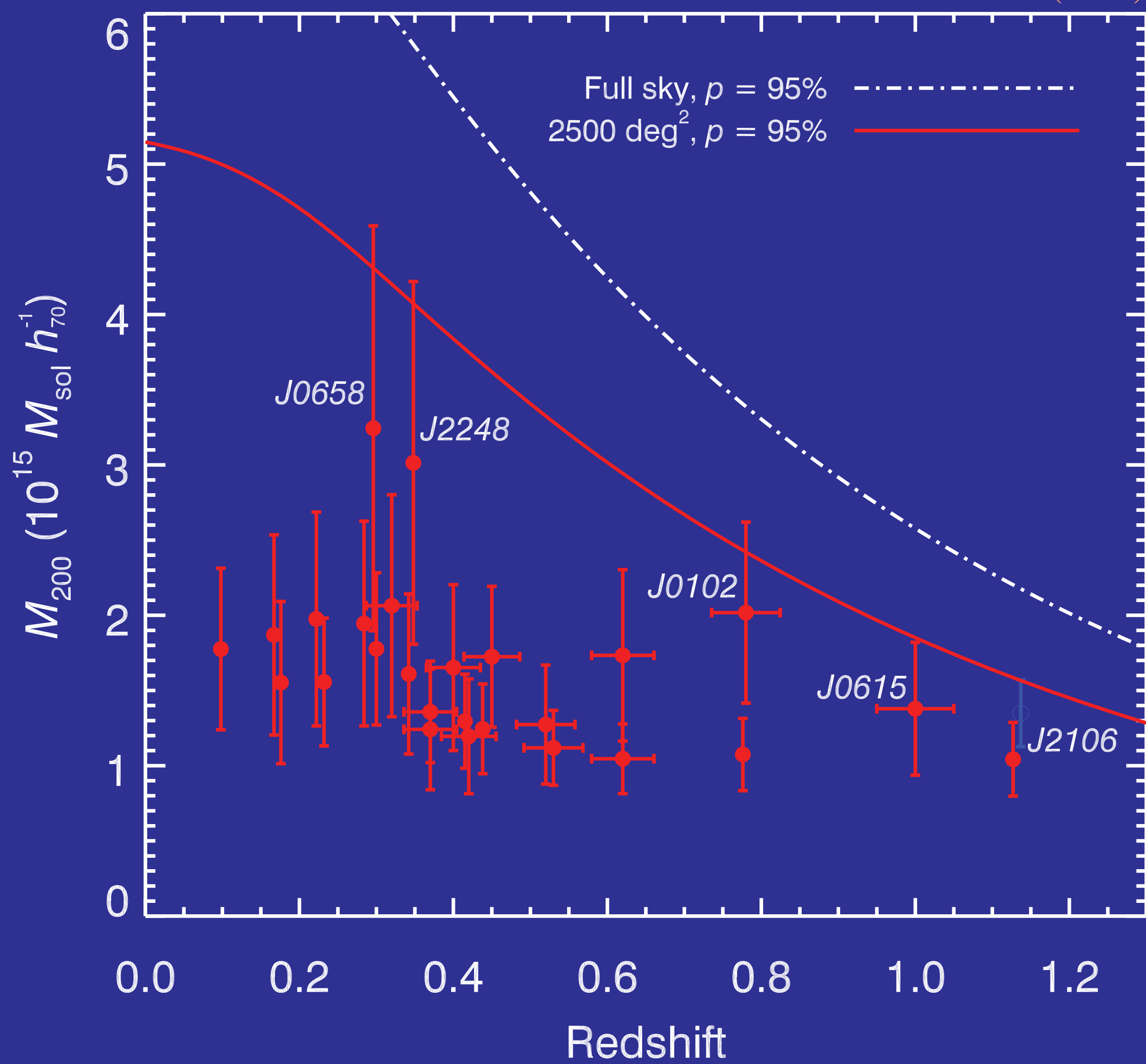




Pink Elephant Parade

- SPT catalogue on 2500 sq degrees

Williamson et al (2011)



Falsify in Favor of What?

Modified Action $f(R)$ Model

- R: Ricci scalar or "curvature"
- $f(R)$: modified action (Starobinsky 1980; Carroll et al 2004)

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_{\rm m} \right]$$

- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2 f/dR^2$: Compton wavelength of f_R squared, inverse mass squared
- B: Compton wavelength of f_R squared in units of the Hubble length

$$B \equiv \frac{f_{RR}}{1 + f_R} R' \frac{H}{H'}$$

- $' \equiv d/d\ln a$: scale factor as time coordinate

Modified Einstein Equation

- In the Jordan frame, gravity becomes 4th order but matter remains minimally coupled and separately conserved

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \Box f_R\right) g_{\alpha\beta} - \nabla_\alpha \nabla_\beta f_R = 8\pi G T_{\alpha\beta}$$

- Trace can be interpreted as a scalar field equation for f_R with a density-dependent effective potential ($p = 0$)

$$3\Box f_R + f_R R - 2f = R - 8\pi G\rho$$

- For small deviations, $|f_R| \ll 1$ and $|f/R| \ll 1$,

$$\Box f_R \approx \frac{1}{3}(R - 8\pi G\rho)$$

the field is sourced by the deviation from GR relation between curvature and density and has a mass

$$m_{f_R}^2 \approx \frac{1}{3}\frac{\partial R}{\partial f_R} = \frac{1}{3 f_{RR}}$$

DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

$$S = \int d^5x \sqrt{-g} \left[\frac{^{(5)}R}{2\kappa^2} + \delta(\chi) \left(\frac{^{(4)}R}{2\mu^2} + \mathcal{L}_m \right) \right]$$

  with crossover scale $r_c = \kappa^2/2\mu^2$

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)
- Matter still minimally coupled and conserved
- Exhibits the 3 regimes of modified gravity
- Weyl tensor anisotropy dominated conserved curvature regime $r > r_c$ (Sawicki, Song, Hu 2006; Cardoso et al 2007)
- Brane bending scalar tensor regime $r_* < r < r_c$ (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)
- Strong coupling General Relativistic regime $r < r_* = (r_c^2 r_g)^{1/3}$ where $r_g = 2GM$ (Dvali 2006)

DGP Field Equations

- DGP field equations

$$G_{\mu\nu} = 4r_c^2 f_{\mu\nu} - E_{\mu\nu}$$

where $f_{\mu\nu}$ is a tensor quadratic in the 4-dimensional Einstein and energy-momentum tensors

$$f_{\mu\nu} \equiv \frac{1}{12} A A_{\mu\nu} - \frac{1}{4} A^{\alpha}_{\mu} A_{\nu\alpha} + \frac{1}{8} g_{\mu\nu} \left(A_{\alpha\beta} A^{\alpha\beta} - \frac{A^2}{3} \right)$$

$$A_{\mu\nu} \equiv G_{\mu\nu} - \mu^2 T_{\mu\nu}$$

and $E_{\mu\nu}$ is the bulk Weyl tensor

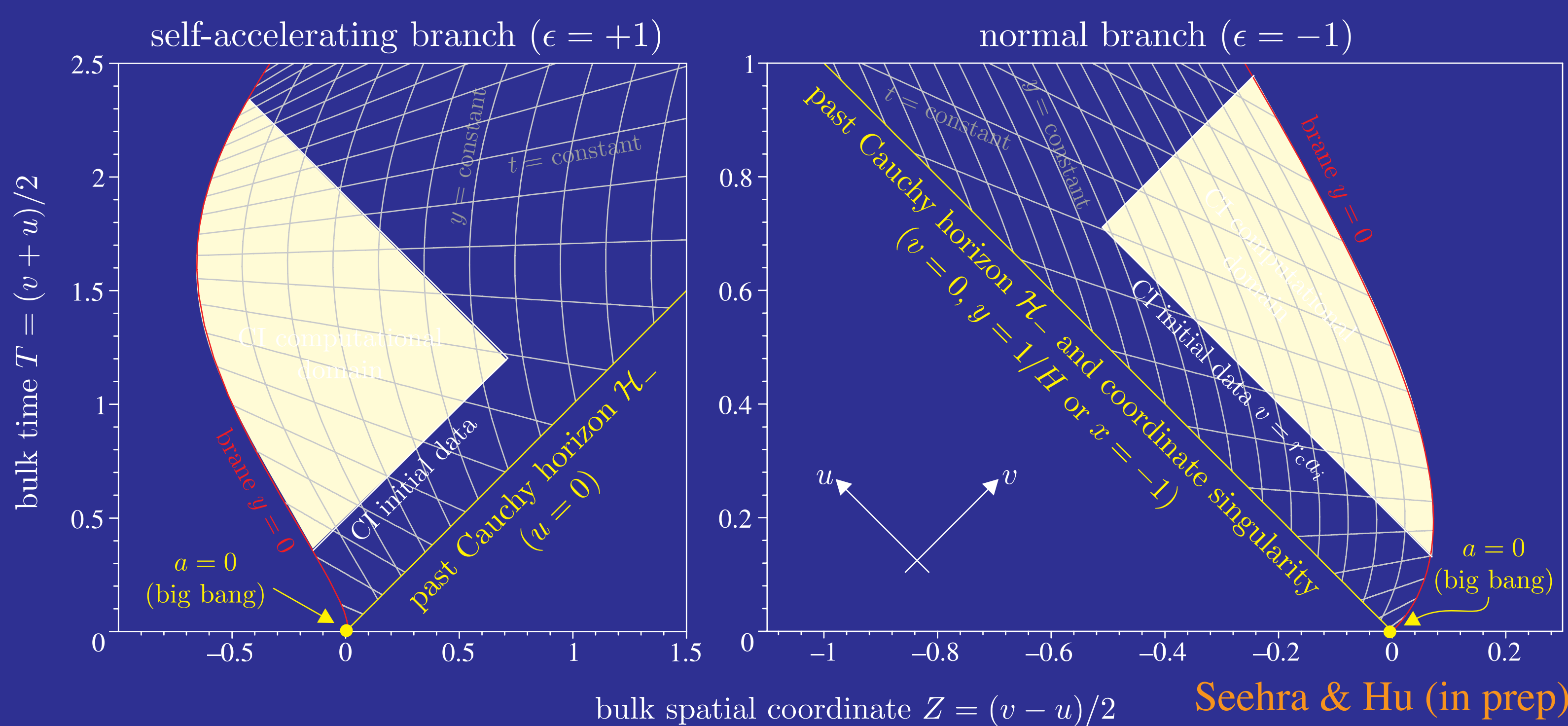
- Background metric yields the modified Friedmann equation

$$H^2 \mp \frac{H}{r_c} = \frac{\mu^2 \rho}{3}$$

- For perturbations, involves solving metric perturbations in the bulk through the “master equation”

Into the Bulk

- Calculation of the metric ratio $g=\Phi+\Psi/\Phi-\Psi$ requires solving for the propagation of metric fluctuations into the bulk
- Encapsulated in the off brane gradient which closes the system (e.g. normal branch $g=-1/(2Hr_c+1)$ until deep in de Sitter)



Seehra & Hu (in prep)

Sawicki, Song, Hu (2007); Cardoso et al (2008)

Three Regimes

- Three regimes with different dynamics
- Examples $f(R)$ and DGP braneworld acceleration
- Parameterized Post-Friedmann description

- Non-linear regime return to General Relativity / Newtonian dynamics

General Relativistic Non-Linear Regime | Scalar-Tensor Regime | Conserved-Curvature Regime

r_* r_c

halos, galaxy | *large scale structure* | *CMB*

$r \longrightarrow$

$f(R)$ Expansion History

Engineering $f(R)$ Models

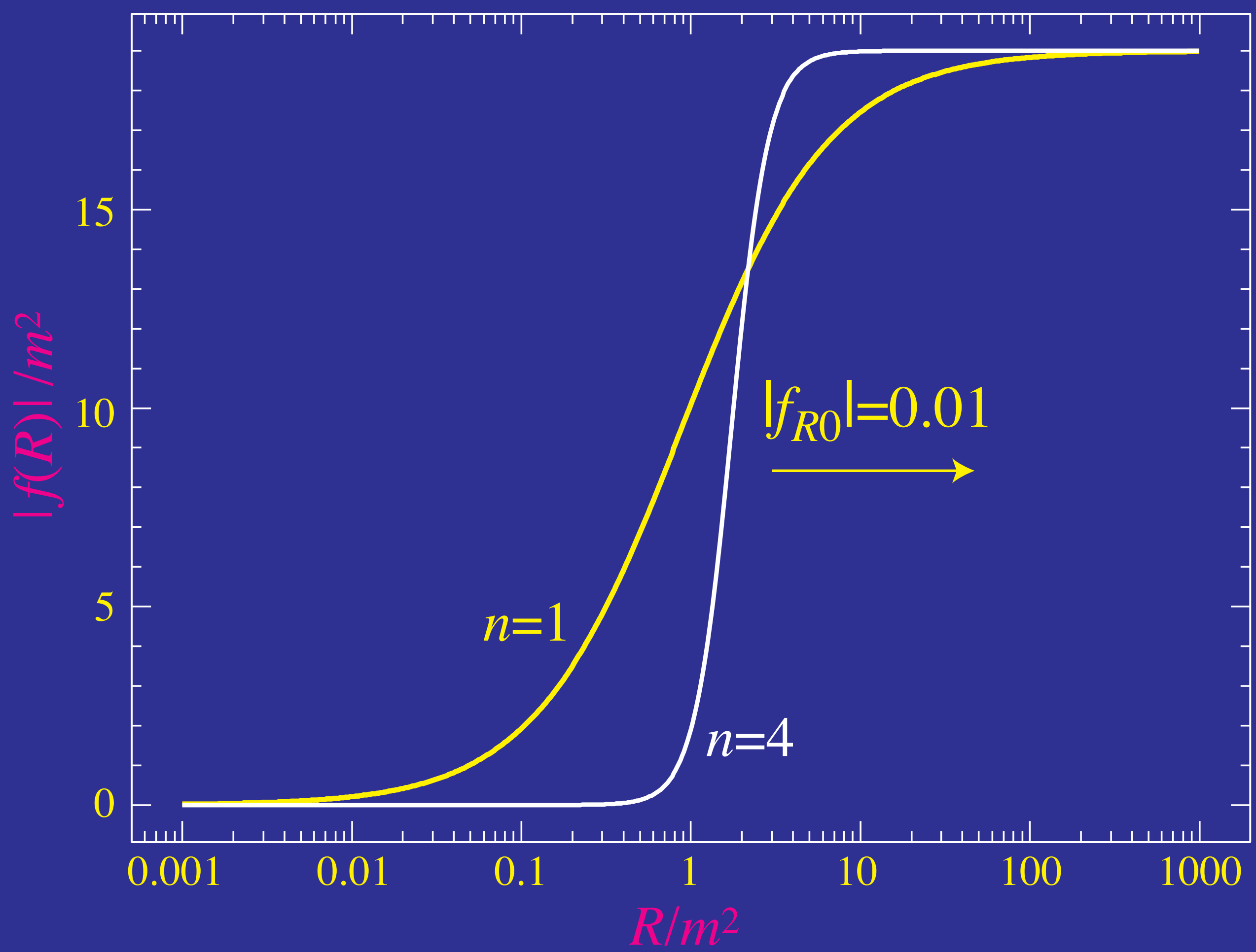
- Mimic ΛCDM at high redshift
- Accelerate the expansion at low redshift without a cosmological constant
- Sufficient freedom to vary expansion history within observationally allowed range
- Contain the phenomenology of ΛCDM in both cosmology and solar system tests as a limiting case for the purposes of constraining small deviations
- Suggests

$$f(R) \propto \frac{R^n}{R^n + \text{const.}}$$

such that modifications vanish as $R \to 0$ and go to a constant as $R \to \infty$

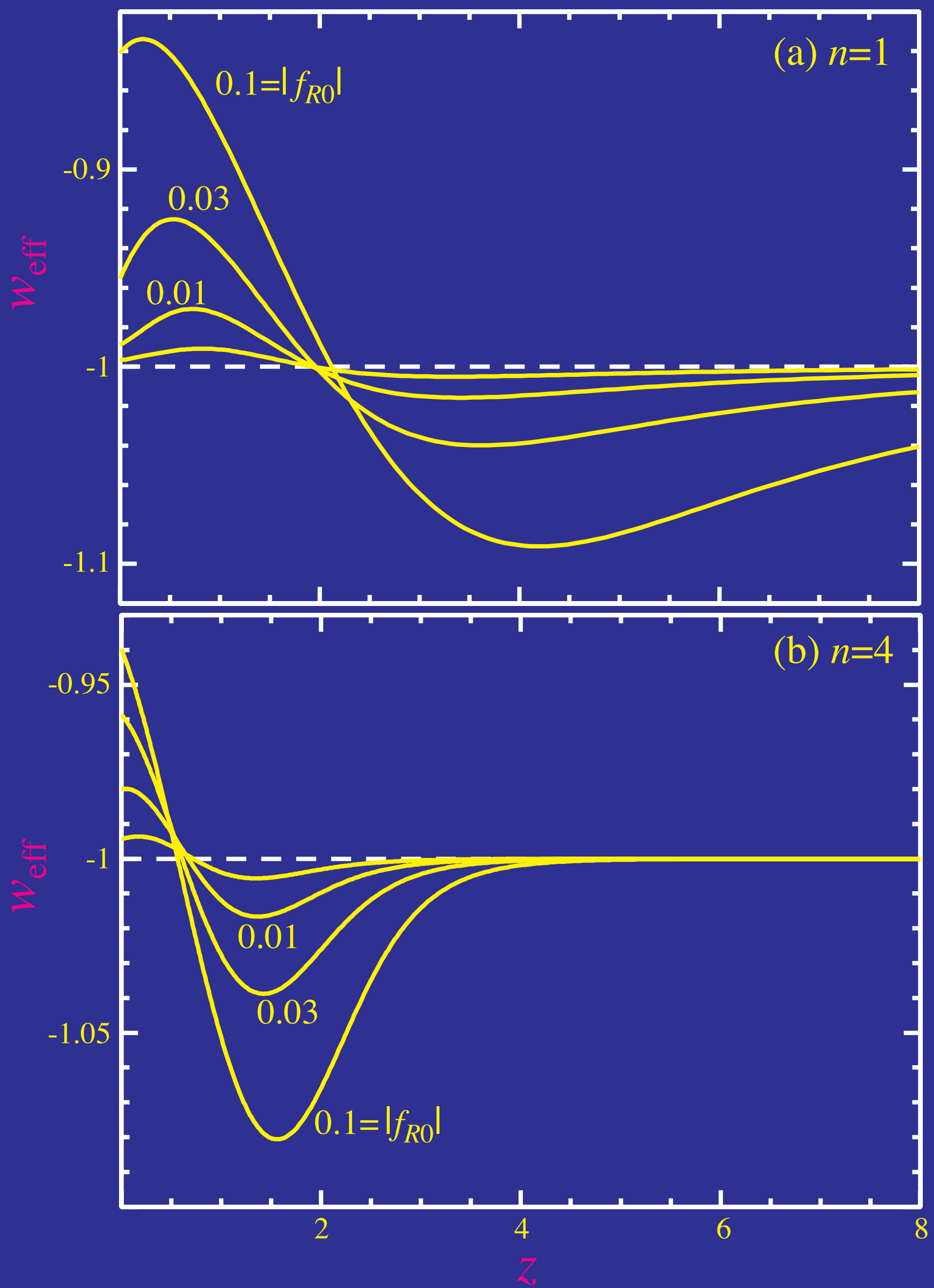
Form of $f(R)$ Models

- Transition from zero to constant across an adjustable curvature scale
- Slope n controls the rapidity of transition, field amplitude f_{R0} position
- Background curvature stops declining during acceleration epoch and thereafter behaves like cosmological constant



Hu & Sawicki (2007)

Expansion History



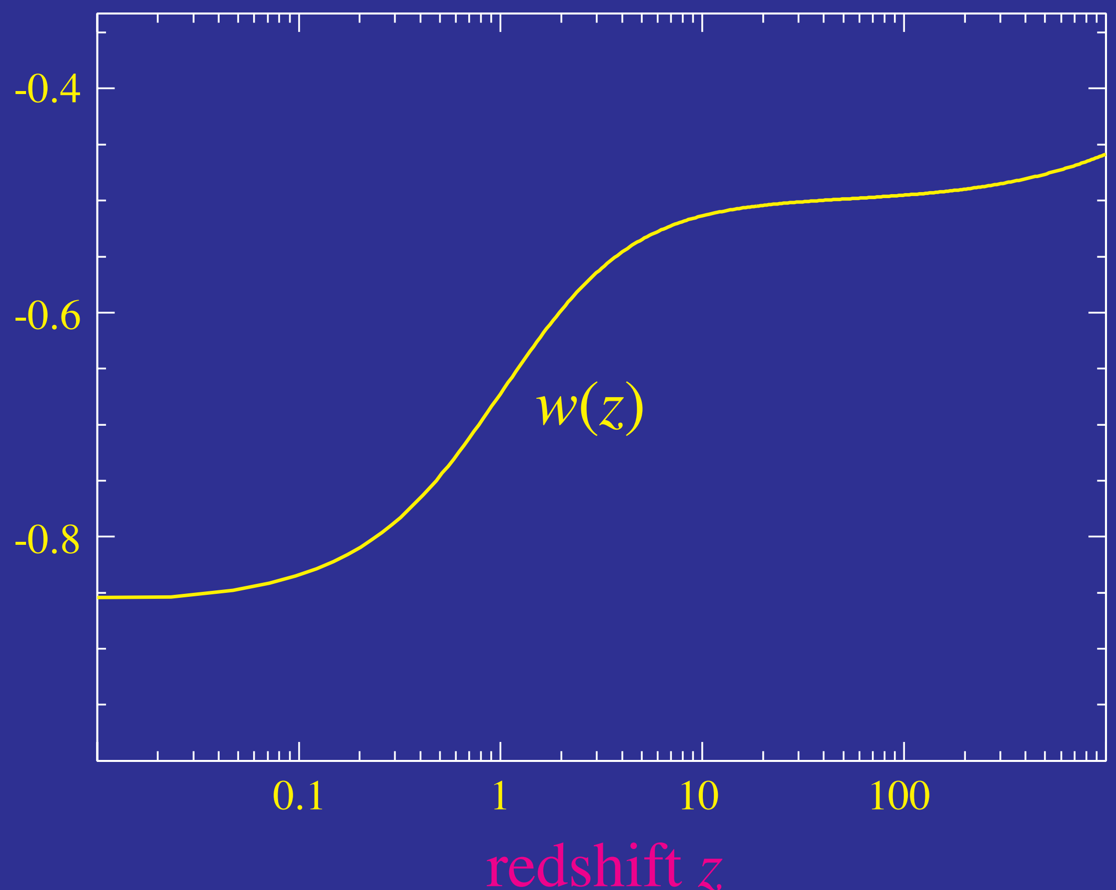
- Effective equation of state $w_{\rm eff}$ scales with field amplitude f_{R0}
- Crosses the phantom divide at a redshift that decreases with n
- Signature of degrees of freedom in dark energy beyond standard kinetic and potential energy of k-essence or quintessence or modified gravity

Hu & Sawicki (2007)

DGP Expansion History

DGP Expansion History

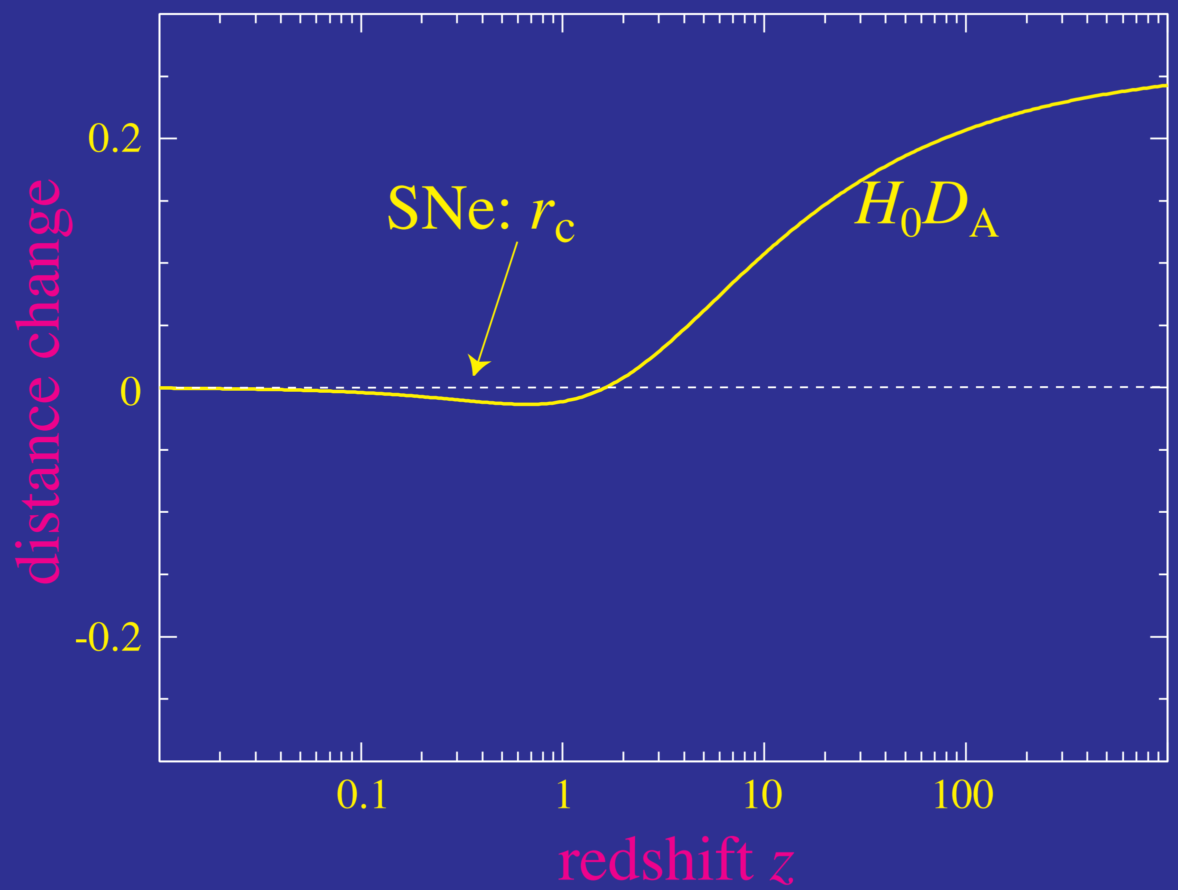
- Matching the DGP expansion history to a dark energy model with the same expansion history
- Effective equation of state $w(z)$ [w_0~-0.85, w_a~0.35]



Song, Sawicki & Hu (2006)

DGP Expansion History

- Crossover scale r_c fit to SN relative distance from z=0: $H_0 D_A$



Song, Sawicki & Hu (2006)

DGP Normal Branch

- On the normal branch, expansion does not self-accelerate and dark energy in the form of a brane tension or scalar field necessary

$$H^2 + \frac{H}{r_c} = \frac{\mu^2}{3}(\rho_m + \rho_{\rm DE})$$

- Gravity is still modified as in the self-accelerated branch (but with attractive forces)
- Ghost free in the quantum theory
- Can choose $\rho_{\rm DE}$ to match any desired expansion history including flat ΛCDM

$$H^2 \equiv \frac{\mu^2}{3}(\rho_m + \rho_\Lambda) \rightarrow \rho_{\rm DE}$$

- Separate out geometrical and dynamical tests of acceleration

Conserved Curvature Regime

Curvature Conservation

- On superhorizon scales, energy momentum conservation and expansion history constrain the evolution of metric fluctuations (Bertschinger 2006)
- For adiabatic perturbations in a flat universe, conservation of comoving curvature applies $\zeta' = 0$ where $' \equiv d/d\ln a$ (Bardeen 1980)
- Gauge transformation to Newtonian gauge

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$$

yields (Hu & Eisenstein 1999)

$$\Phi'' - \Psi' - \frac{H''}{H'}\Phi' - \left(\frac{H'}{H} - \frac{H''}{H'}\right)\Psi = 0$$

- Modified gravity theory supplies the closure relationship $\Phi = -\gamma(\ln a)\Psi$ between and expansion history $H = \dot{a}/a$ supplies rest.

Linear Theory for $f(R)$

- In $f(R)$ model, "superhorizon" behavior persists until Compton wavelength smaller than fluctuation wavelength $B^{1/2}(k/aH) < 1$
- Once Compton wavelength becomes larger than fluctuation

$$B^{1/2}(k/aH) > 1$$

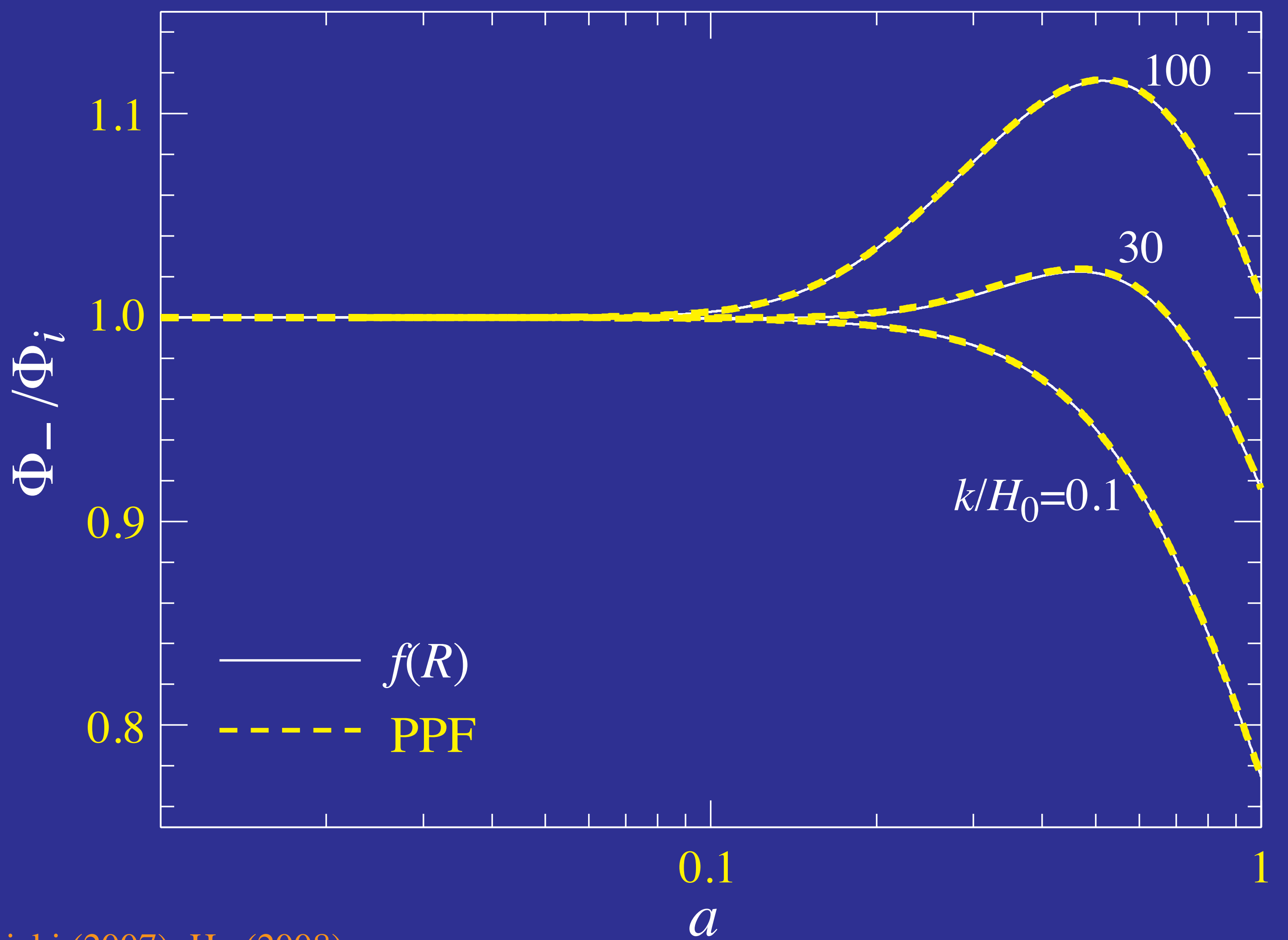
  perturbations are in scalar-tensor regime described by $\gamma = 1/2$.
- Small scale density growth enhanced and

$$8\pi G\rho > R$$

  low curvature regime with order unity deviations from GR
- Transitions in the non-linear regime where the Compton wavelength can shrink via chameleon mechanism
- Given $k_{\rm NL}/aH \gg 1$, even very small f_R have scalar-tensor regime

PPF $f(R)$ Description

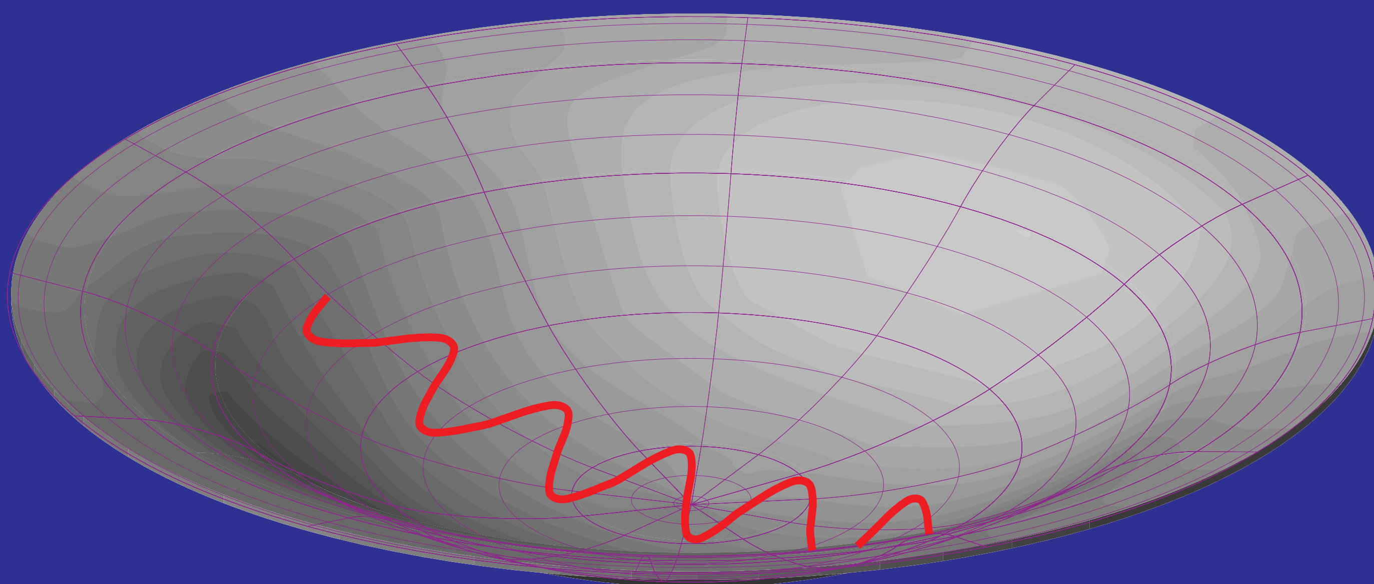
- Metric and matter evolution well-matched by PPF description
- Standard GR tools apply (CAMB), self-consistent, gauge invar.



Hu & Sawicki (2007); Hu (2008)

Integrated Sachs-Wolfe Effect

- CMB photons transit gravitational potentials of large-scale structure
- If potential decays during transit, gravitational blueshift of infall not cancelled by gravitational redshift of exit
- Spatial curvature of gravitational potential leads to additional effect $\Delta T/T = -\Delta(\Phi - \Psi)$



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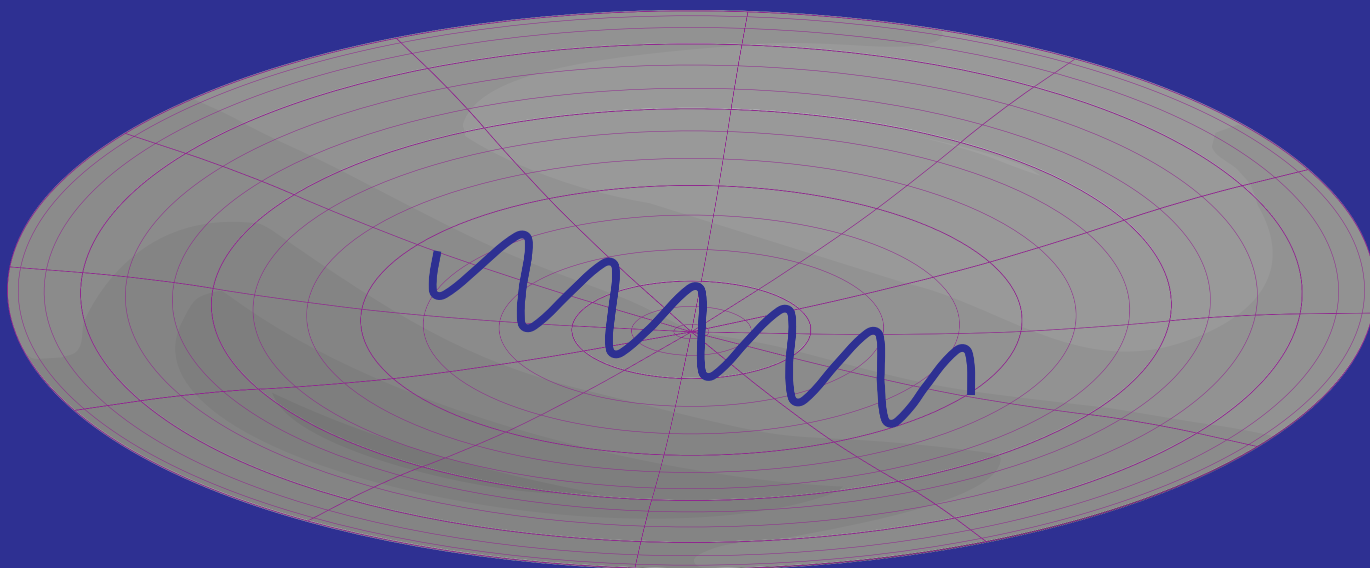
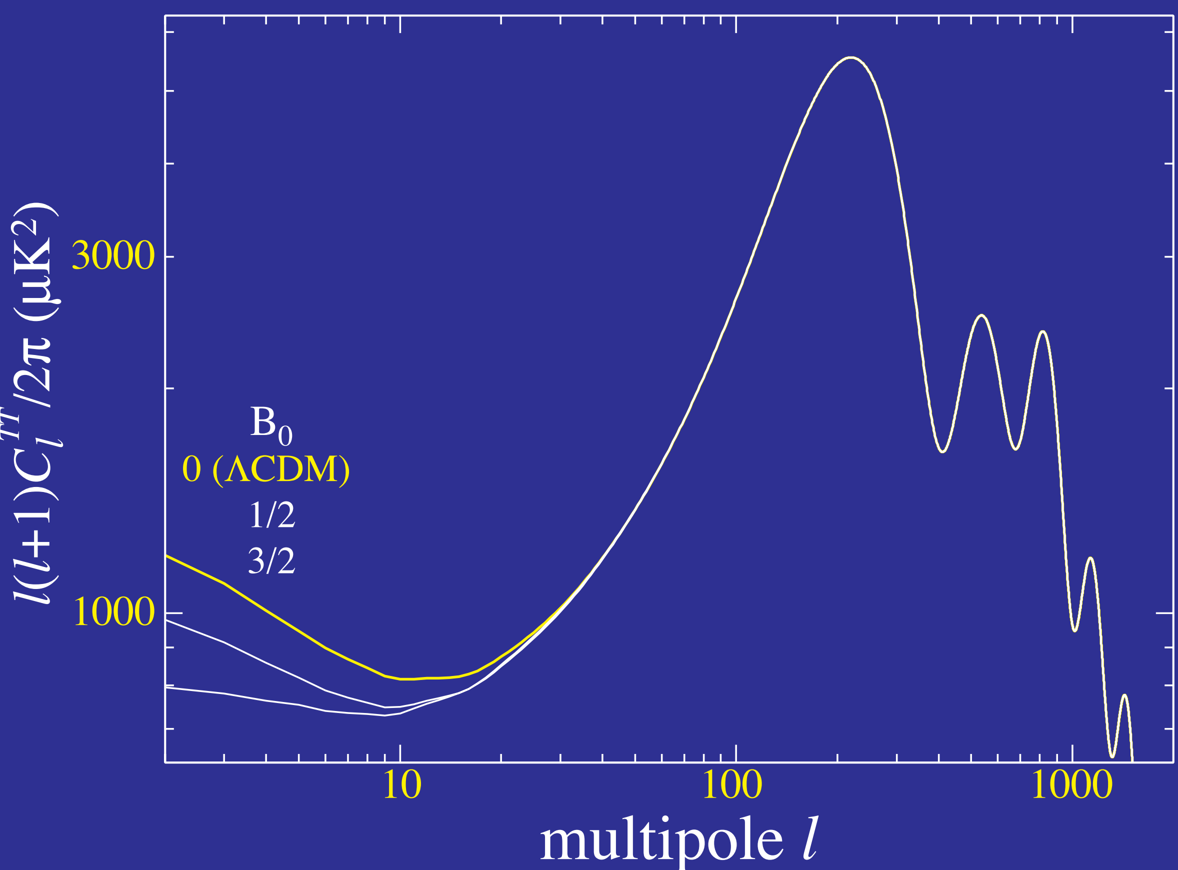


ISW Quadrupole

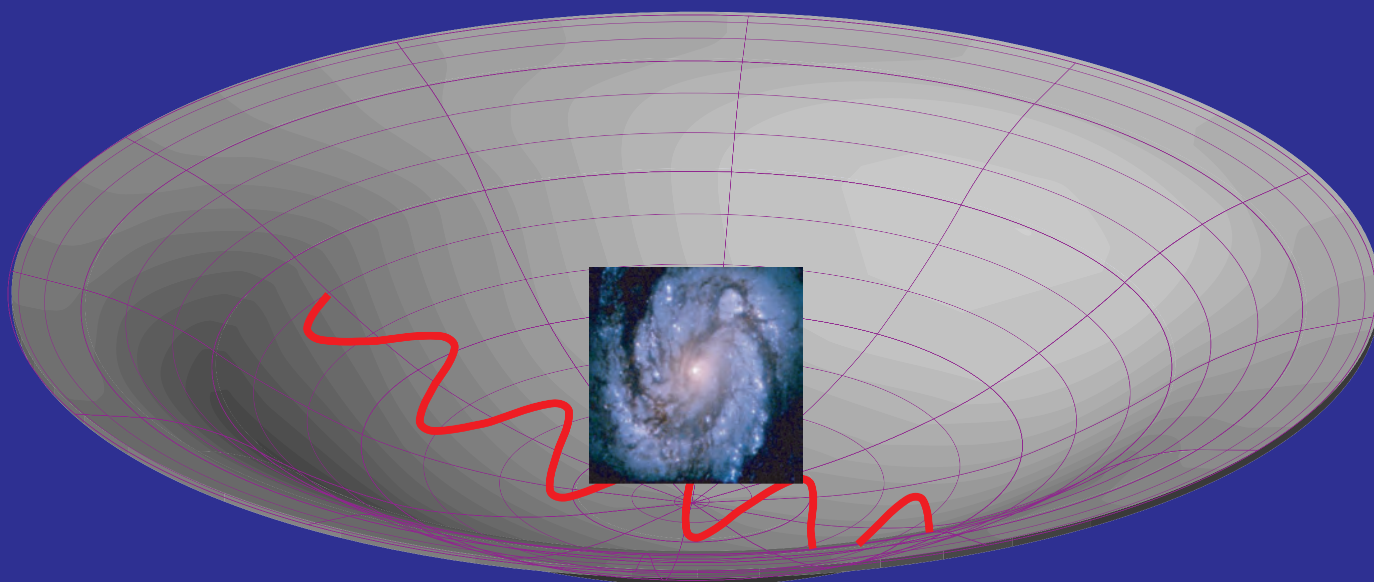
- Reduction of large angle anisotropy for B_0~1 for same expansion history and distances as ΛCDM
- Well-tested small scale anisotropy unchanged



Song, Hu & Sawicki (2006)

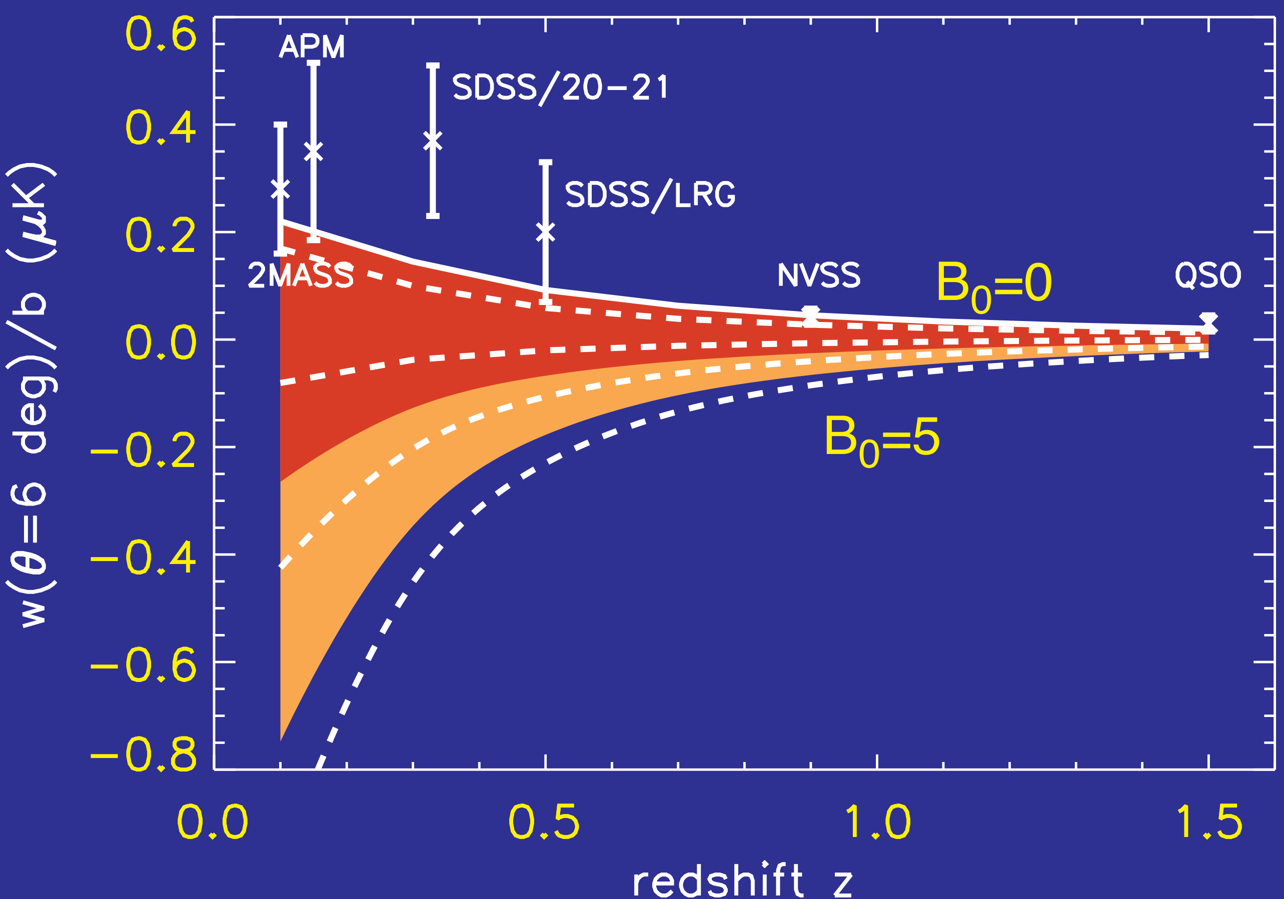
ISW-Galaxy Correlation

- Decaying potential: galaxy positions correlated with CMB
- Growing potential: galaxy positions anticorrelated with CMB
- Observations indicate correlation



Galaxy-ISW Anti-Correlation

- Large Compton wavelength $B^{1/2}$ creates potential growth which can anti-correlate galaxies and the CMB
- In tension with detections of positive correlations across a range of redshifts



Song, Peiris & Hu (2007); Lombriser et al (2010) B_0<0.43

DGP Horizon Scales

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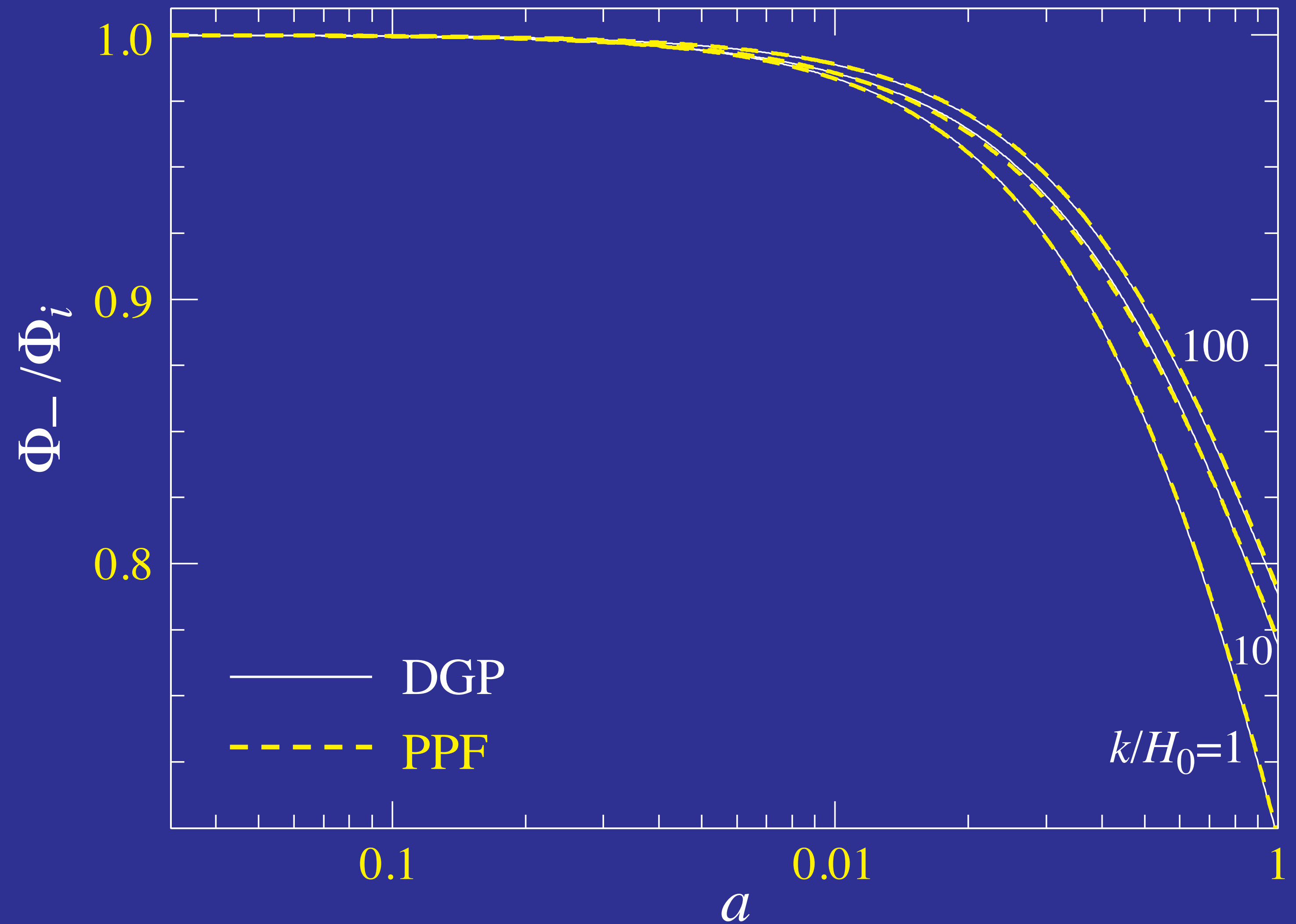
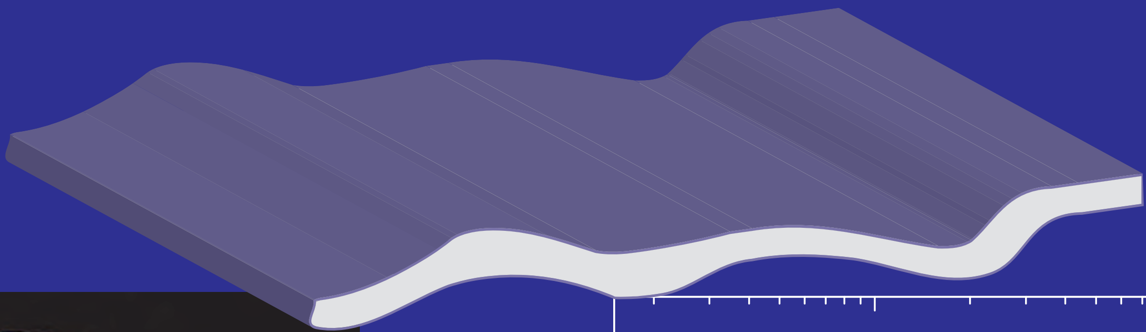
- Standard GR tools apply (CAMB), self-consistent, gauge invar.

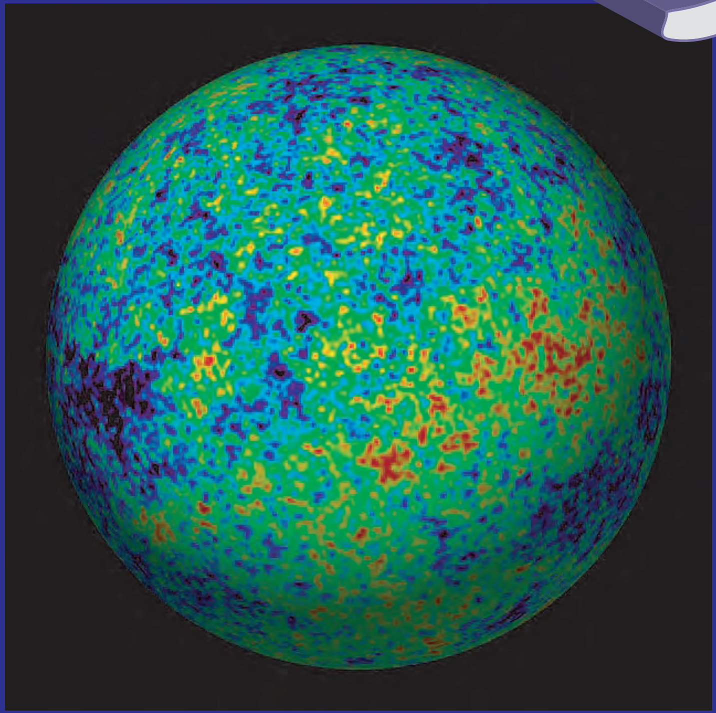


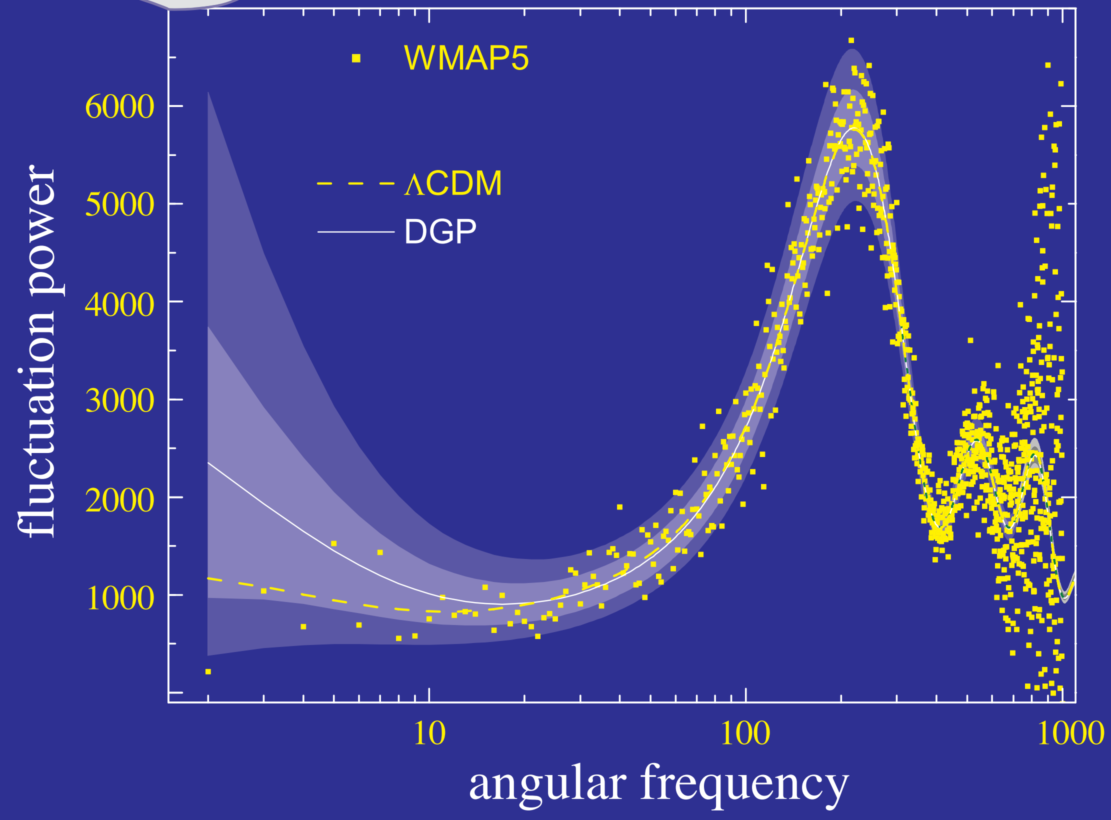
Hu & Sawicki (2007); Hu (2008)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background

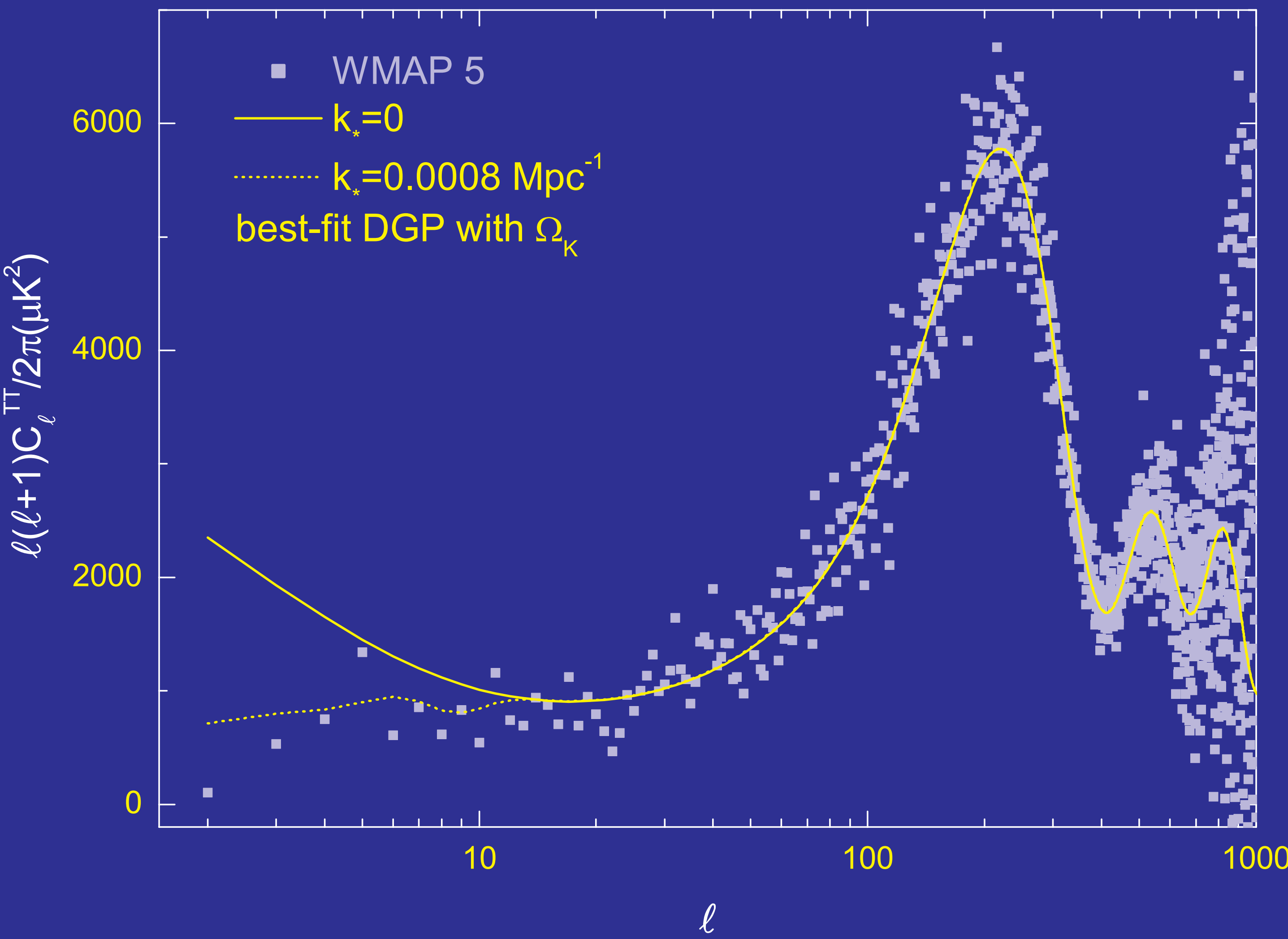






CMB in DGP

- Adding cut off as an epicycle can fix distances, ISW problem
- Suppresses polarization in violation of EE data - cannot save DGP!



Fang et al (2008)

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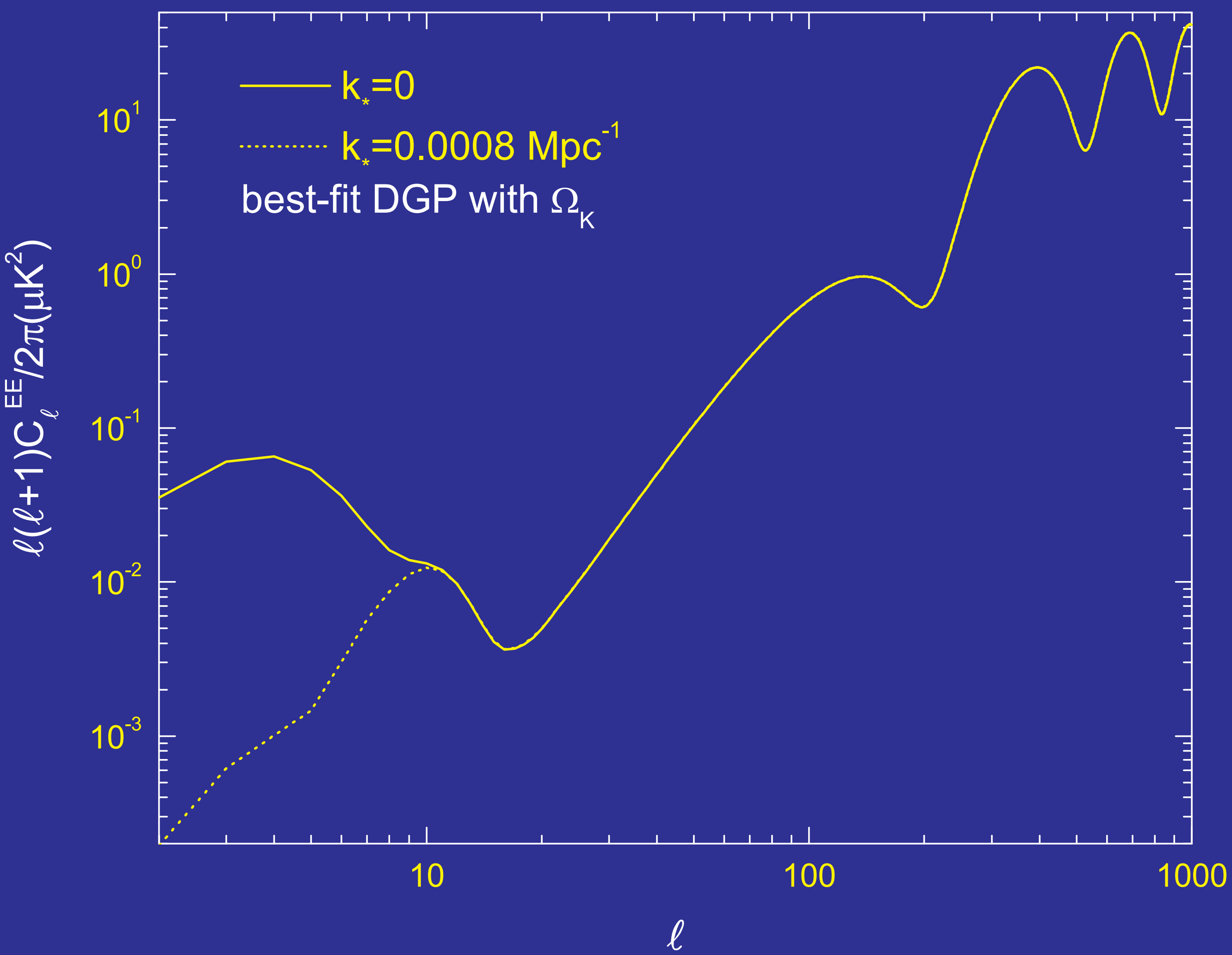
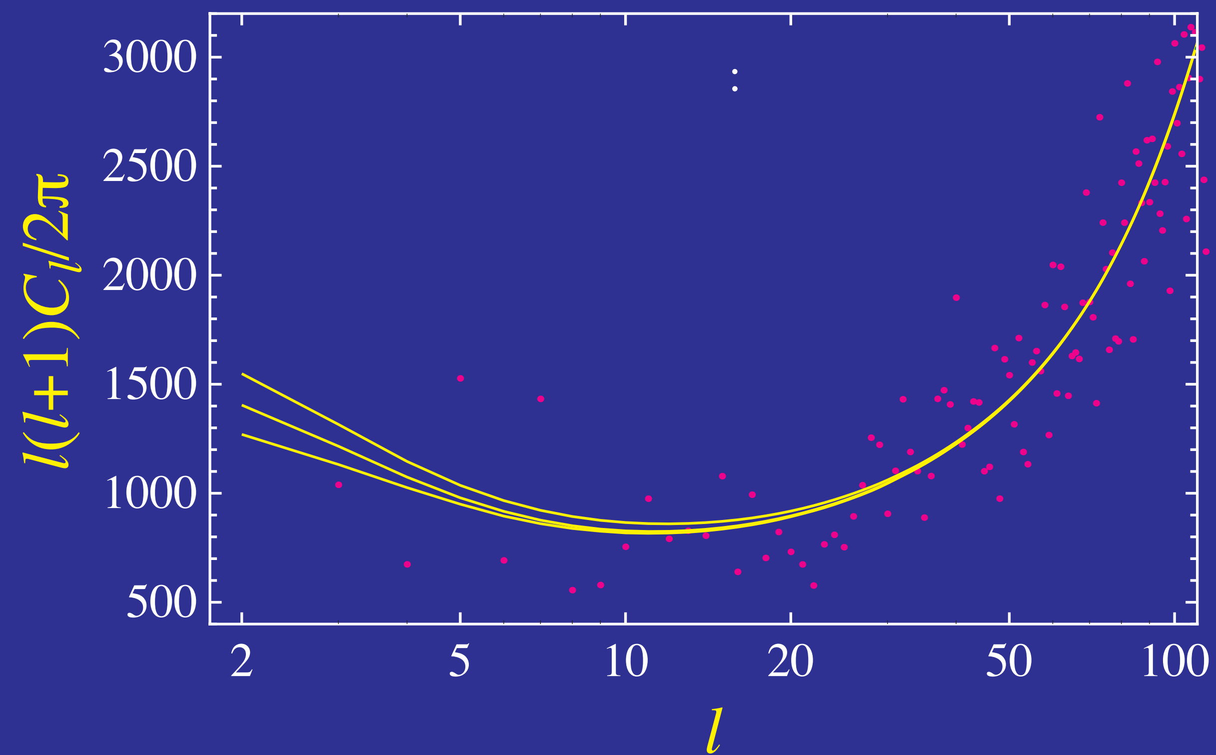


Fang et al (2008)

DGP Normal Branch

- Brane tension (cosmological constant) on normal branch allows models to pass ISW test
- Joint expansion history constraints require $Hr_c > 3$ at 95% CL



Lombriser et al (2009)

Linear Scalar Tensor Regime

Three Regimes

- Metric: $ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)dx^2$
- Superhorizon regime: ζ =const., $g(a) = (\Phi+\Psi)/(\Phi-\Psi)$
- Linear regime - closure ↔ "smooth" dark energy density:

$$\nabla^2(\Phi-\Psi)/2 = -4\pi G a^2 \Delta\rho$$

G can be promoted to $G(a)$, $G(a,k)$ but for scalar degrees of freedom conformal invariance requires $G = G_N$ and

- Non-linear regime:

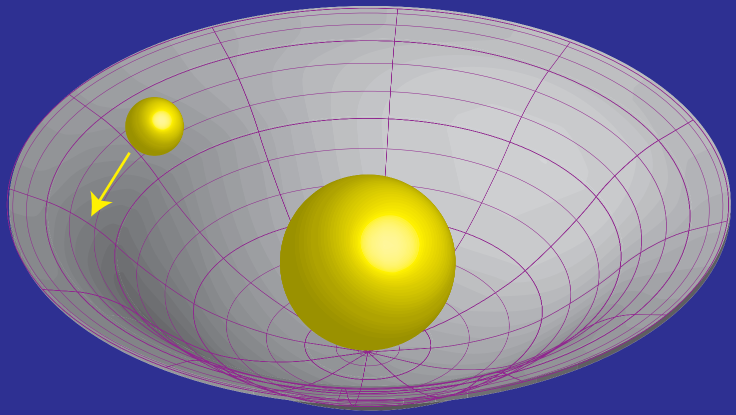
$$\begin{aligned}\nabla^2(\Phi-\Psi)/2 &= -4\pi G a^2 \Delta\rho \\ \nabla^2\Psi &= 4\pi G a^2 \Delta\rho + \frac{1}{2}\nabla^2\phi\end{aligned}$$

with non-linearity in the field equation

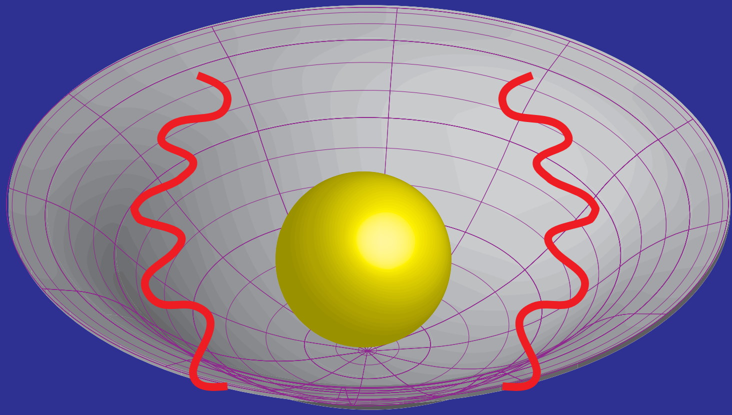
$$\nabla^2\phi = g_{\rm lin}(a) a^2 \left(8\pi G\Delta\rho - N[\phi]\right)$$

Dynamical vs Lensing Mass

- Newtonian potential: $\Psi=\delta g_{00}/2g_{00}$ which non-relativistic particles feel



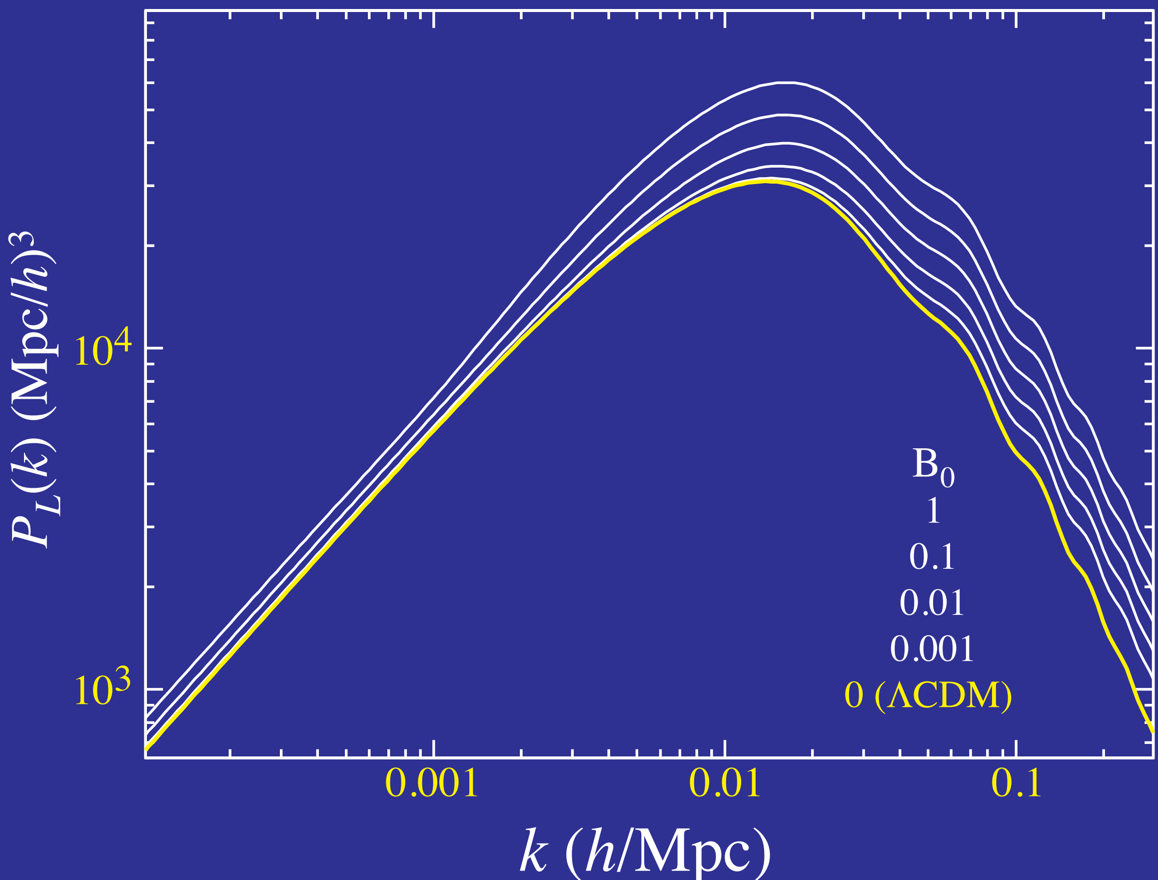
- Space curvature: $\Phi=\delta g_{ii}/2g_{ii}$ which also deflects photons



- Most of the incisive tests of gravity reduce to testing the space curvature per unit dynamical mass

Linear Power Spectrum

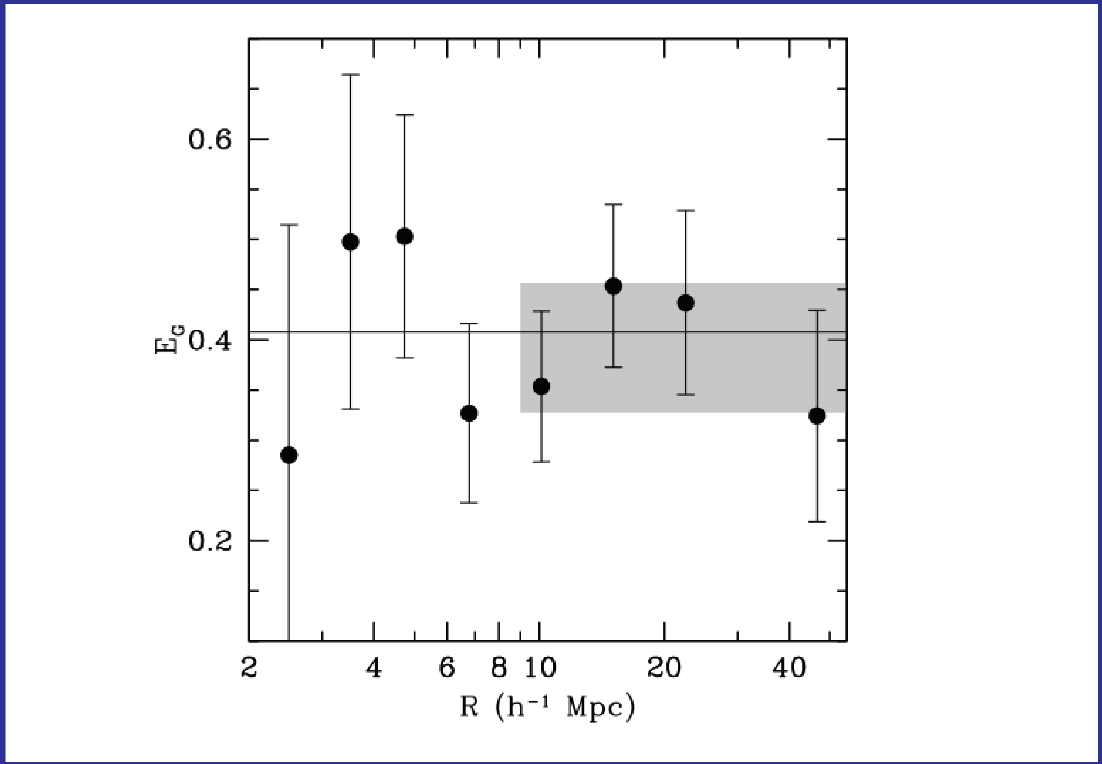
- Linear real space power spectrum enhanced on small scales
- Degeneracy with galaxy bias and lack of non-linear predictions leave constraints from shape of power spectrum



Lensing v Dynamical Comparison

- Gravitational lensing around galaxies vs. linear velocity field (through redshift space distortions and galaxy autocorrelation)
- Consistent with GR + smooth dark energy beginning to test interesting models

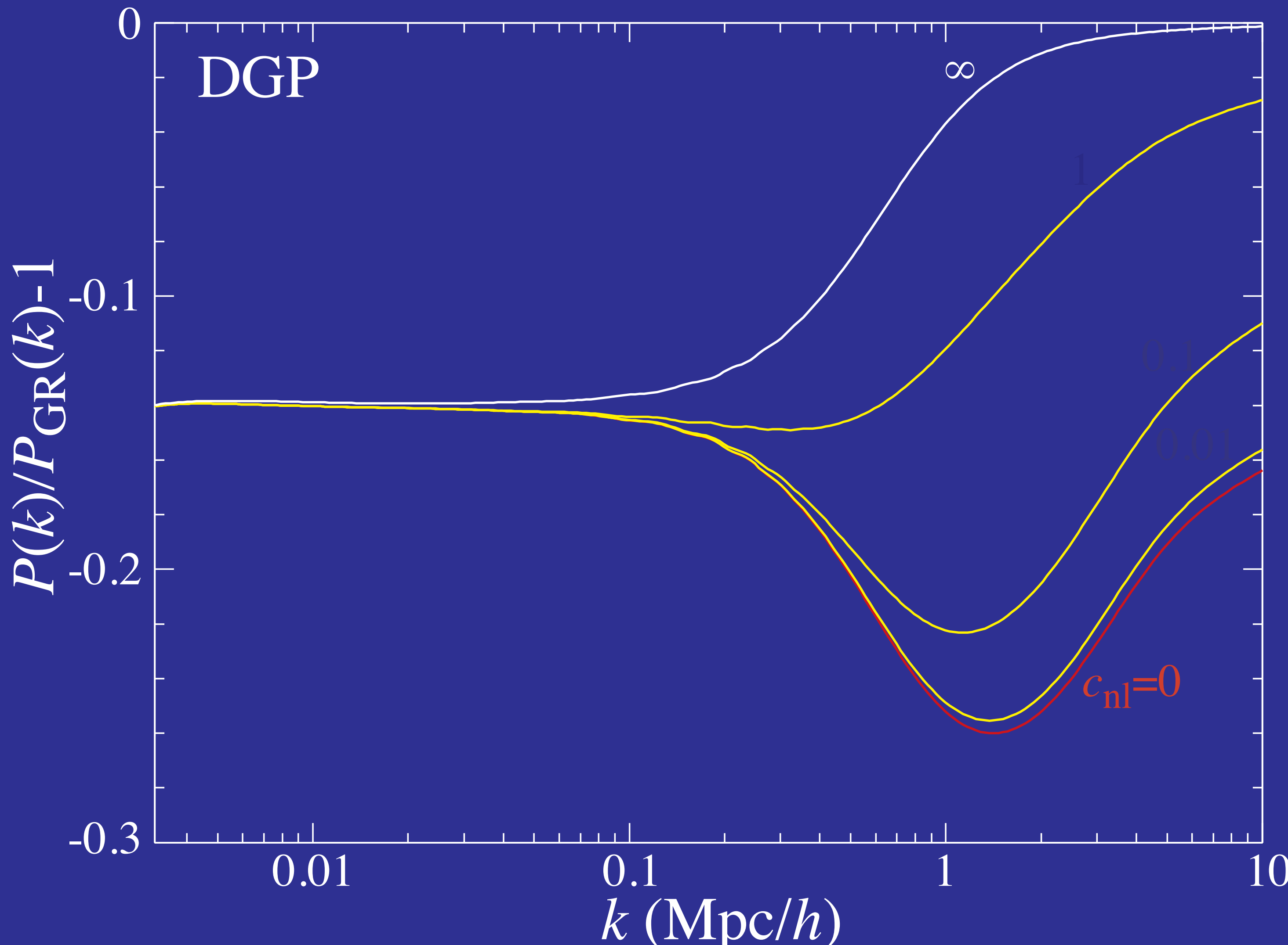
Reyes et al (2010); Lombriser et al (2010)



Zhang et al (2007); Jain & Zhang (2008)

DGP Power Spectrum

- Constant suppression in the linear regime for self-acceleration



Lue, Scoccimarro, Starkman (2004); Hu & Sawicki (2007)

Non-Linear GR Regime

Three Regimes

- Fully worked $f(R)$ and DGP examples show 3 regimes
- Superhorizon regime: ζ =const., $g(a)$
- Linear regime - closure condition - analogue of "smooth" dark energy density:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ g(a, \mathbf{x}) &\leftrightarrow g(a, k) \end{aligned}$$

G can be promoted to $G(a)$ but conformal invariance relates fluctuations to field fluctuation that is small

- Non-linear regime:

$$\begin{aligned} \nabla^2(\Phi - \Psi)/2 &= -4\pi G a^2 \Delta\rho \\ \nabla^2\Psi &= 4\pi G a^2 \Delta\rho - \frac{1}{2}\nabla^2\phi \end{aligned}$$

Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2\phi = g_{\rm lin}(a)a^2\left(8\pi G\Delta\rho - N[\phi]\right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a nonlinear function of the field

Linked to gravitational potential

- For DGP, ϕ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4}\left[(\nabla^2\phi)^2 - (\nabla_i\nabla_j\phi)^2\right]$$

a nonlinear function of second derivatives of the field

Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

Non-Linear Chameleon

- For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G\delta\rho)$$

  is the non-linear equation that returns general relativity

- High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

- Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \le \frac{2}{3}\Phi\,,$$

  else required field gradients too large despite $\delta R = 8\pi G\delta\rho$ being the local minimum of effective potential

Non-Linear Dynamics

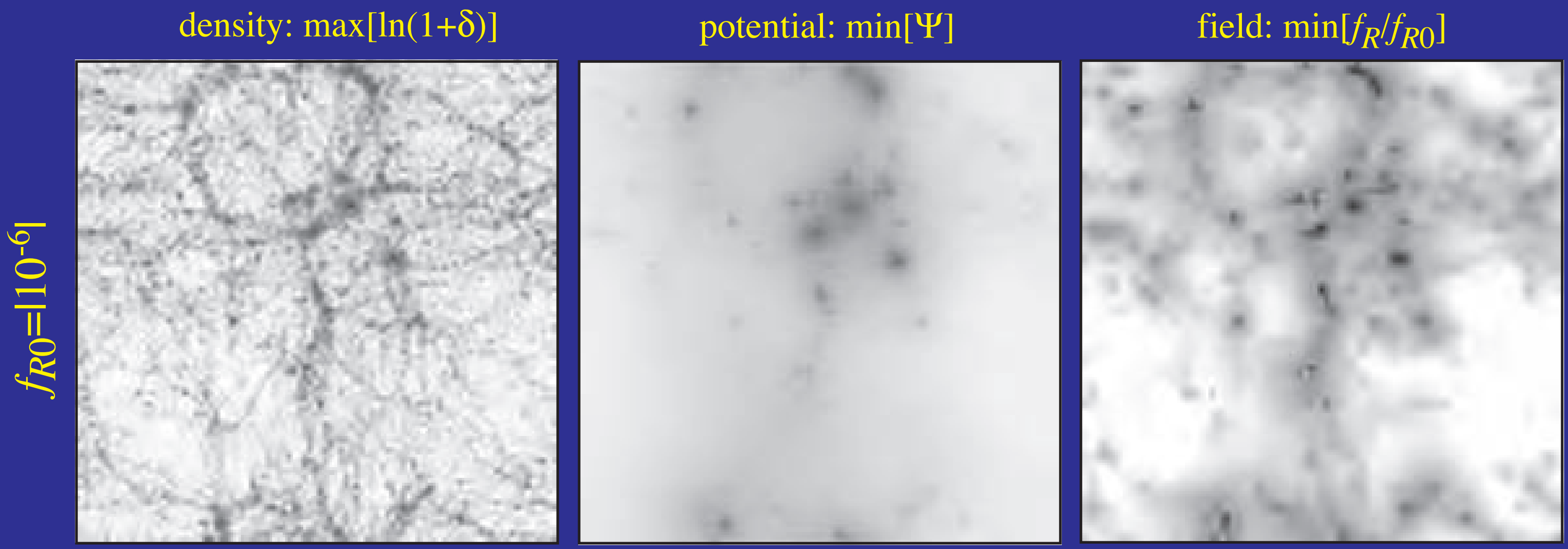
- Supplement that with the modified Poisson equation

$$\nabla^2\Psi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R(f_R)$$

- Matter evolution given metric unchanged: usual motion of matter in a gravitational potential Ψ
- Prescription for N-body code
- Particle Mesh (PM) for the Poisson equation
- Field equation is a non-linear Poisson equation: relaxation method for f_R
- Initial conditions set to GR at high redshift

Environment Dependent Force

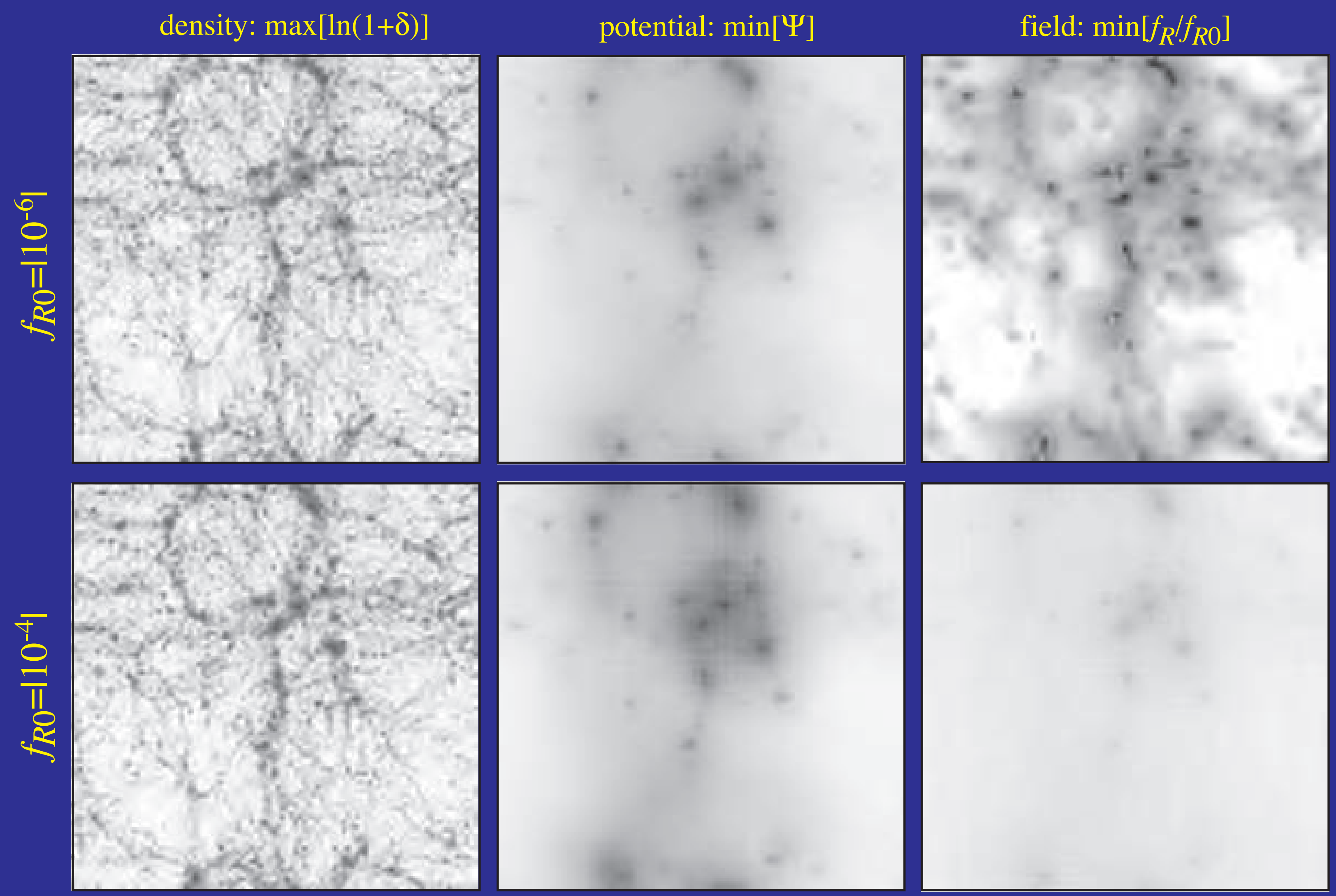
- Chameleon suppresses extra force (scalar field) in high density, deep potential regions



Oyaizu, Lima, Hu (2008)

Environment Dependent Force

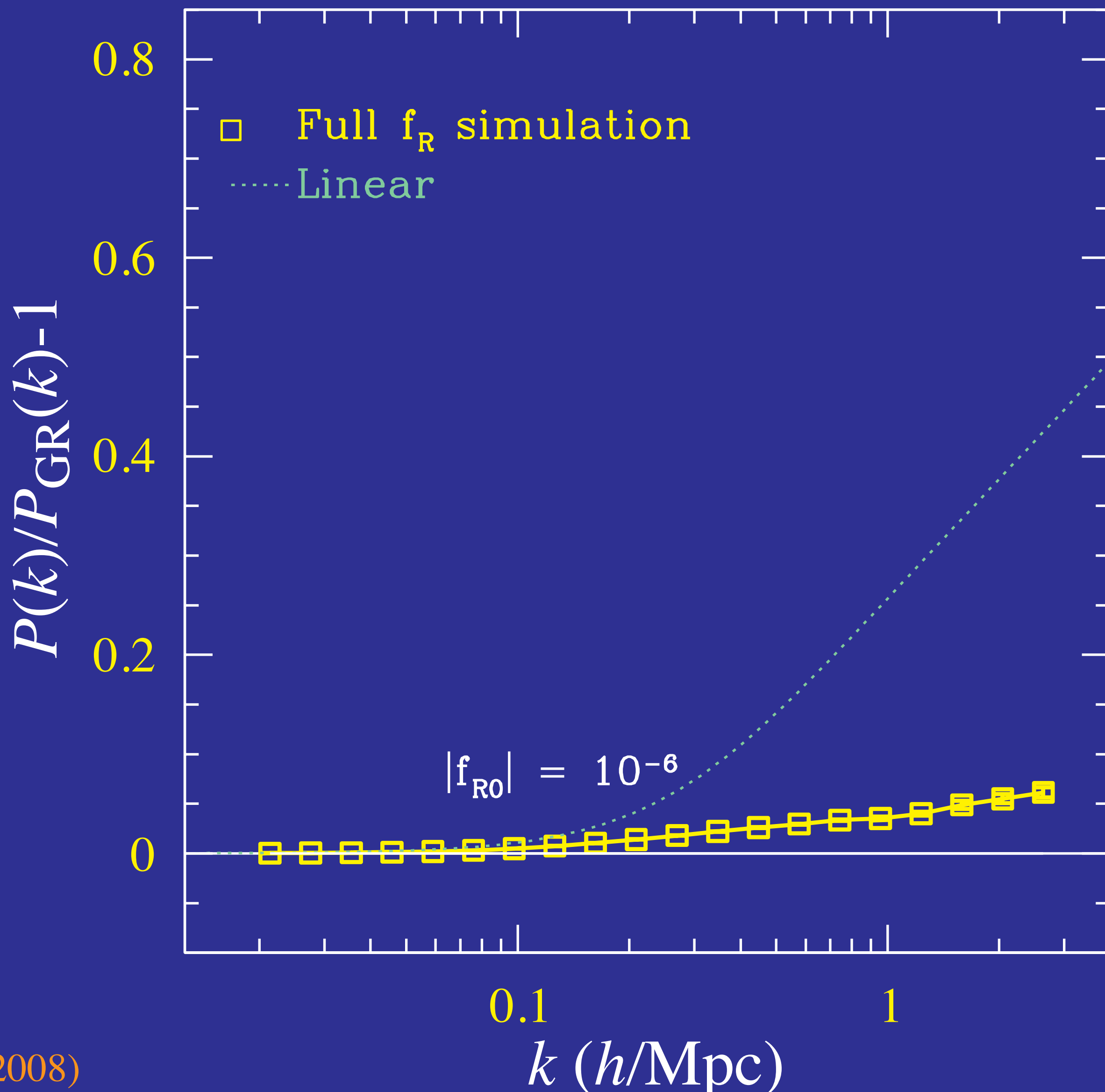
- For large background field, gradients in the scalar prevent the chameleon from appearing



Oyaizu, Lima, Hu (2008)

N-body Power Spectrum

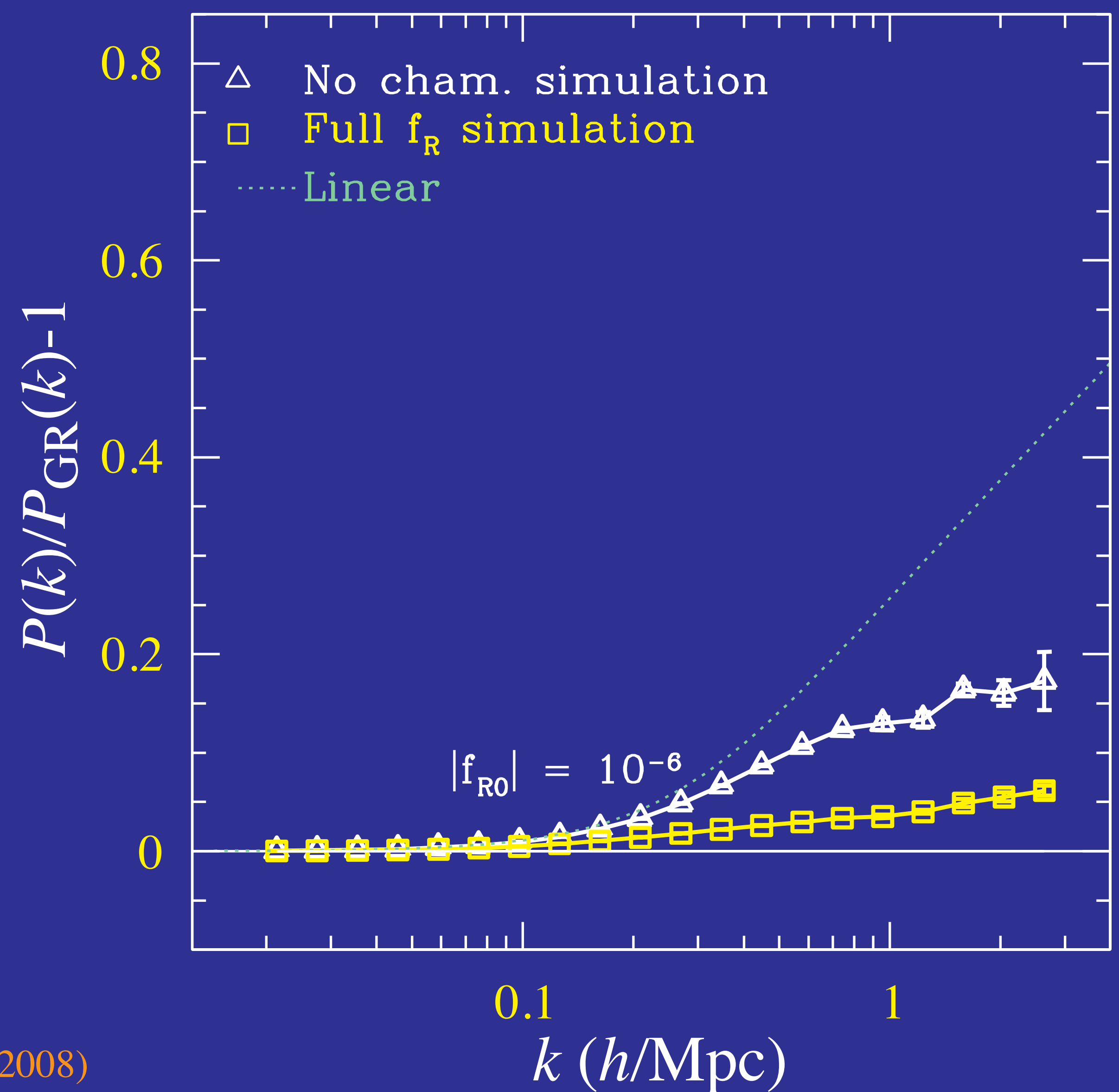
- 512^3 PM-relaxation code resolves the chameleon transition to GR: greatly reduced non-linear effect



Oyaizu, Lima, Hu (2008)

N-body Power Spectrum

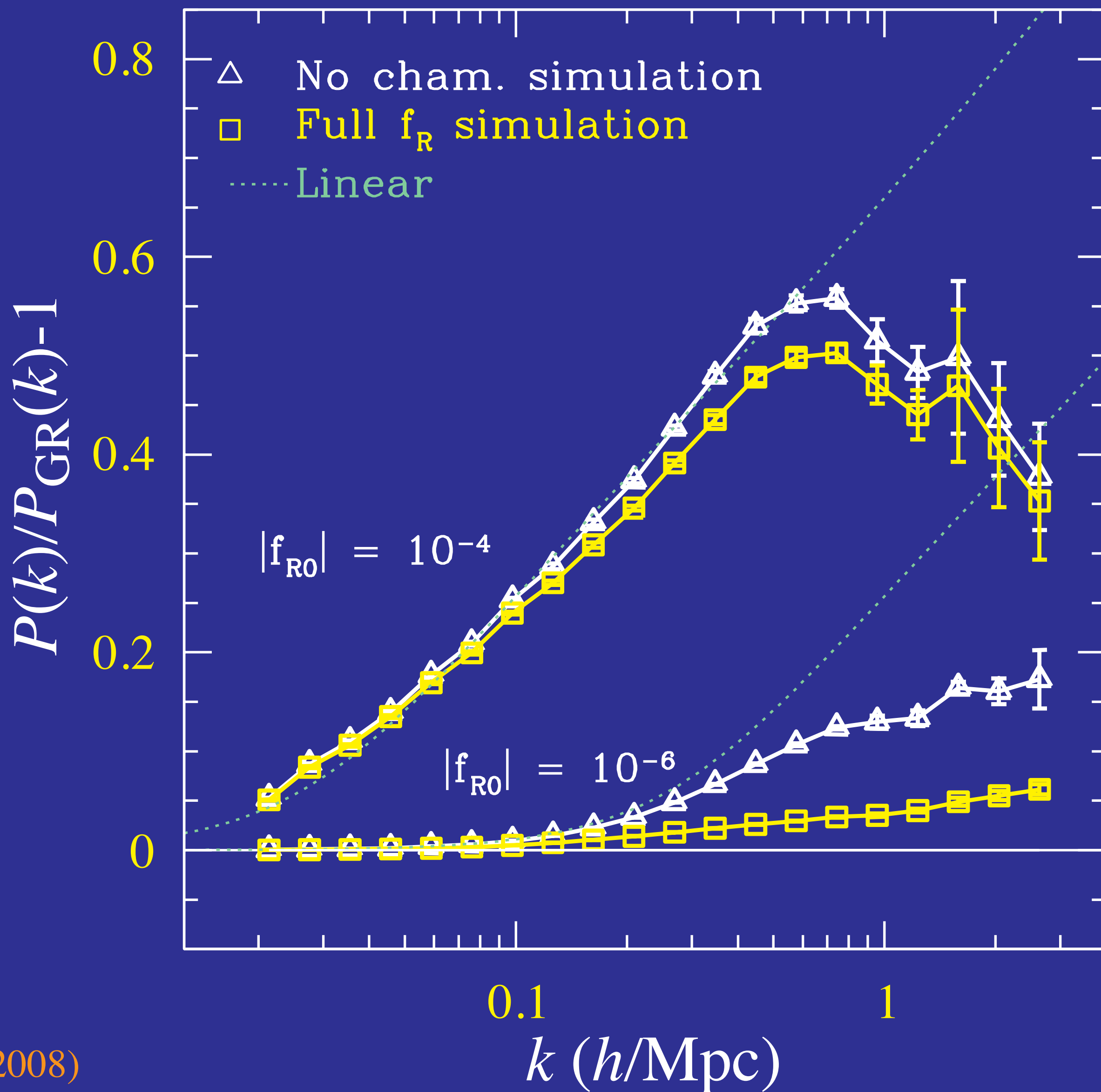
- Artificially turning off the chameleon mechanism restores much of enhancement



Oyaizu, Lima, Hu (2008)

N-body Power Spectrum

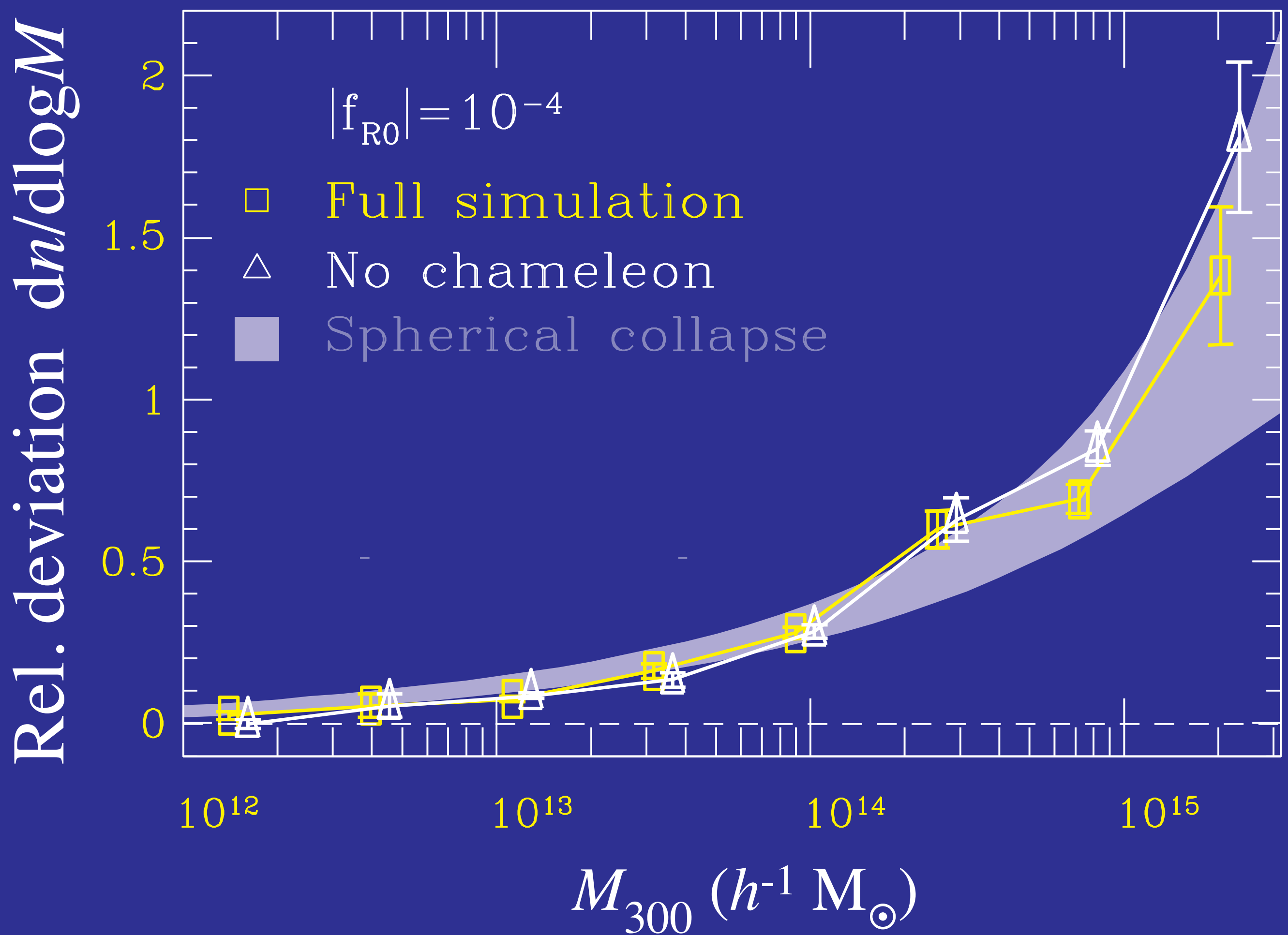
- Models where the chameleon absent today (large field models) show residual effects from a high redshift chameleon



Oyaizu, Lima, Hu (2008)

Cluster Abundance

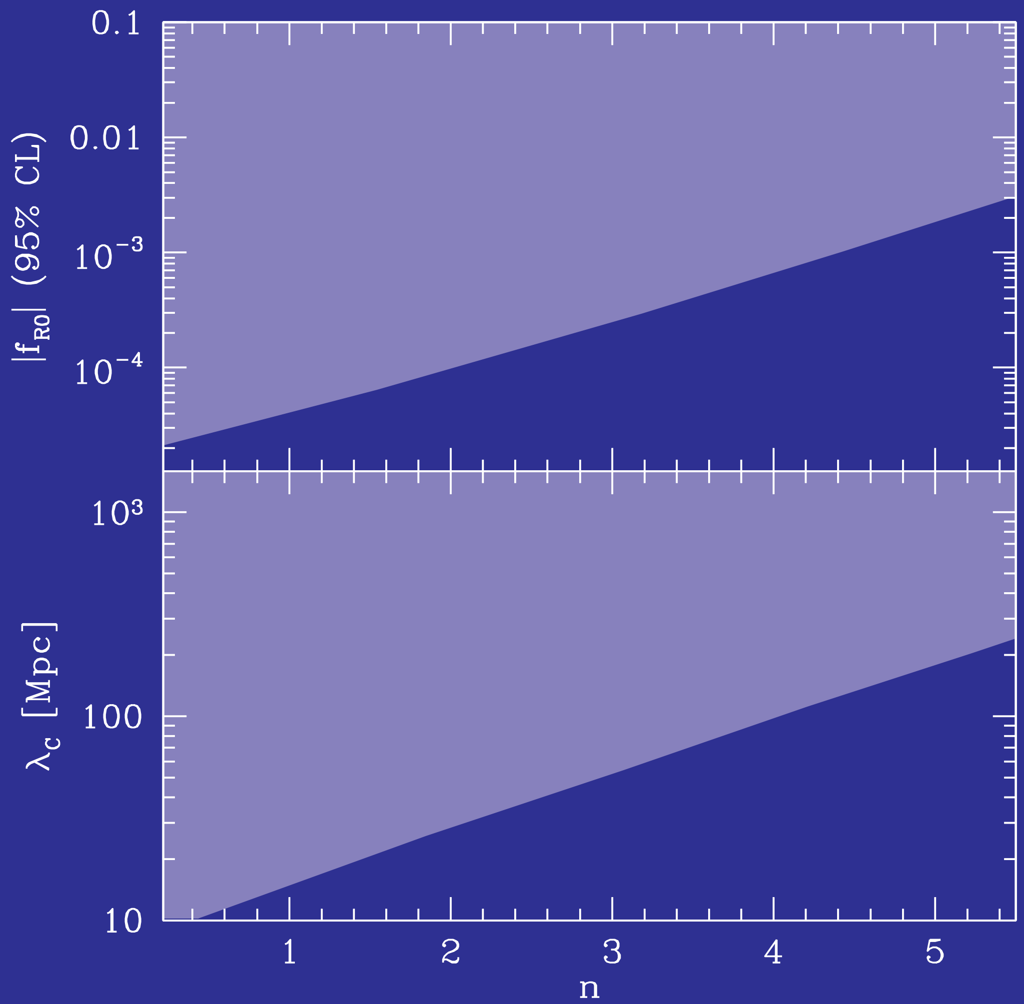
- Enhanced abundance of rare dark matter halos (clusters) with extra force



Lima, Schmidt, Oyaizu, Hu (2008)

Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index n



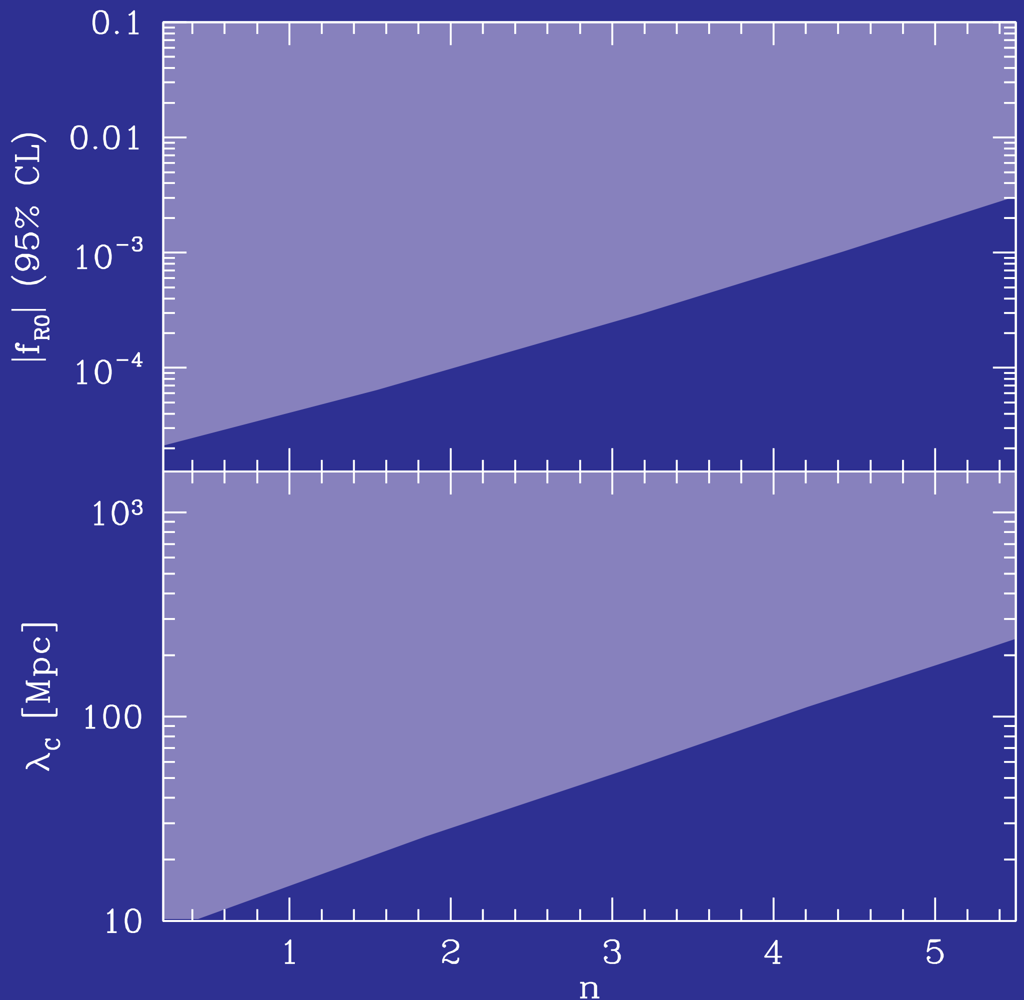
field amplitude

range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Cluster $f(R)$ Constraints

- Approaching competitiveness with solar system + Galaxy constraints of few 10^{-6} at low n
- Vastly different scale



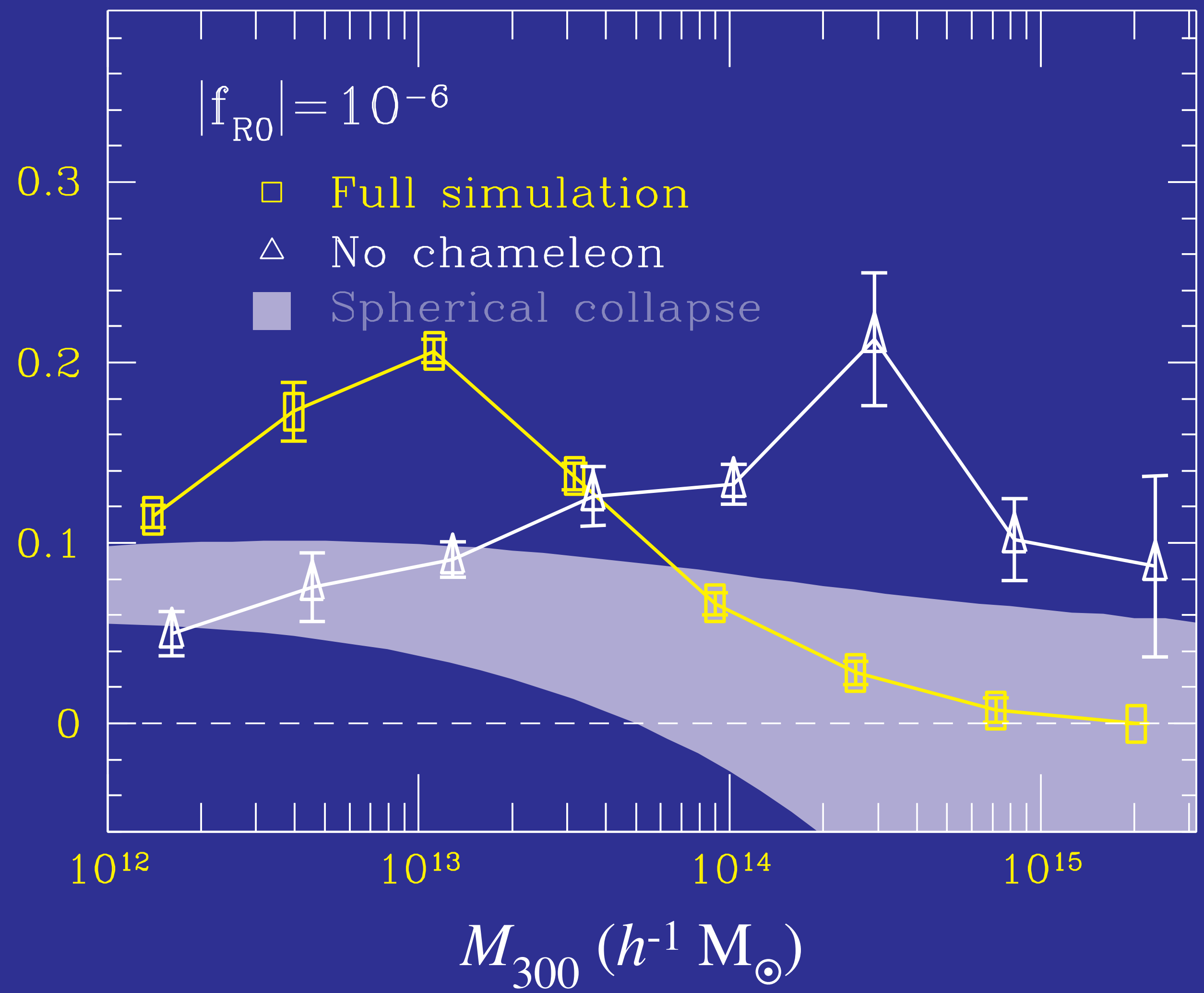
field amplitude

range of force

Schmidt, Vikhlinin, Hu (2009); Ferraro, Schmidt, Hu (2010)

Chameleon Mass Function

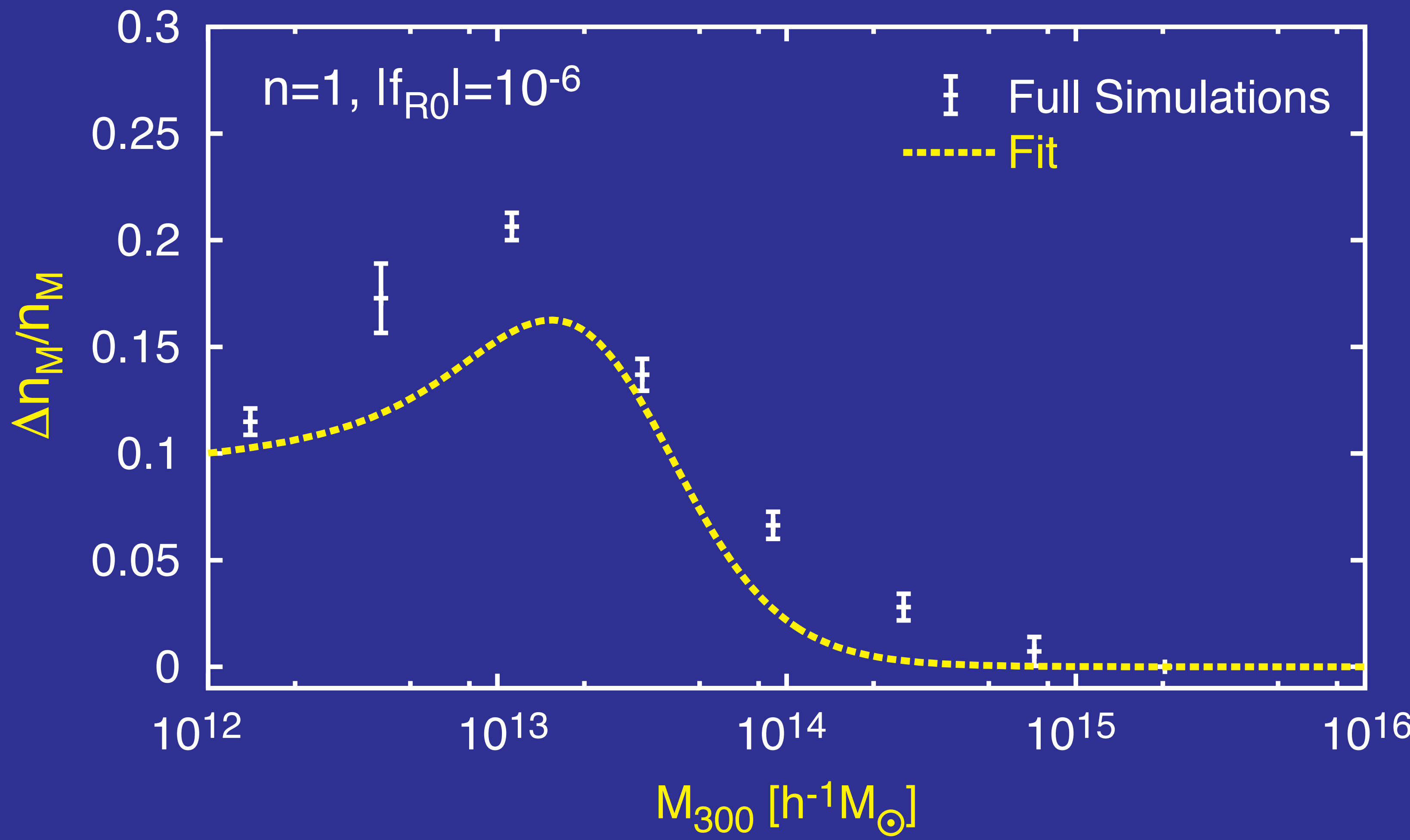
- Chameleon effect suppresses the enhancement at high masses
- Pile up of abundance at intermediate group scale



Lima, Schmidt, Oyaizu, Hu (2008)

Chameleon Mass Function

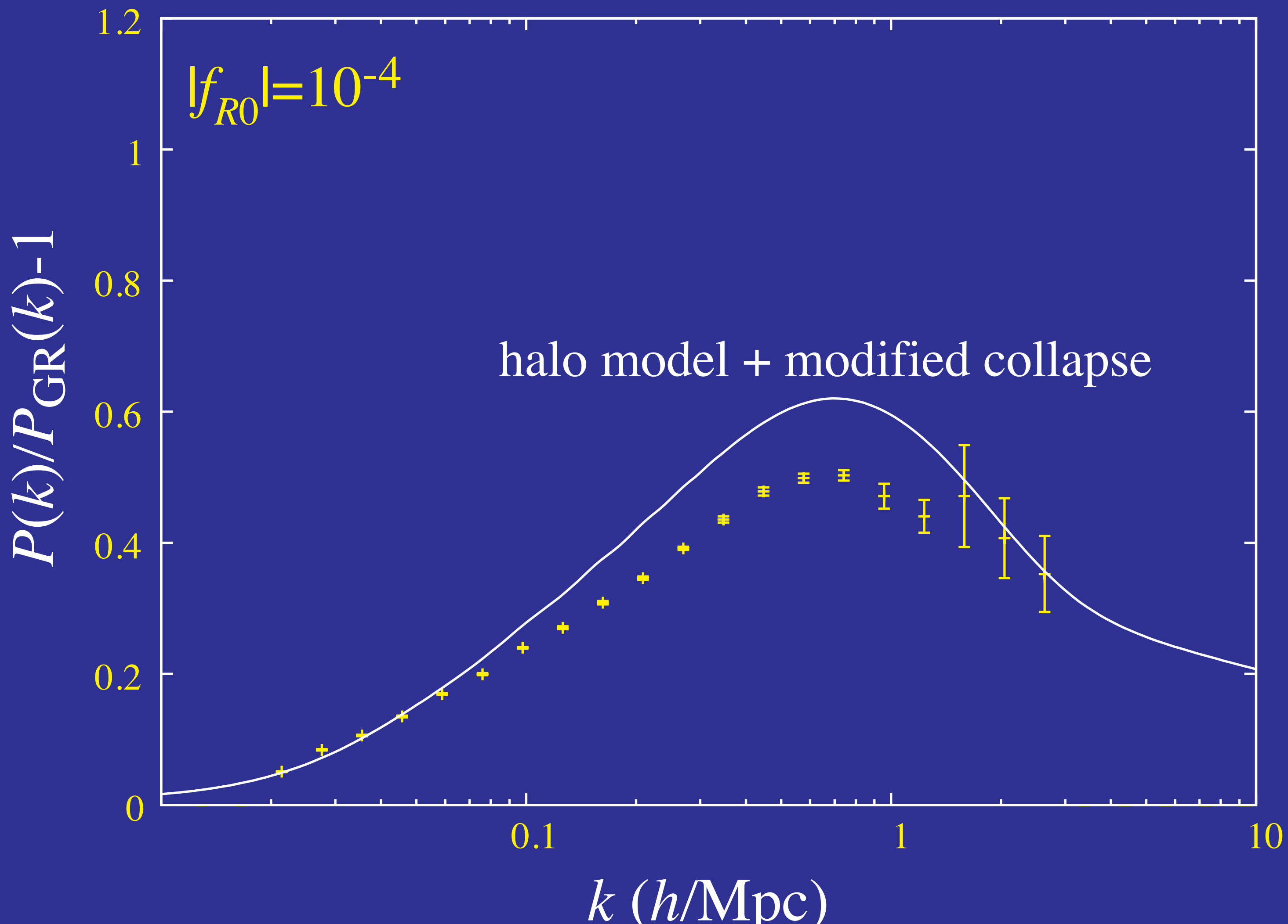
- Simple single parameter extention covers variety of models
- Basis of a halo model based post Friedmann parameterization of chameleon



Li & Hu (2011)

Halo Model

- Power spectrum trends also consistent with halos and modified collapse



Schmidt, Lima, Oyaizu, Hu (2008)

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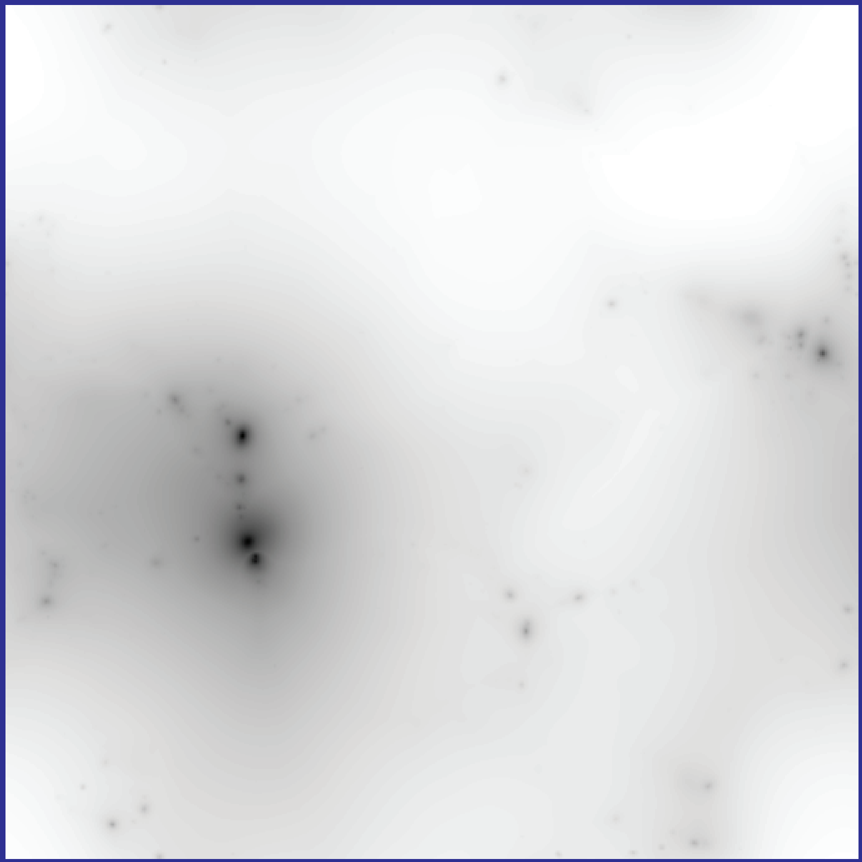
a nonlinear function of second derivatives of the field

Linked to density fluctuation - Galileon invariance - no self-shielding of external forces

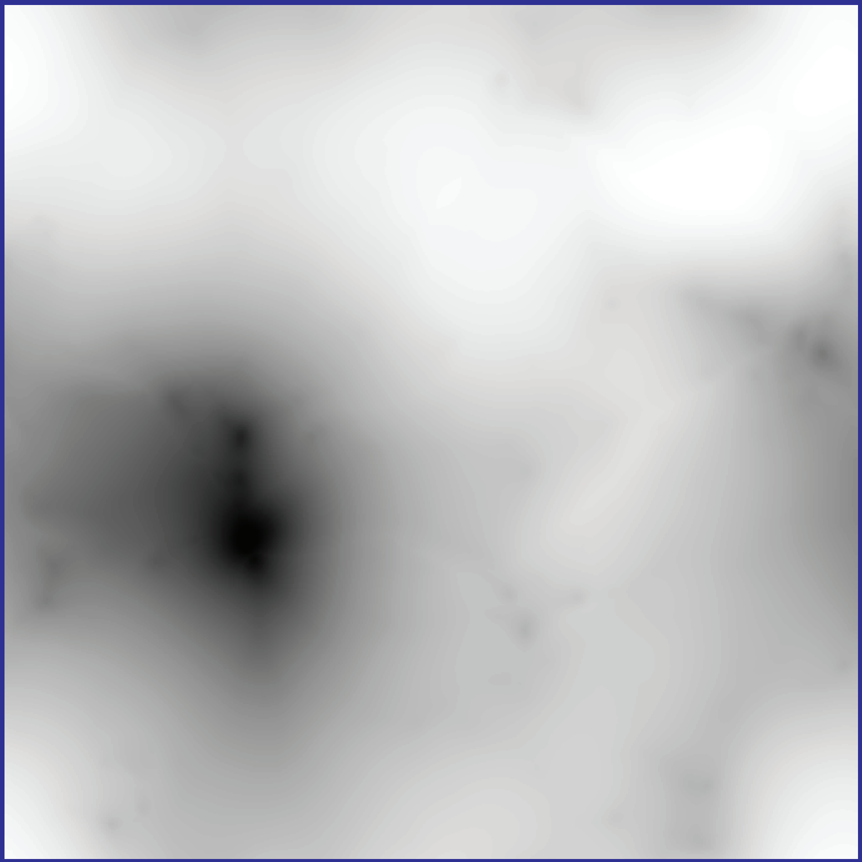
DGP N-Body

- DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism

Newtonian Potential



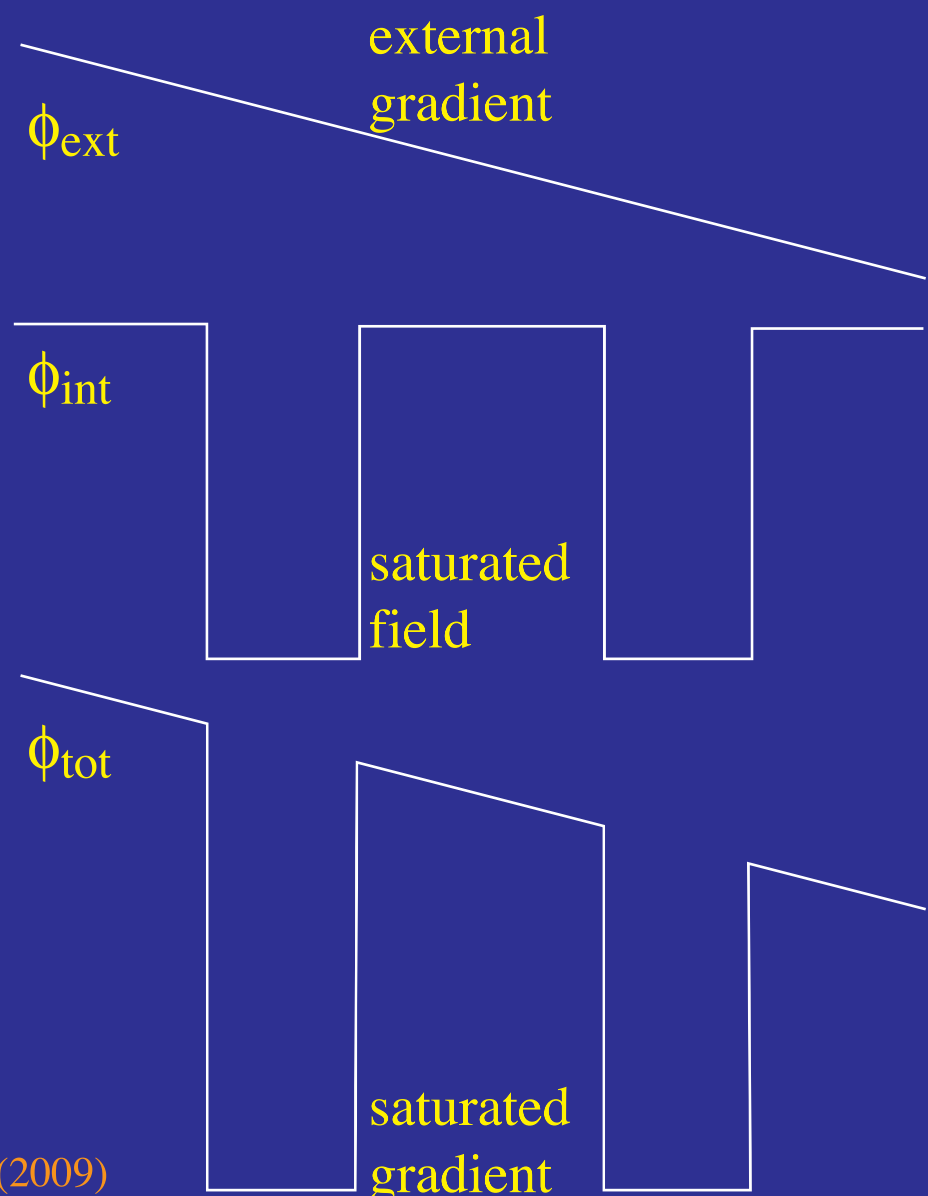
Brane Bending Mode



Schmidt (2009); Chan & Scoccimarro (2009) (cf. Khoury & Wyman 2009)

Apparent Equivalence Prinicple Violation

- Self-field of a "test mass" can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)



Hui, Nicolis, Stubbs (2009); Hu (2009)

Summary

- Lessons from the $f(R)$ and DGP worked examples – 3 regimes:
  - large scales: conservation determined
  - intermediate scales: scalar-tensor
  - small scales: GR in high density regions, modified in low
- Large scales: expansion history and metric ratio $g = (\Phi + \Psi)/(\Phi - \Psi)$ through curvature conservation
- Intermediate scales: scalar tensor modified Newtonian regime, g and Poisson equation
- Small scales: nonlinear interaction of modification field makes g depend on local environment (not scale) - density or potential - suppressing deviations
- N-body (PM-relaxation) simulations show halo model framework can describe observables in the nonlinear regime