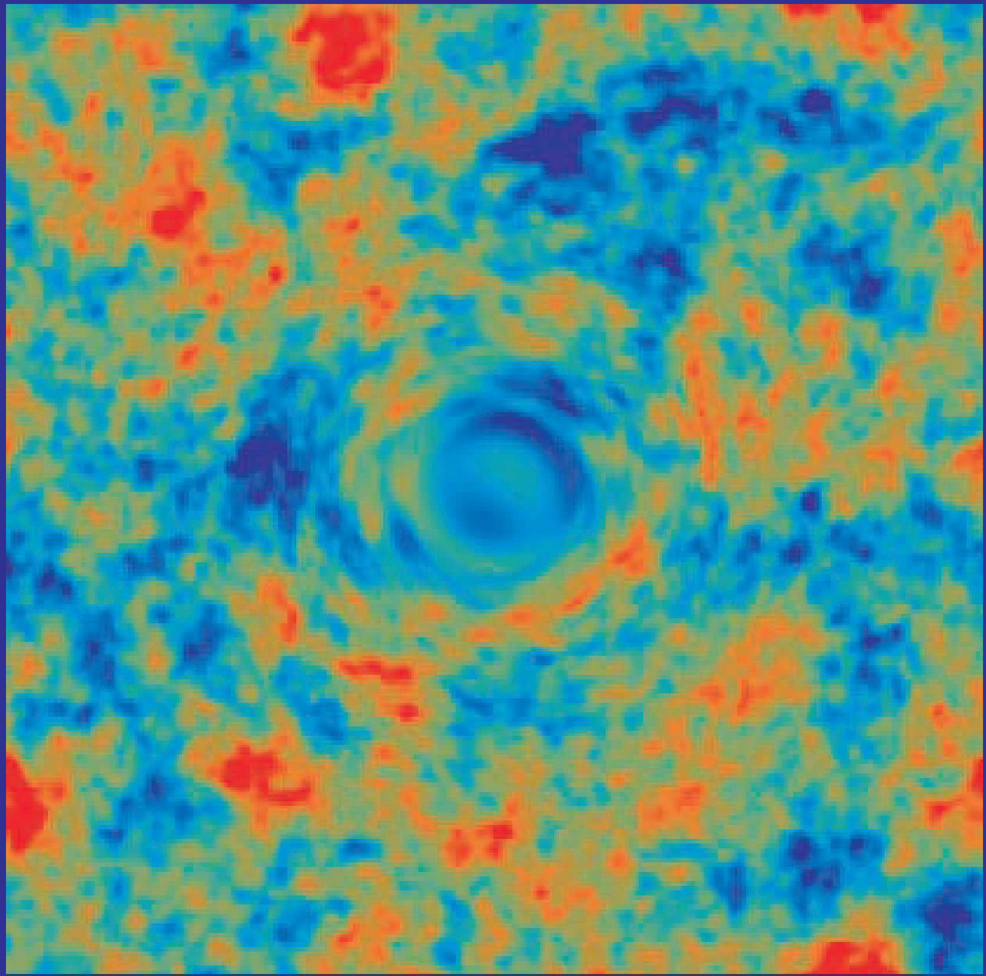
Gravitational Secondaries



in the CMB

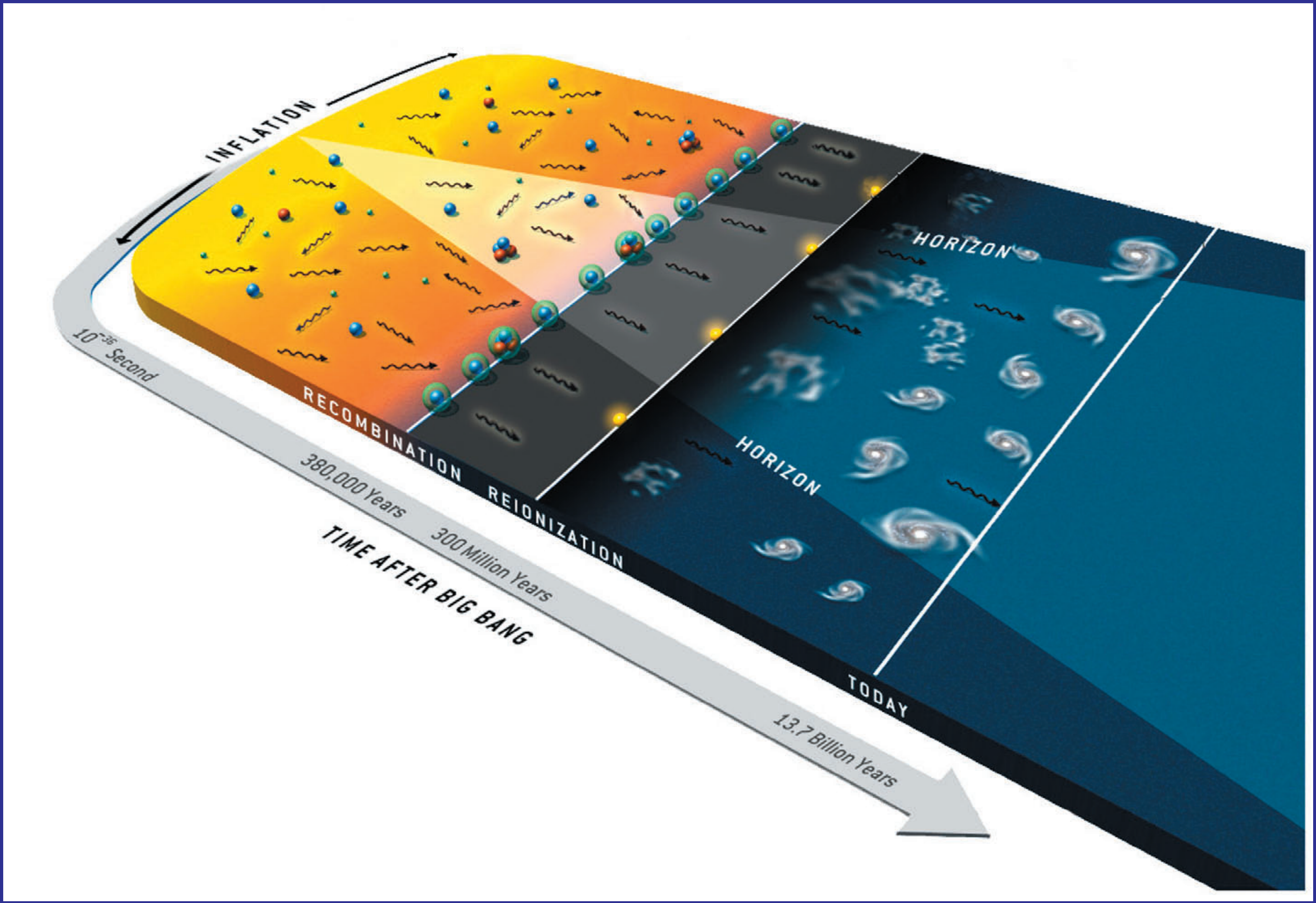
Wayne Hu

KIAA, June 2011

Gravitational Secondaries

- CMB secondary anisotropy: temperature and polarization anisotropy generated after recombination
- Gravitational Lensing:
  - Temperature power spectrum
  - Polarization power spectrum and B-modes
  - Mass reconstruction
- Gravitational Redshift:
  - Integrated Sachs-Wolfe effect
  - Dark energy smoothness
  - Testing gravity

Across the Horizon

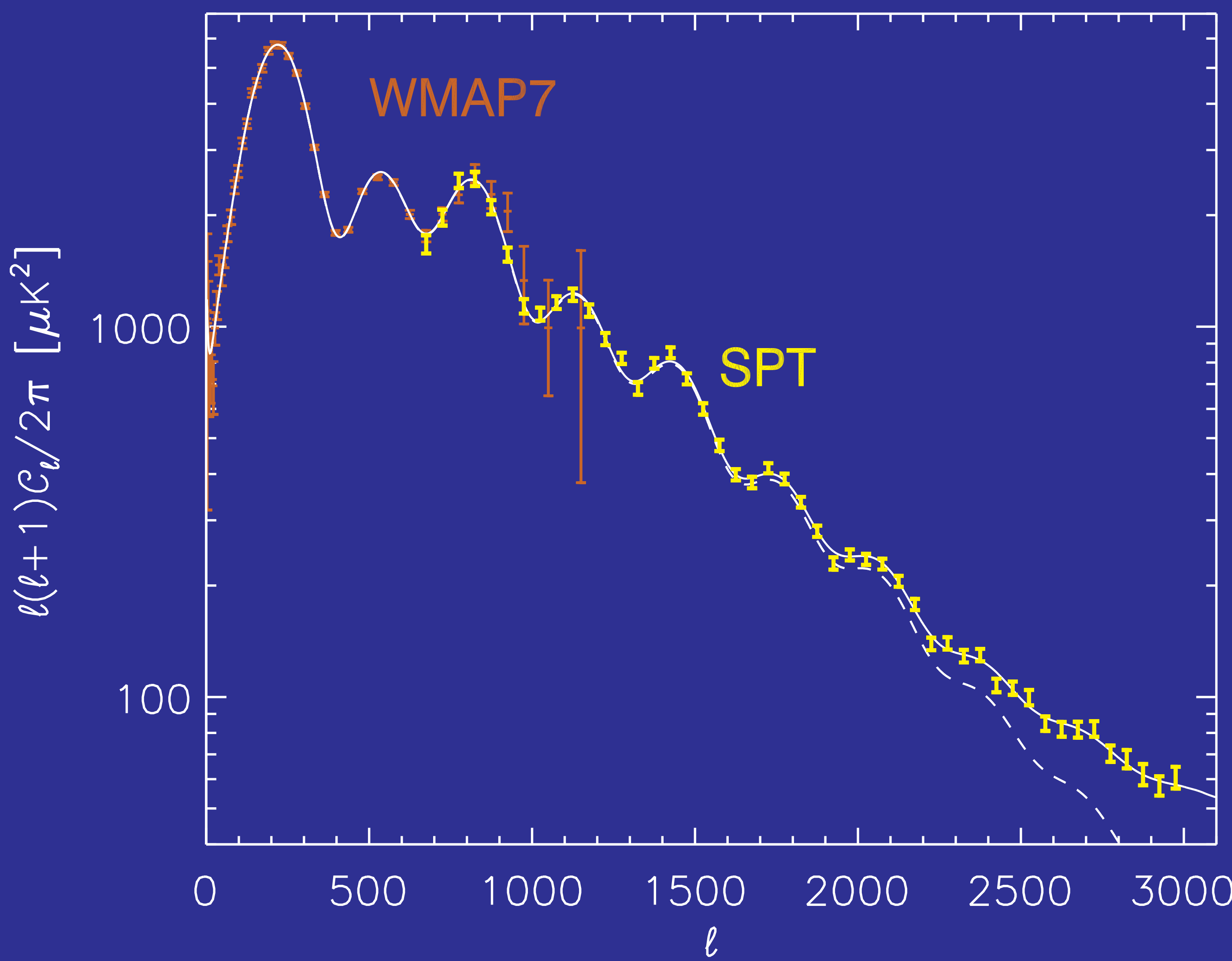


Hu & White (2004); artist:B. Christie/SciAm; available at http://background.uchicago.edu

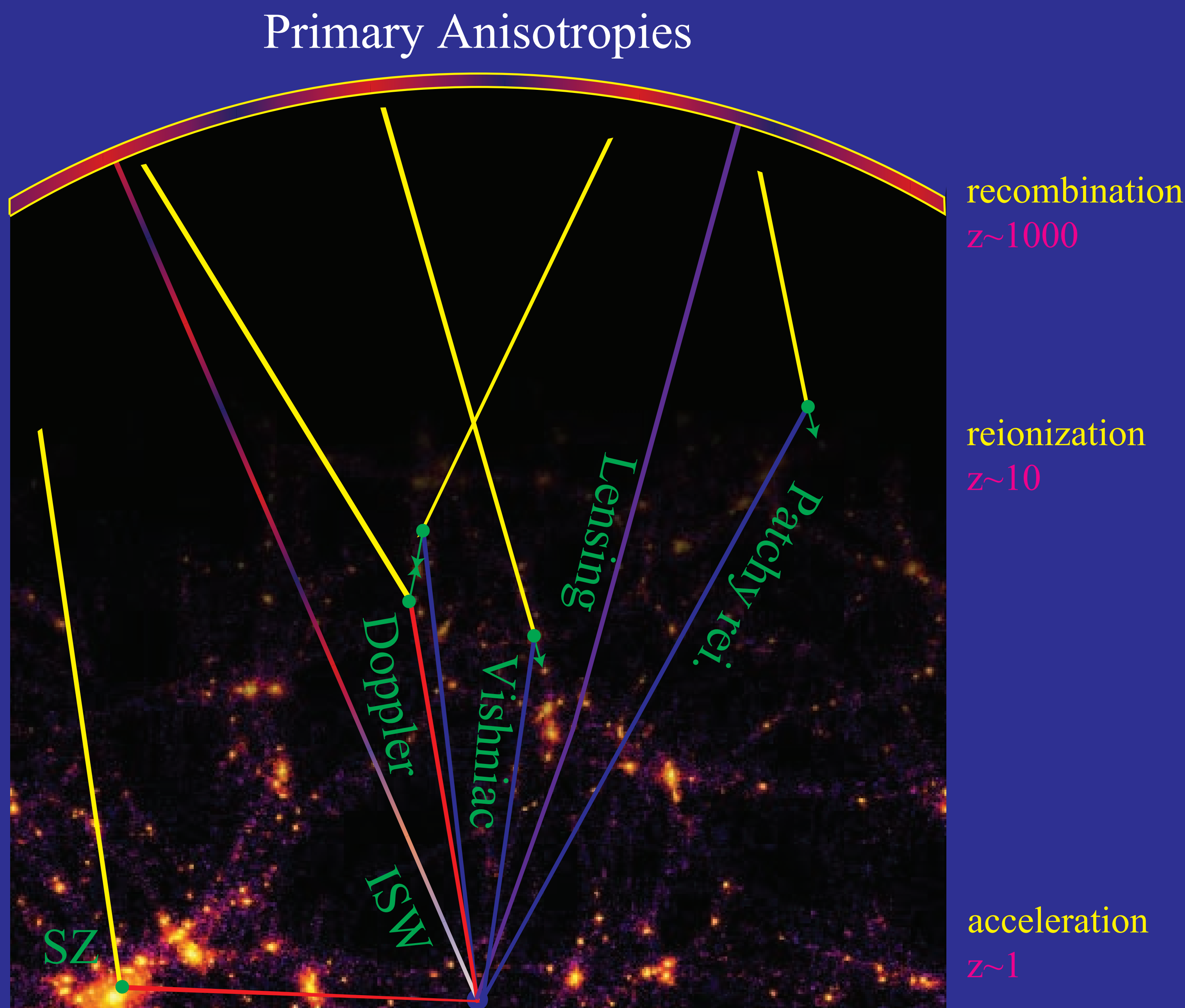
Primary CMB Anisotropy

- Exceedingly well-observed; 7-8 acoustic oscillations in temperature

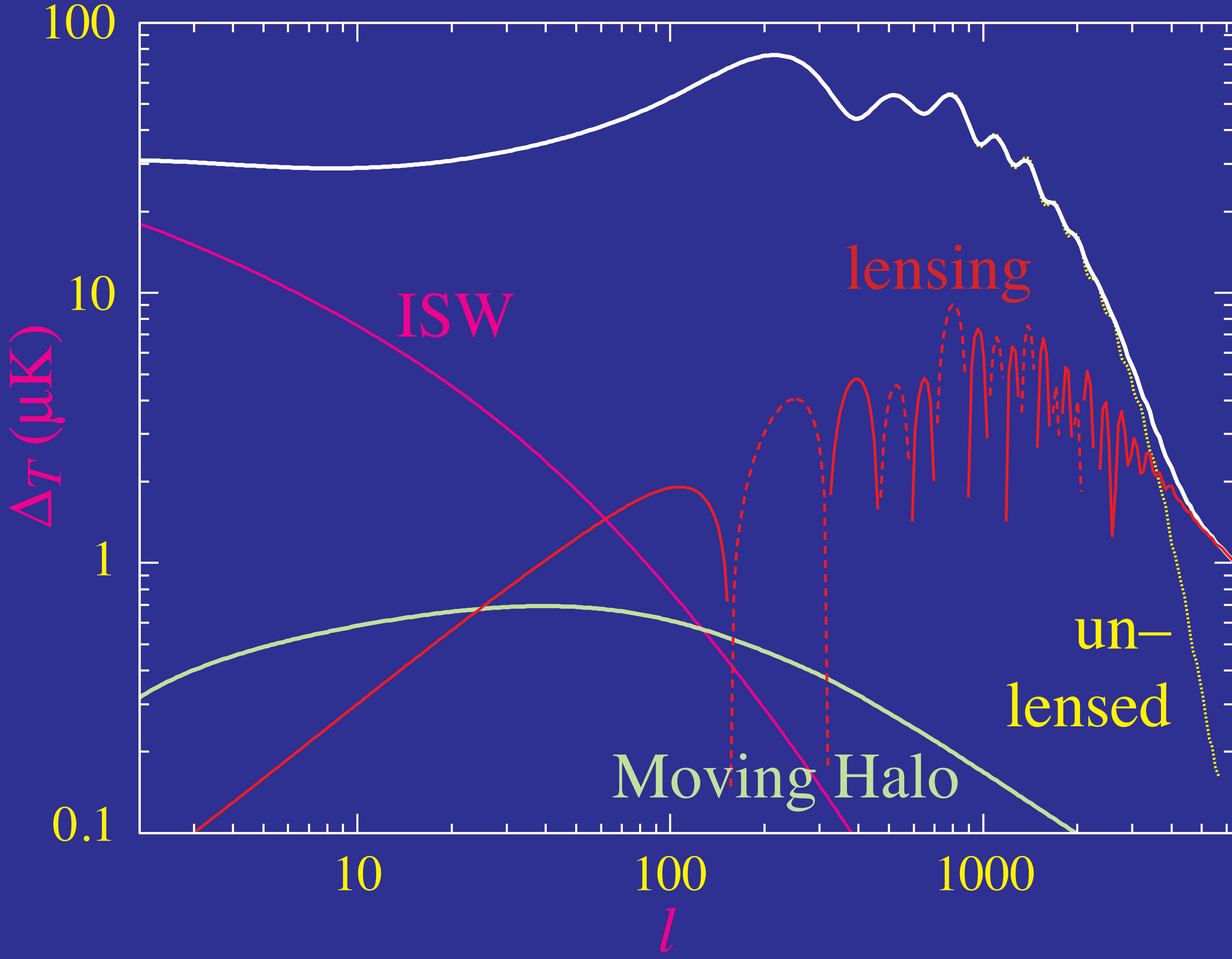
SPT - Keisler et al (2011)



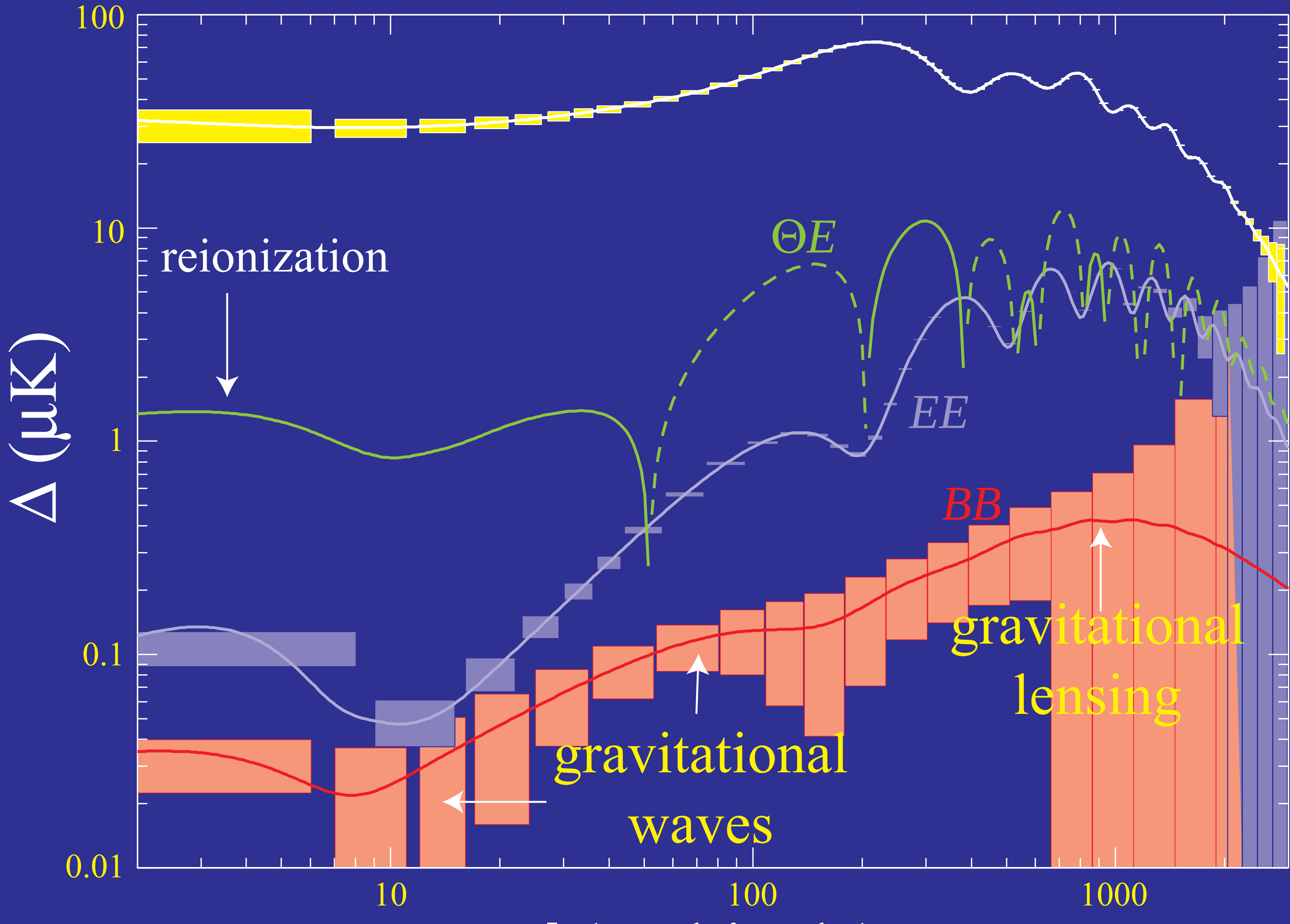
Physics of Secondary Anisotropies



Gravitational Secondaries



Polarized Landscape



Hu & Dodelson (2002)

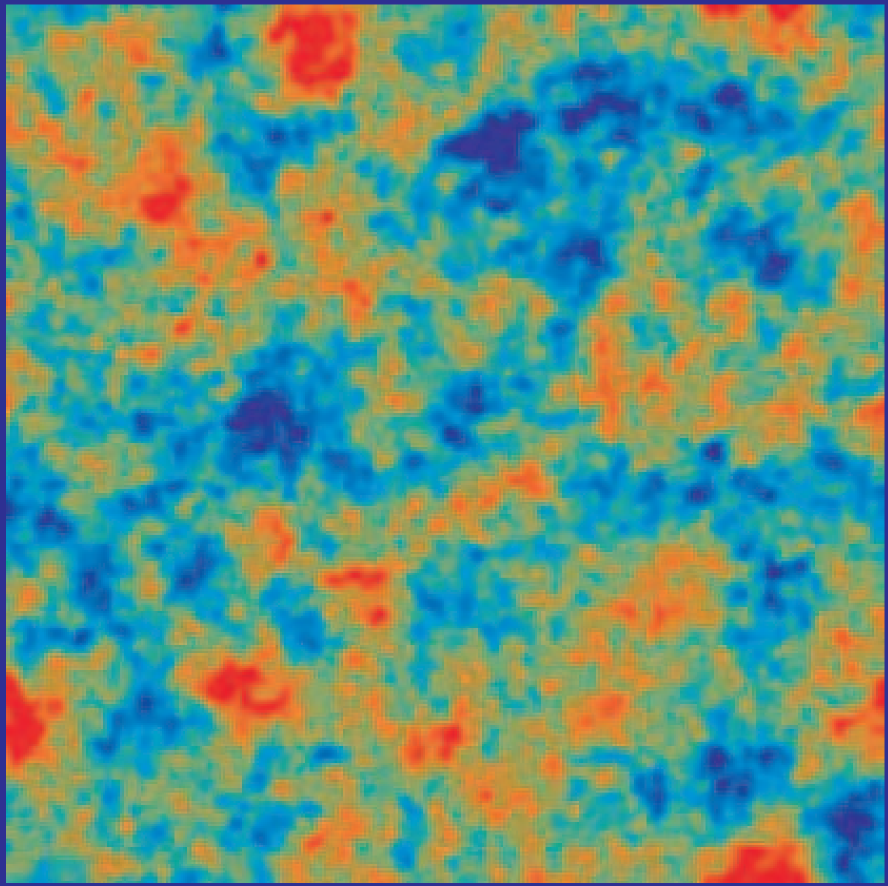
Gravitational Lensing

Example of CMB Lensing

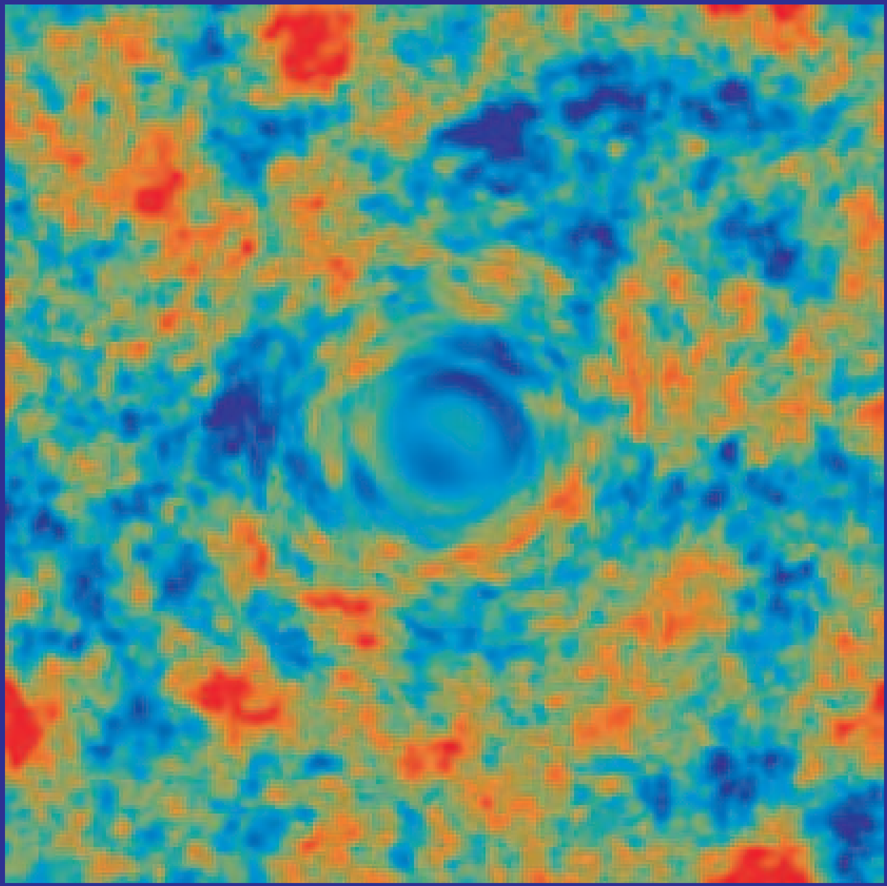
- Toy example of lensing of the CMB primary anisotropies
- Shearing of the image

Gravitational Lensing

- Gravitational lensing by large scale structure distorts the observed temperature and polarization fields
- Exaggerated example for the temperature



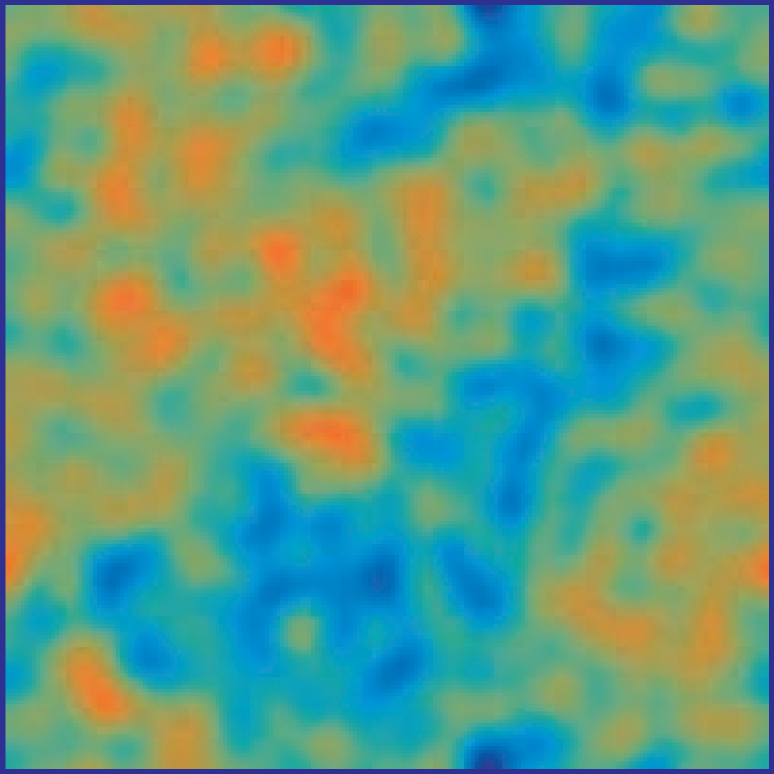
Original



Lensed

Lensing by a Gaussian Random Field

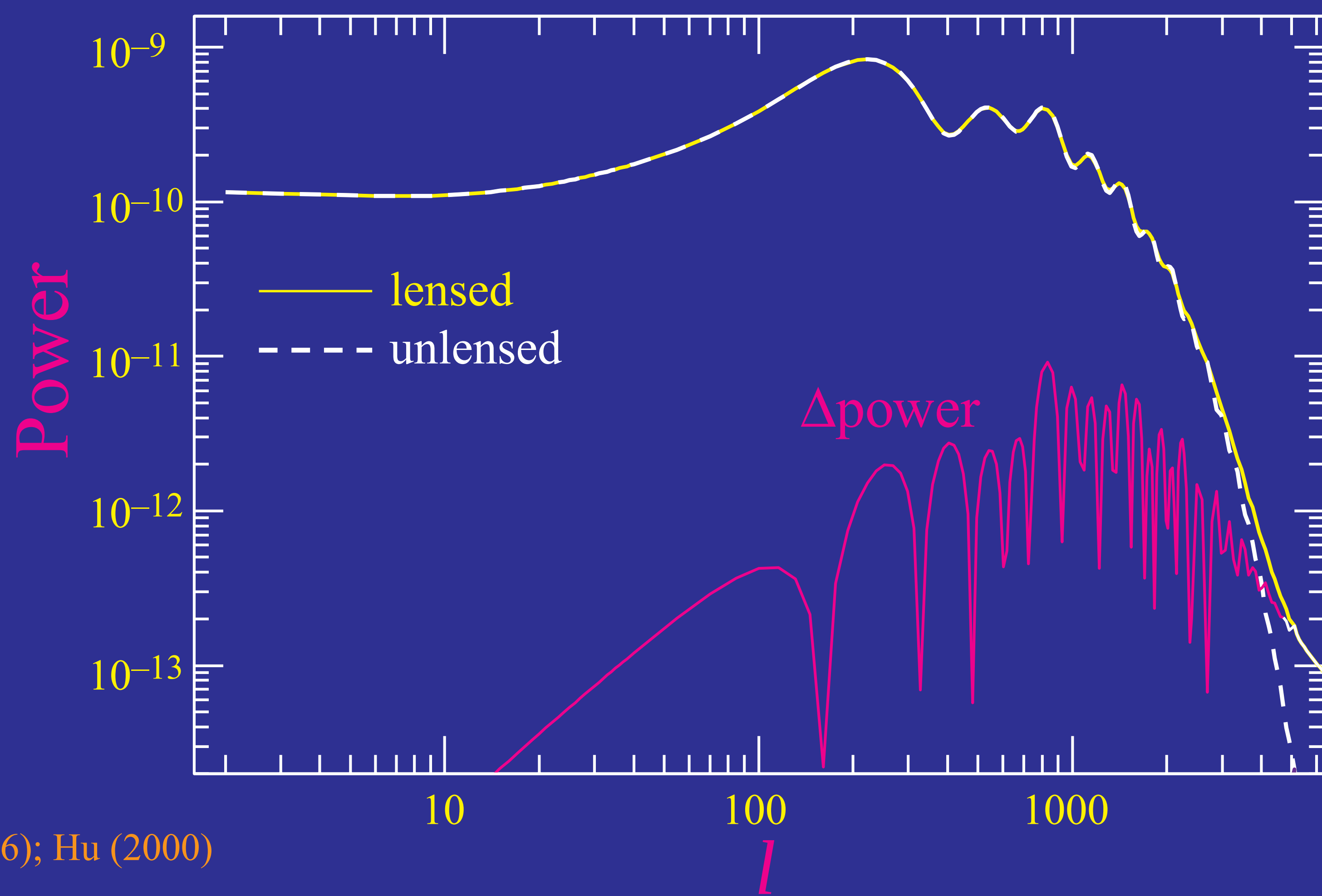
- Mass distribution at large angles and high redshift in in the linear regime
- Projected mass distribution (low pass filtered reflecting deflection angles): 1000 sq. deg



rms deflection
2.6'
deflection coherence
10°

Lensing in the Power Spectrum

- Lensing smooths the power spectrum with a width $\Delta l \sim 60$
- Convolution with specific kernel: higher order correlations between multipole moments – not apparent in power



Seljak (1996); Hu (2000)

Gravitational Lensing

- Lensing is a surface brightness conserving remapping of source to image planes by the gradient of the projected potential

$$\phi(\hat{\mathbf{n}}) = 2 \int_{\eta_*}^{\eta_0} d\eta \, \frac{(D_* - D)}{D\, D_*} \Phi(D\hat{\mathbf{n}}, \eta) \,.$$

  such that the fields are remapped as

$$x(\hat{\mathbf{n}}) \rightarrow x(\hat{\mathbf{n}} + \nabla\phi) \,,$$

  where $x \in \{\Theta, Q, U\}$ temperature and polarization.

- Taylor expansion leads to product of fields and Fourier mode-coupling

Flat-sky Treatment

- Taylor expand

$$\Theta(\hat{\mathbf{n}}) = \tilde{\Theta}(\hat{\mathbf{n}} + \nabla\phi)$$
$$= \tilde{\Theta}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{\Theta}(\hat{\mathbf{n}}) + \frac{1}{2}\nabla_i\phi(\hat{\mathbf{n}})\nabla_j\phi(\hat{\mathbf{n}})\nabla^i\nabla^j\tilde{\Theta}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition

$$\phi(\hat{\mathbf{n}}) = \int \frac{d^2l}{(2\pi)^2}\phi(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$
$$\tilde{\Theta}(\hat{\mathbf{n}}) = \int \frac{d^2l}{(2\pi)^2}\tilde{\Theta}(\mathbf{l})e^{i\mathbf{l}\cdot\hat{\mathbf{n}}}$$

Flat-sky Treatment

- Mode coupling of harmonics

$$\Theta(\mathbf{l}) = \int d\hat{\mathbf{n}}\, \Theta(\hat{\mathbf{n}}) e^{-il\cdot\hat{\mathbf{n}}}$$
$$= \tilde{\Theta}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} \tilde{\Theta}(\mathbf{l}_1) L(\mathbf{l}, \mathbf{l}_1)\,,$$

where

$$L(\mathbf{l}, \mathbf{l}_1) = \phi(\mathbf{l} - \mathbf{l}_1)\,(\mathbf{l} - \mathbf{l}_1)\cdot \mathbf{l}_1$$
$$+ \frac{1}{2}\int \frac{d^2\mathbf{l}_2}{(2\pi)^2} \phi(\mathbf{l}_2)\phi^*(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\,(\mathbf{l}_2\cdot \mathbf{l}_1)(\mathbf{l}_2 + \mathbf{l}_1 - \mathbf{l})\cdot \mathbf{l}_1\,.$$

- Represents a coupling of harmonics separated by $L \approx 60$ peak of deflection power

Power Spectrum

- Power spectra

$$\langle \Theta^*(\mathbf{l})\Theta(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\, C_l^{\Theta\Theta}\,,$$

$$\langle \phi^*(\mathbf{l})\phi(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')\, C_l^{\phi\phi}\,,$$

becomes

$$C_l^{\Theta\Theta} = \left(1 - l^2 R\right)\tilde{C}_l^{\Theta\Theta} + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}\tilde{C}_{|\mathbf{l}-\mathbf{l}_1|}^{\Theta\Theta} C_{l_1}^{\phi\phi}[(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1]^2\,,$$

where

$$R = \frac{1}{4\pi}\int \frac{dl}{l}\, l^4 C_l^{\phi\phi}\,.$$

Smoothing Power Spectrum

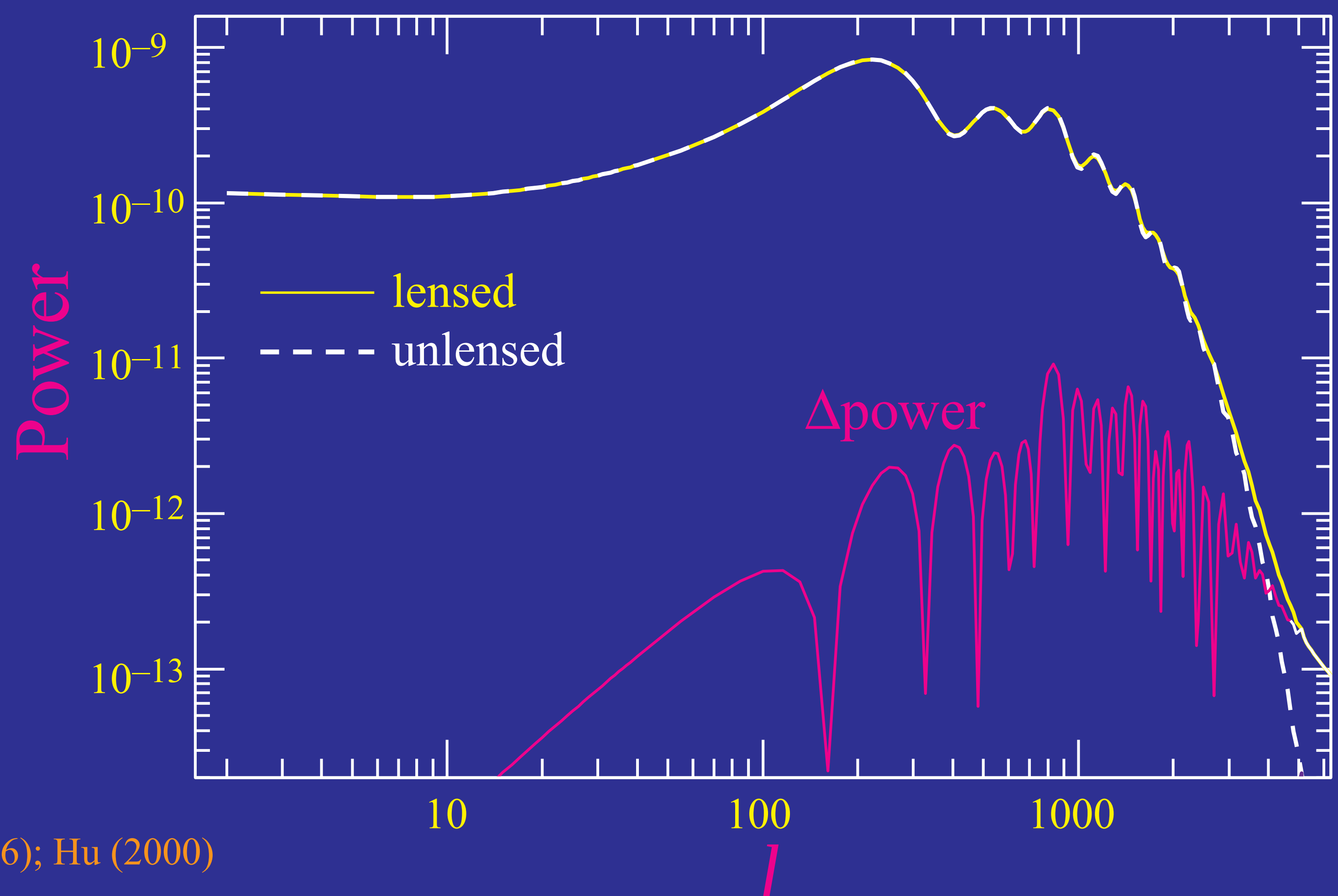
- If $\tilde{C}_l^{\Theta\Theta}$ slowly varying then two term cancel

$$\tilde{C}_l^{\Theta\Theta} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} C_l^{\phi\phi} (\mathbf{l} \cdot \mathbf{l}_1)^2 \approx l^2 R \tilde{C}_l^{\Theta\Theta} \, .$$

- So lensing acts to smooth features in the power spectrum. Smoothing kernel is $\Delta L \sim 60$ the peak of deflection power spectrum
- Because acoustic feature appear on a scale $l_A \sim 300$, smoothing is a subtle effect in the power spectrum.
- Lensing generates power below the damping scale which directly reflect power in deflections on the same scale

Lensing in the Power Spectrum

- Lensing smooths the power spectrum with a width $\Delta l \sim 60$
- Convolution with specific kernel: higher order correlations between multipole moments – not apparent in power



Seljak (1996); Hu (2000)

Generation of Power

- On scales below the damping scale, primary CMB looks like a smooth gradient
- Lensing effects modulate the gradient ($l_1 \ll l$):

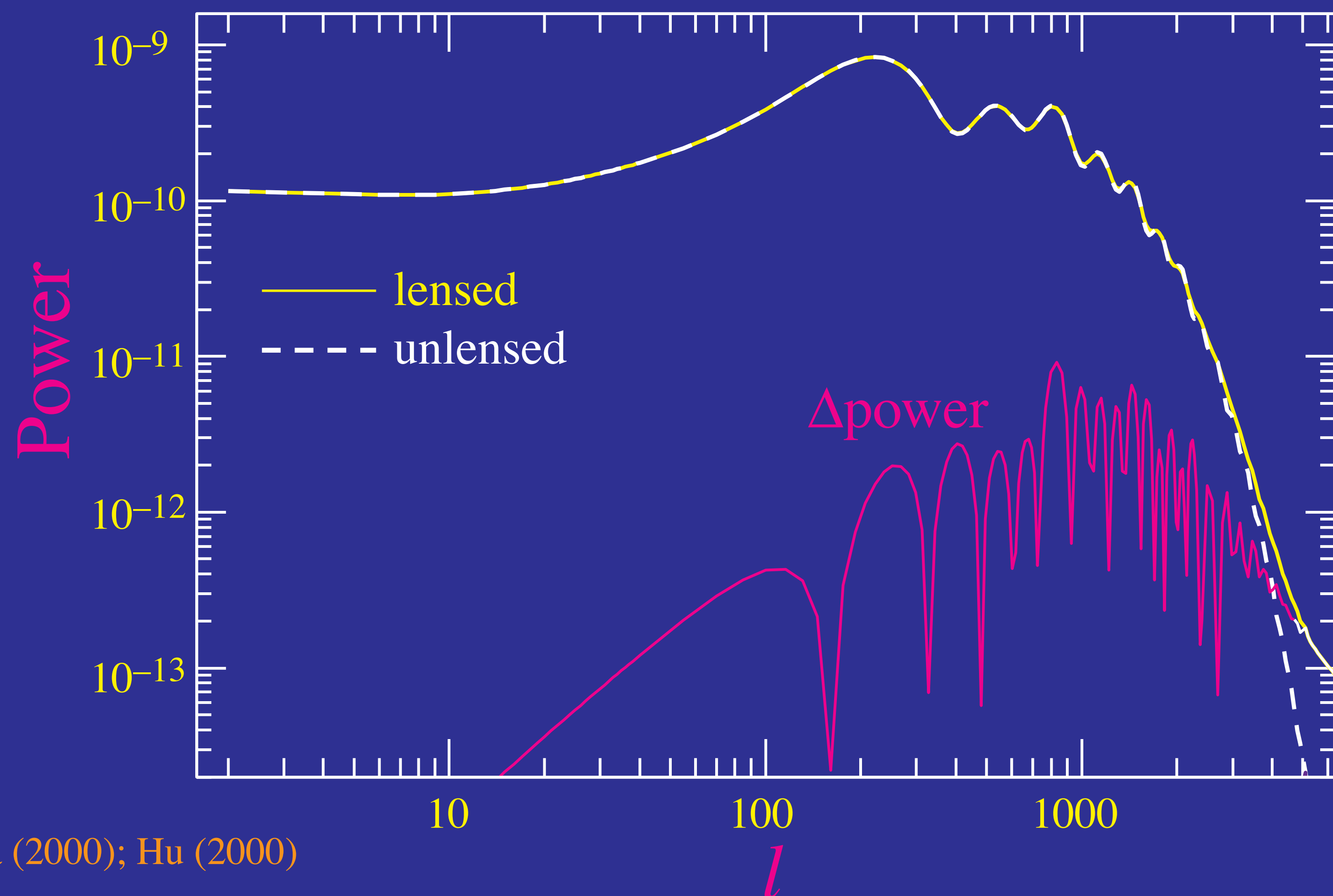
$$C_l^{\Theta\Theta} \approx \int \frac{d^2 \mathbf{l}_1}{(2\pi)^2} \tilde{C}_{l_1}^{\Theta\Theta} C_{|\mathbf{l}-\mathbf{l}_1|}^{\phi\phi} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2$$

$$\approx \frac{1}{2} l^2 C_l^{\phi\phi} \int \frac{d^2 \mathbf{l}_1}{(2\pi)^2} l_1^2 \tilde{C}_{l_1}^{\Theta\Theta}$$

and produce power on the same scale from power in the primary gradient (Zaldarriaga 2000)

Lensing in the Power Spectrum

- Small scale lenses modulate the large scale temperature field
- Generates power below damping scale from gradient power

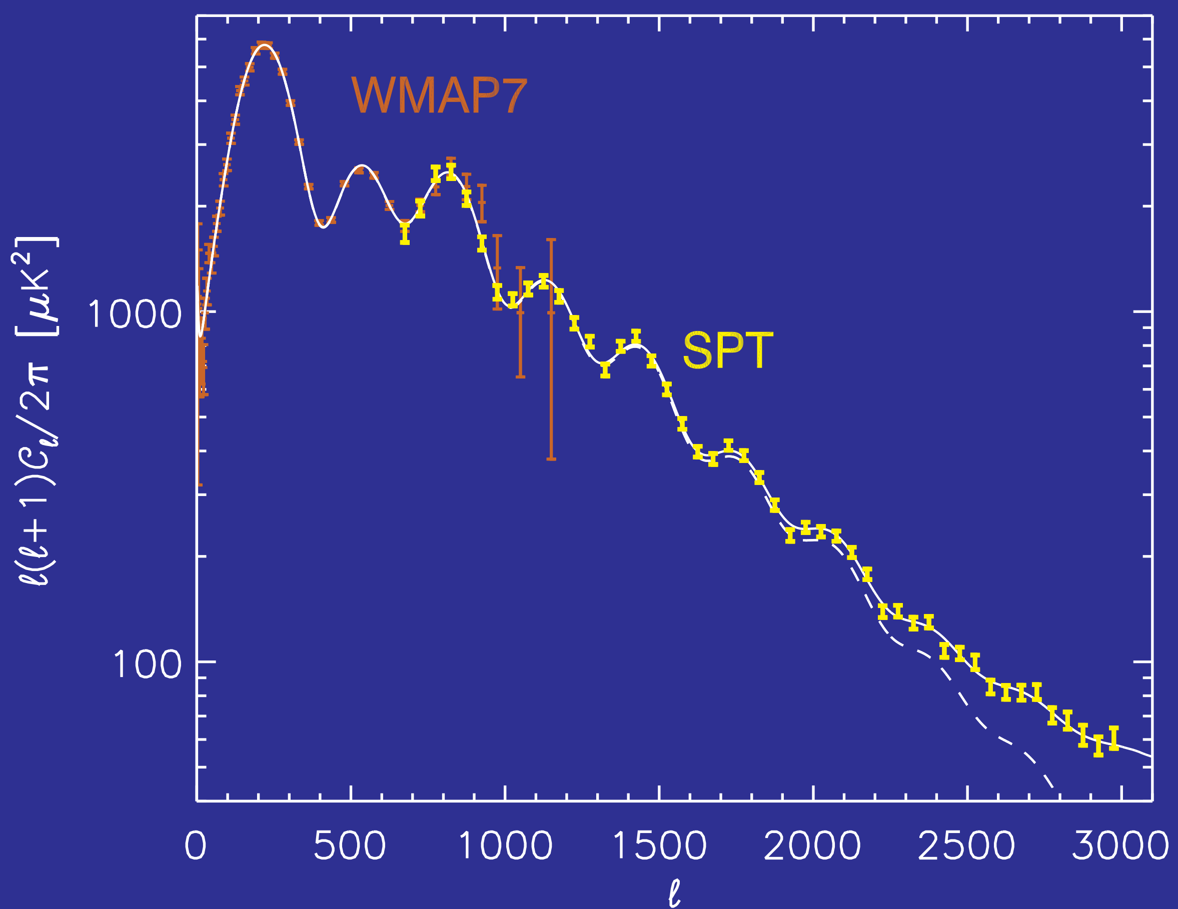


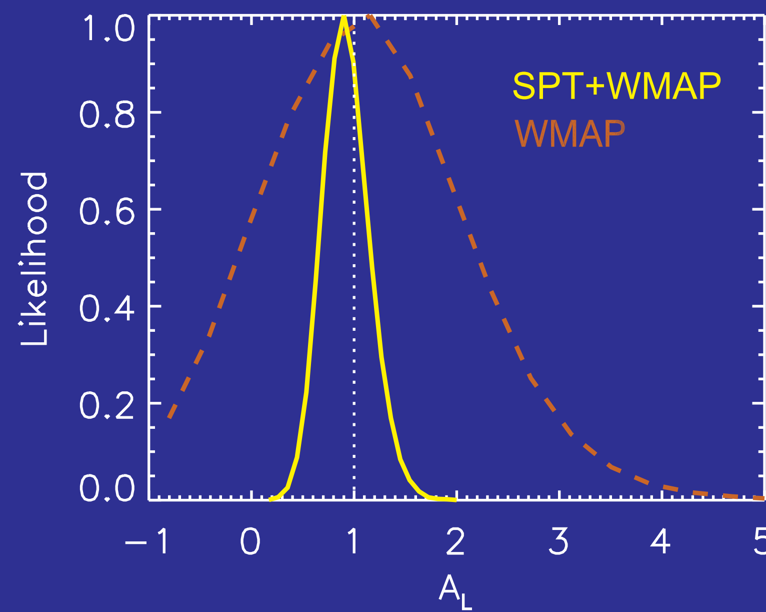
Zaldarriaga (2000); Hu (2000)

SPT Lensing Detection

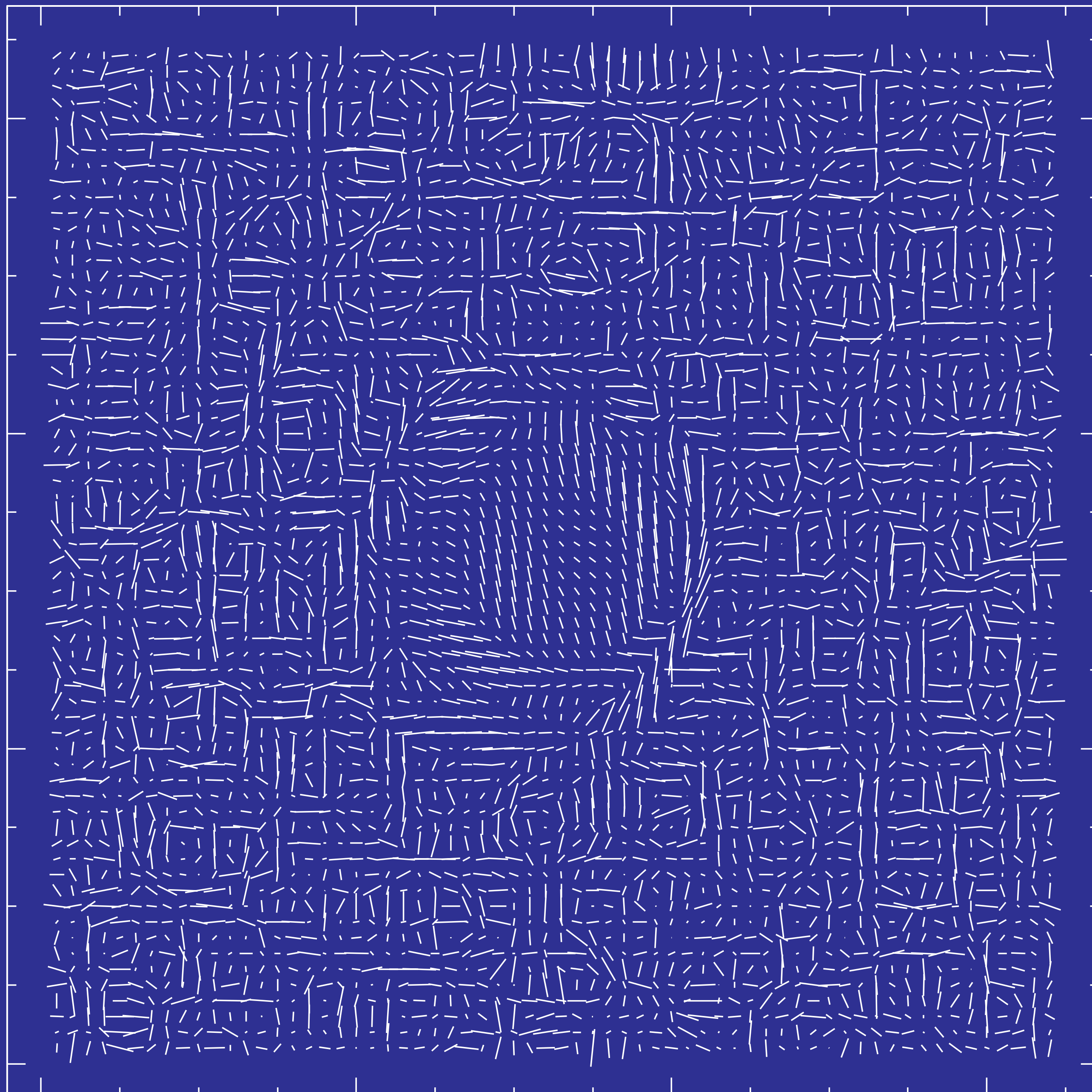
- SPT 4.9σ detection of lensing in the temperature power spectrum

Keisler et al (2011)





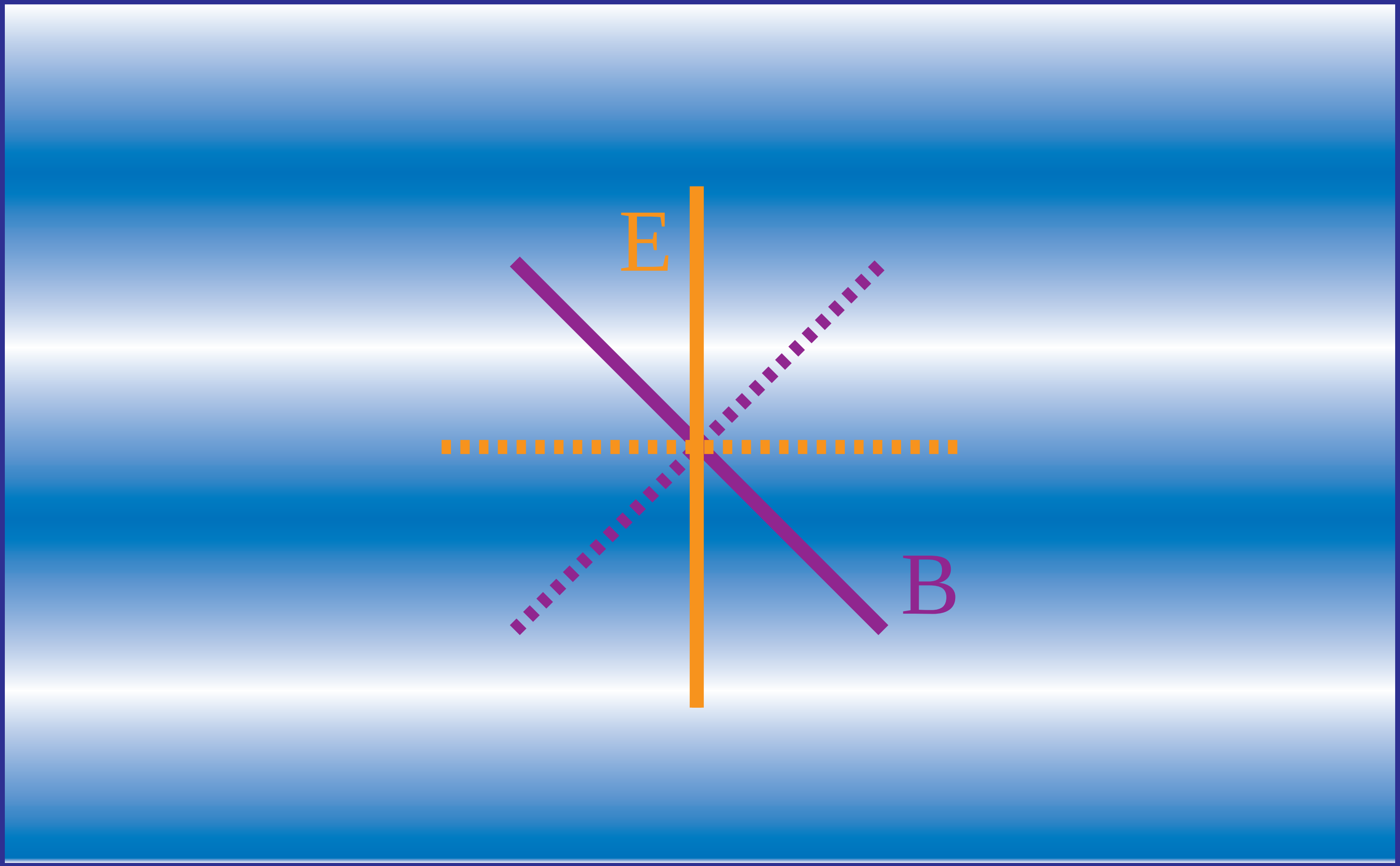
Polarization Lensing



Electric & Magnetic Polarization

(a.k.a. gradient & curl)

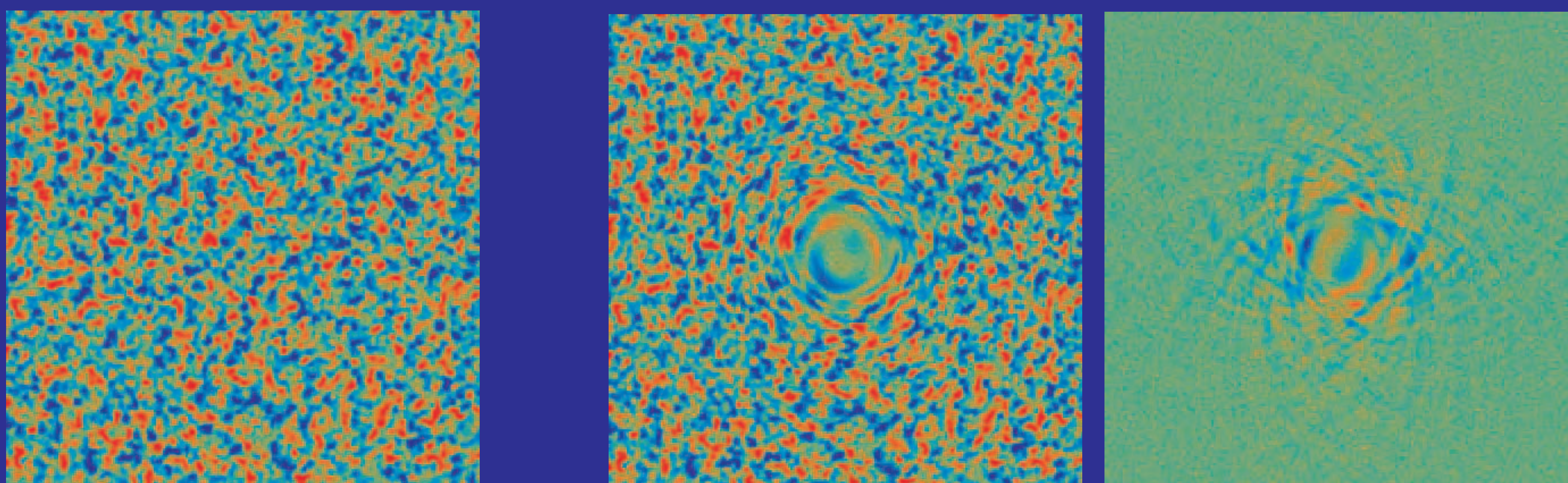
- Alignment of principal vs polarization axes
(curvature matrix vs polarization direction)



Kamionkowski, Kosowsky, Stebbins (1997)
Zaldarriaga & Seljak (1997)

Polarization Lensing

- Since E and B denote the relationship between the polarization amplitude and direction, warping due to lensing creates B-modes



Original   Lensed E   Lensed B

Zaldarriaga & Seljak (1998) [figure: Hu & Okamoto (2001)]

Polarization Lensing

- Polarization field harmonics lensed similarly

$$[Q \pm iU](\hat{\mathbf{n}}) = -\int \frac{d^2 l}{(2\pi)^2} [E \pm iB](\mathbf{l}) e^{\pm 2i\phi_{\mathbf{l}}} e^{\mathbf{l}\cdot\hat{\mathbf{n}}}$$

so that

$$\begin{aligned}
[Q \pm iU](\hat{\mathbf{n}}) &= [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}} + \nabla\phi) \\
&\approx [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) + \nabla_i \phi(\hat{\mathbf{n}}) \nabla^i [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) \\
&\quad + \frac{1}{2} \nabla_i \phi(\hat{\mathbf{n}}) \nabla_j \phi(\hat{\mathbf{n}}) \nabla^i \nabla^j [\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}})
\end{aligned}$$

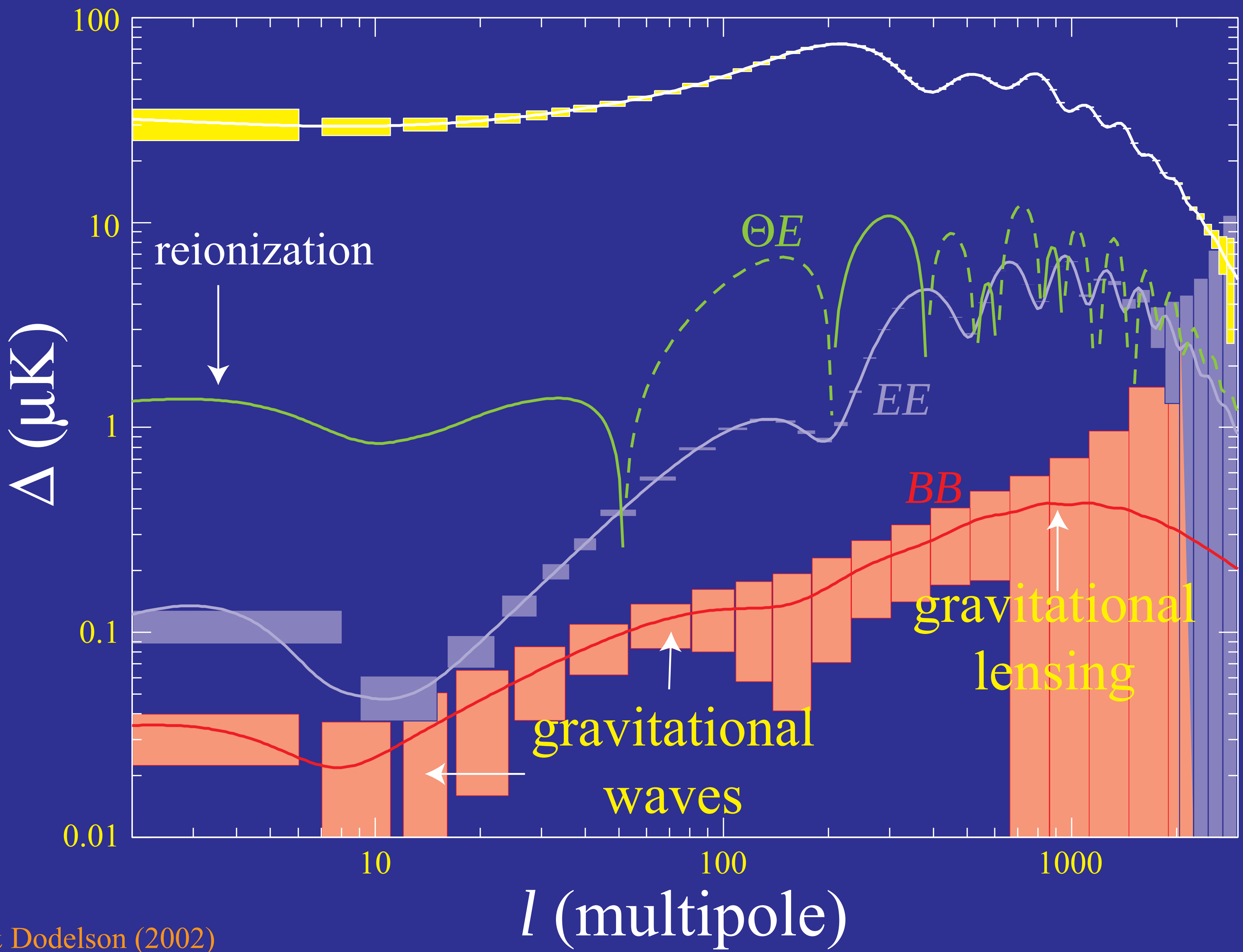
Polarization Power Spectra

- Carrying through the algebra to the power spectrum

$$C_l^{EE} = \left(1 - l^2 R\right) \tilde{C}_l^{EE} + \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \left[(\tilde{C}_{l_1}^{EE} + \tilde{C}_{l_1}^{BB}) + \cos(4\varphi_{l_1})(\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_1}^{BB})\right],$$
$$C_l^{BB} = \left(1 - l^2 R\right) \tilde{C}_l^{BB} + \frac{1}{2} \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \left[(\tilde{C}_{l_1}^{EE} + \tilde{C}_{l_1}^{BB}) - \cos(4\varphi_{l_1})(\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_1}^{BB})\right],$$
$$C_l^{\Theta E} = \left(1 - l^2 R\right) \tilde{C}_l^{\Theta E} + \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} [(\mathbf{l} - \mathbf{l}_1) \cdot \mathbf{l}_1]^2 C^{\phi\phi}_{|\mathbf{l}-\mathbf{l}_1|}$$
$$\times \tilde{C}_{l_1}^{\Theta E} \cos(2\varphi_{l_1}),$$

- Lensing generates B-modes out of the acoustic polaraization E-modes contaminates gravitational wave signature if $E_i < 10^{16}$GeV.

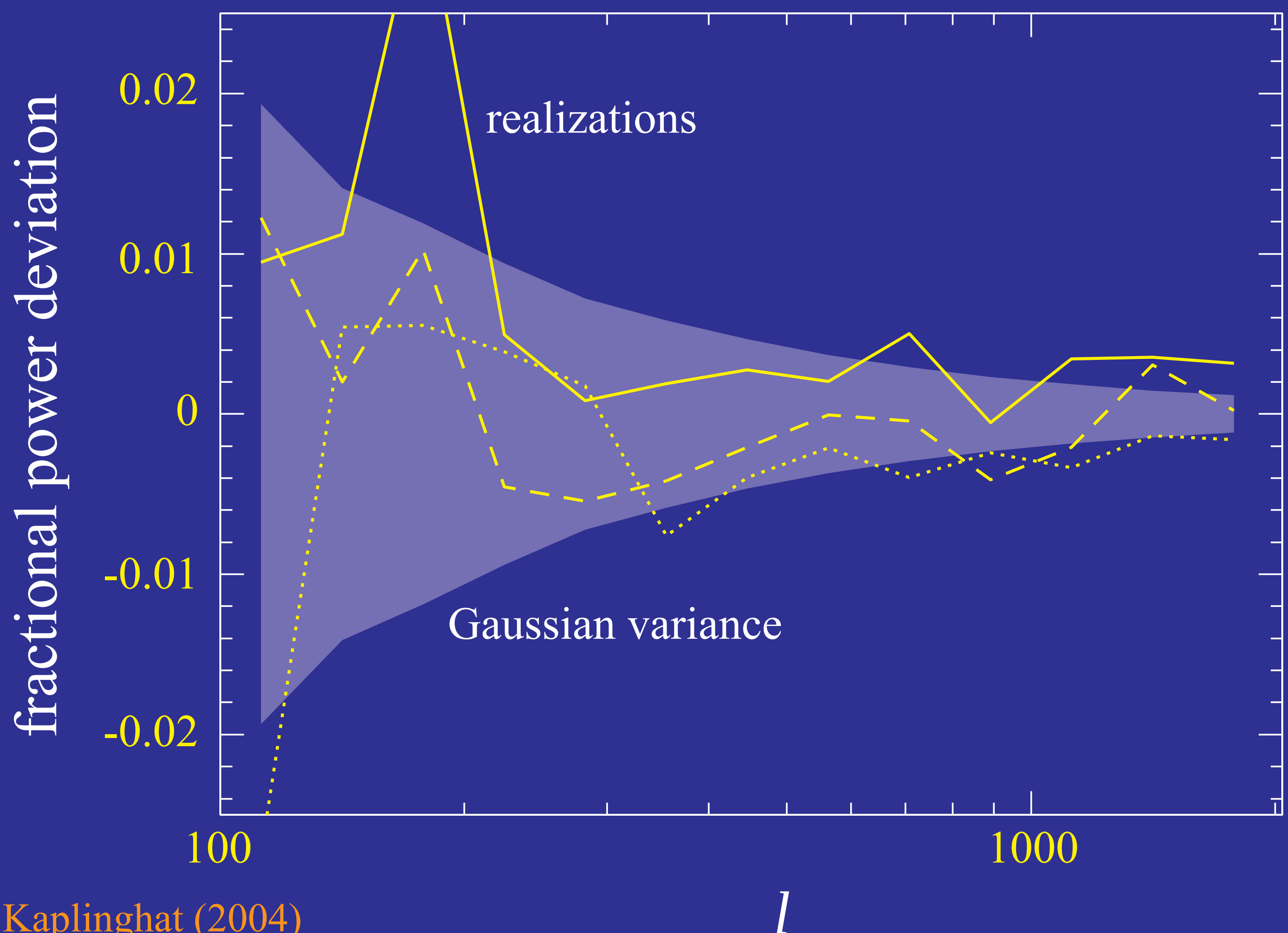
Polarized Landscape



Hu & Dodelson (2002)

Power Spectrum Measurements

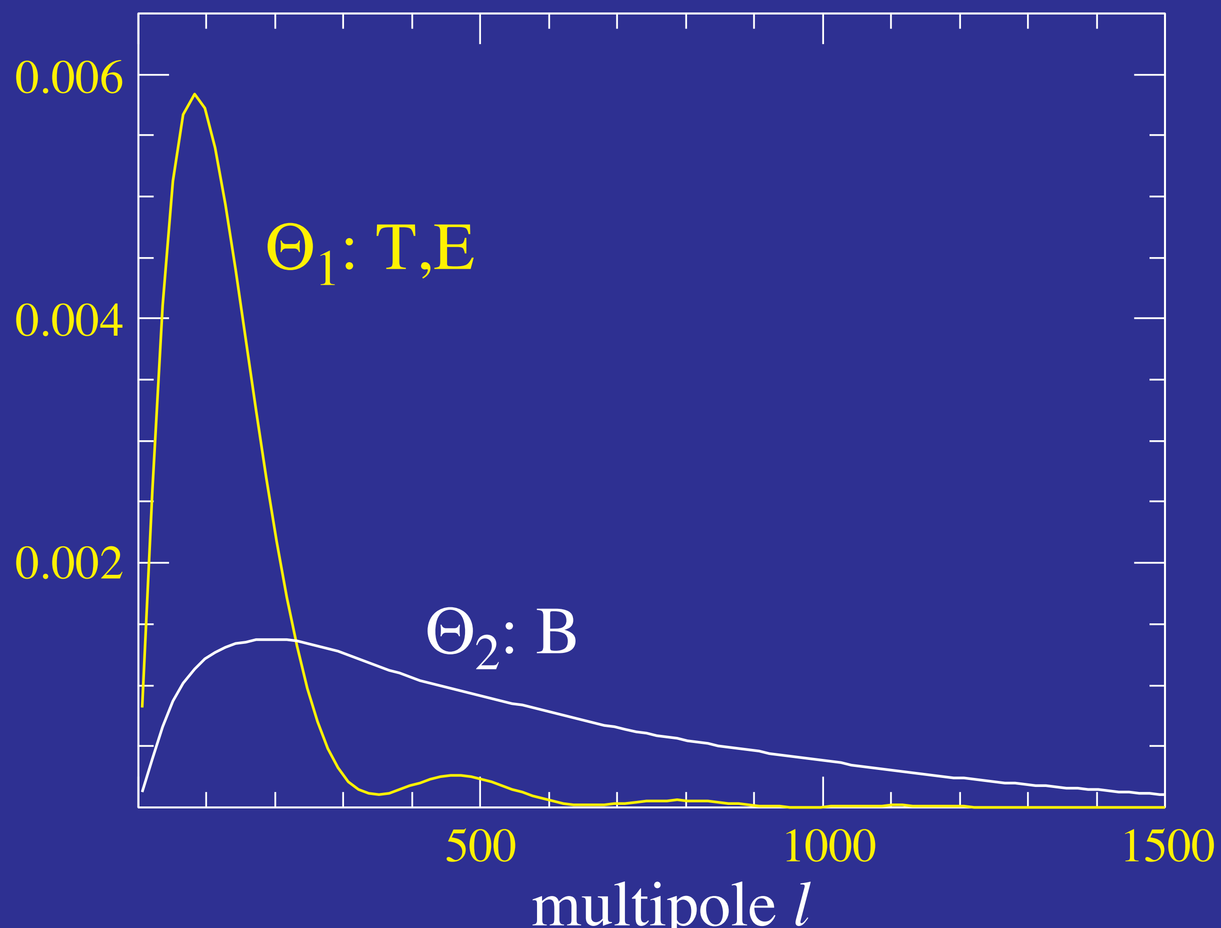
- Lensed field is non-Gaussian in that a single degree scale lens controls the polarization at arcminutes
- Increased variance and covariance implies that 10x as much sky needed compared with Gaussian fields



Smith, Hu & Kaplinghat (2004)

Lensed Power Spectrum Observables

- Principal components show two observables in lensed power spectra
- Temperature and E-polarization: deflection power at l~100
  B-polarization: deflection power at l~500
- Normalized so that observables error = fractional lens power error

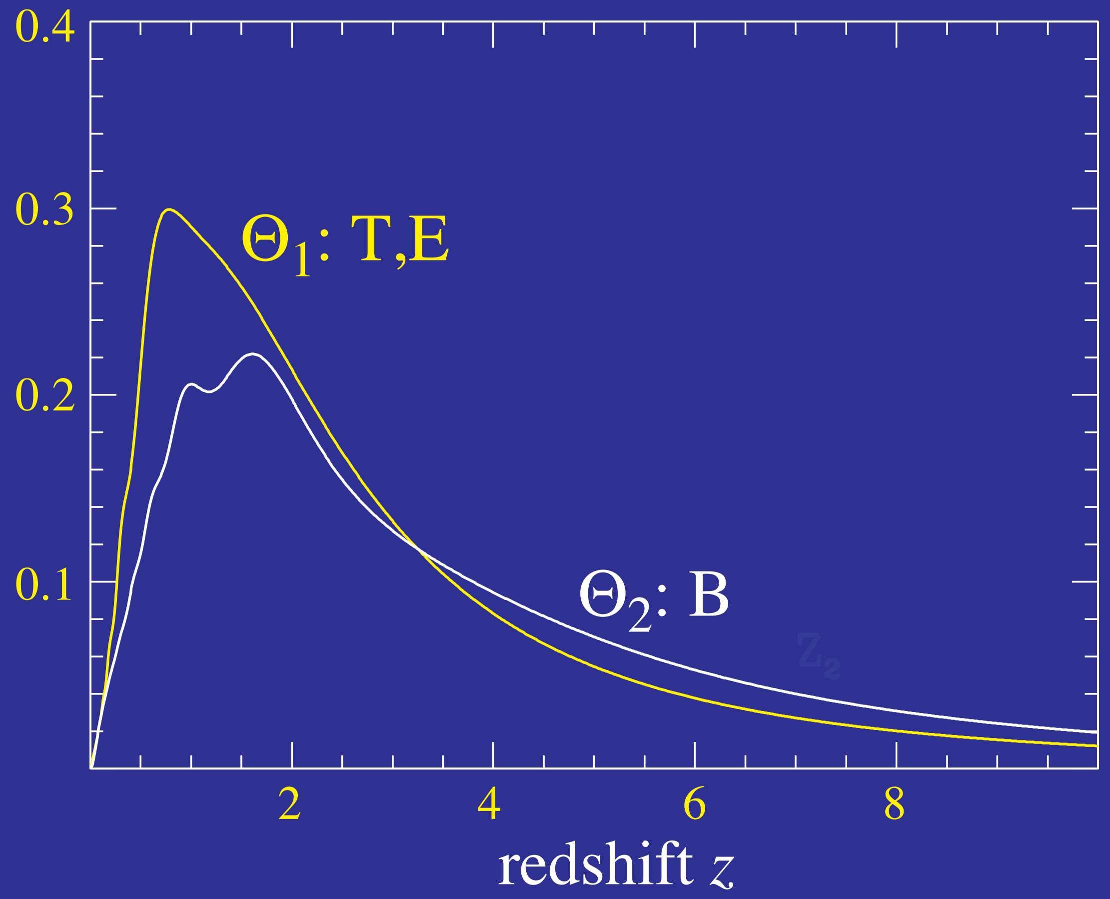


Smith, Hu & Kaplinghat (2006)

Redshift Sensitivity

- Lensing observables probe distance and structure at high redshift

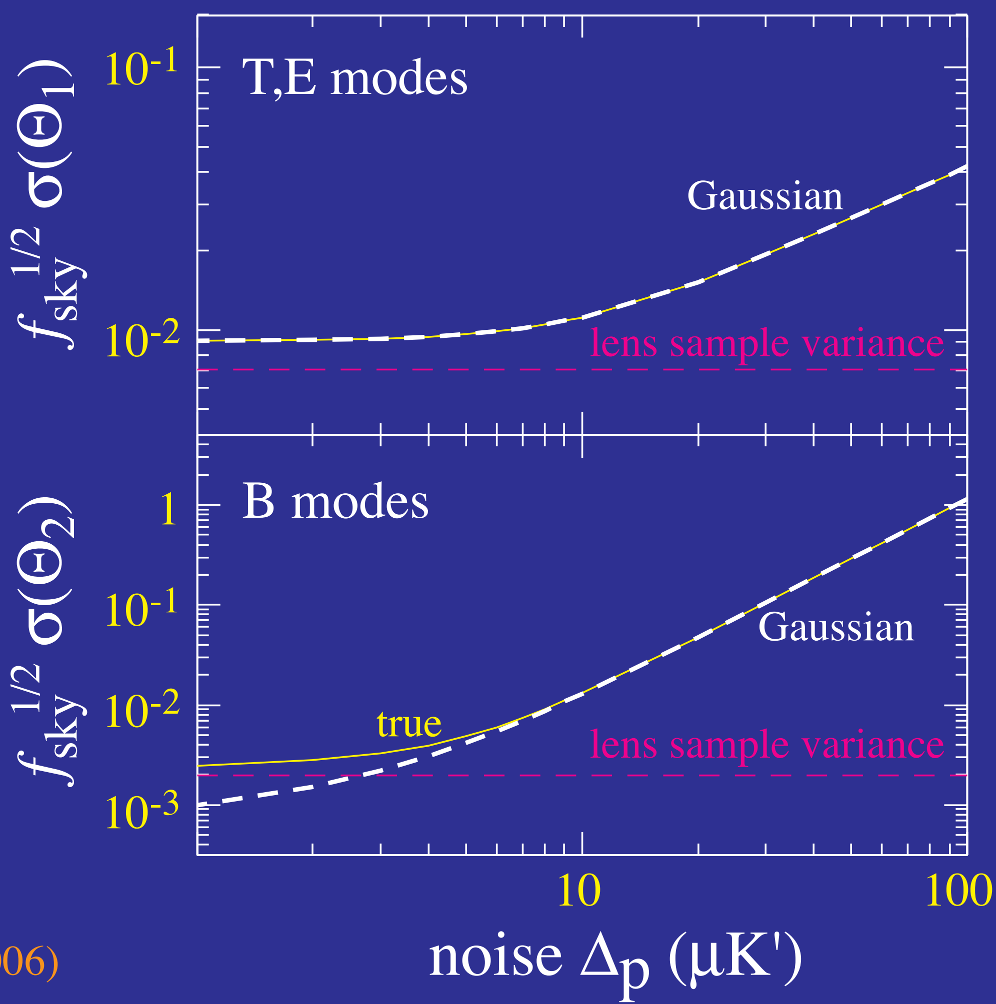
$$\frac{\delta\Theta_i}{\Theta_i} = \left[\left(3 - \frac{d\ln\Delta_m^2}{d\ln k} \right) \frac{\delta D_A}{D_A} - \frac{\delta H}{H} + 2\frac{\delta G}{G} + 2\frac{\delta D_A(D_s - D)}{D_A(D_s - D)} \right]$$



Smith, Hu & Kaplinghat (2006)

Constraints on Lensing Observables

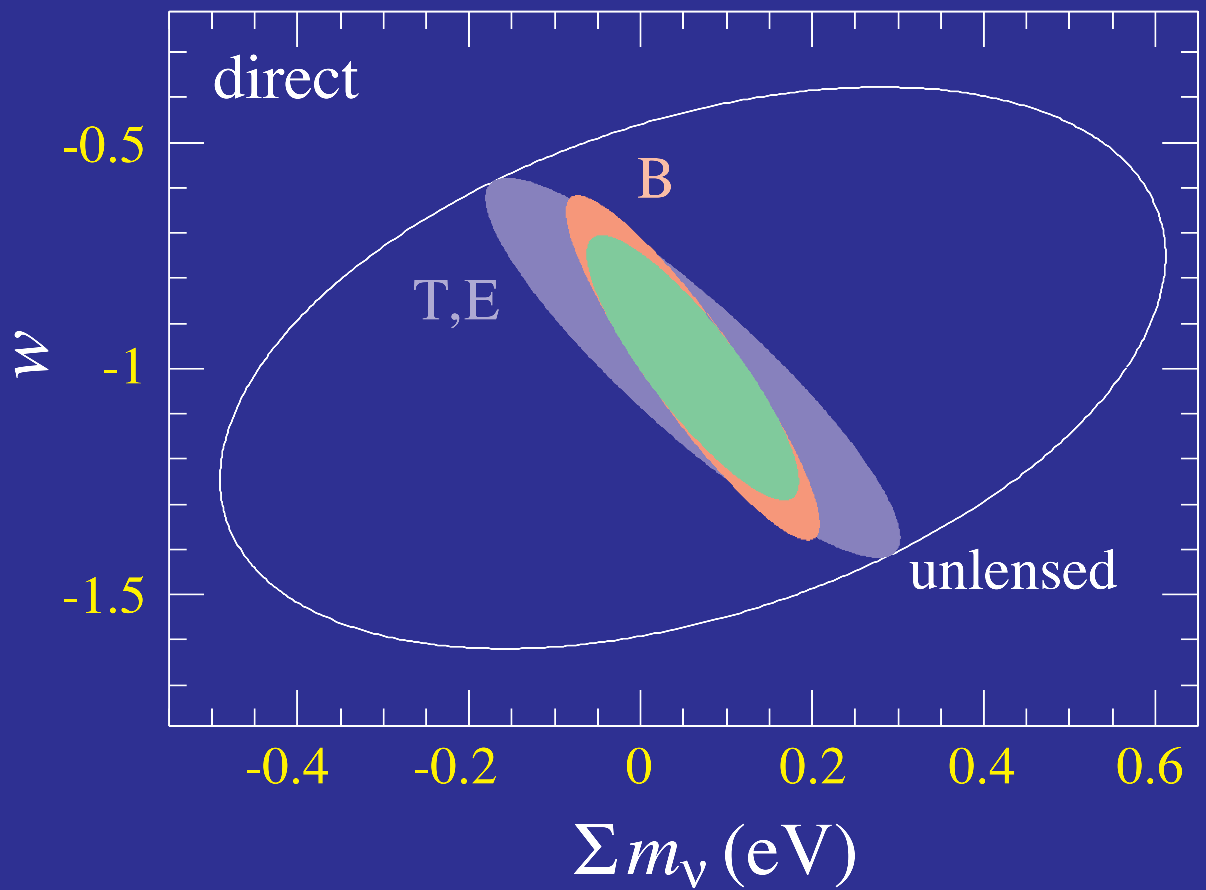
- Lensing observables in T,E are limited by CMB sample variance
- Lensing observables in B are limited by lens sample variance
- B-modes require 10x as much sky at high signal-to-noise or 3x as much sky at the optimal signal-to-noise with Δ_P=4.7uK'



Smith, Hu & Kaplinghat (2006)

Lensing Observables

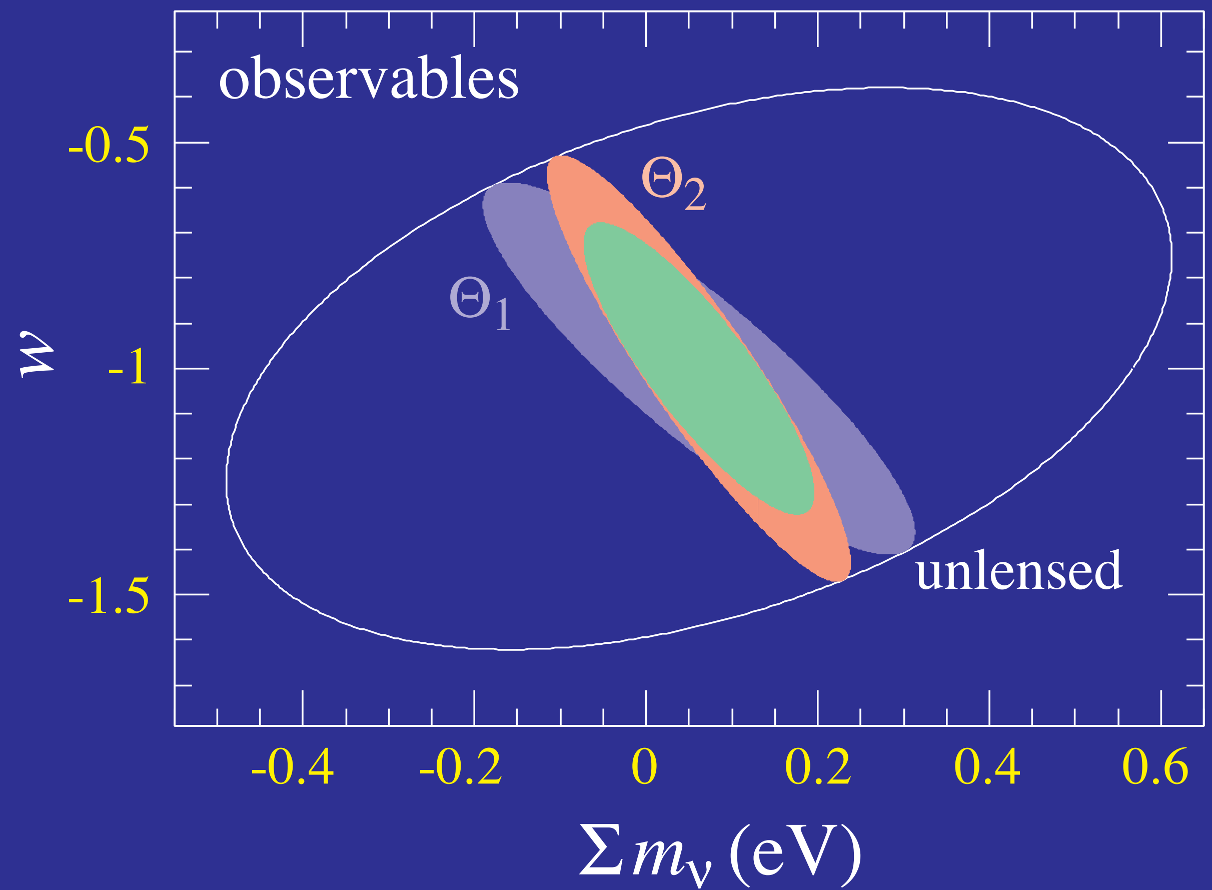
- Lensing observables provide a simple way of accounting for non-Gaussianity and parameter degeneracies
- Direct forecasts for Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006) [see also: Kaplinghat et. al 2003, Acquaviva & Baccigalupi 2005, Smith et al 2005, Li et al 2006]

Lensing Observables

- Lensing observables provide a simple way of accounting for non-Gaussianity and parameter degeneracies
- Observables forecasts for Planck + 10% sky with noise Δ_P=1.4uK'



Smith, Hu, Kaplinghat (2006)

Lensing Reconstruction

Quadratic Estimator

- Taylor expand mapping

$$\Theta(\hat{\mathbf{n}}) = \tilde{\Theta}(\hat{\mathbf{n}} + \nabla\phi)$$
$$= \tilde{\Theta}(\hat{\mathbf{n}}) + \nabla_i\phi(\hat{\mathbf{n}})\nabla^i\tilde{\Theta}(\hat{\mathbf{n}}) + \ldots$$

- Fourier decomposition $\rightarrow$ mode coupling of harmonics

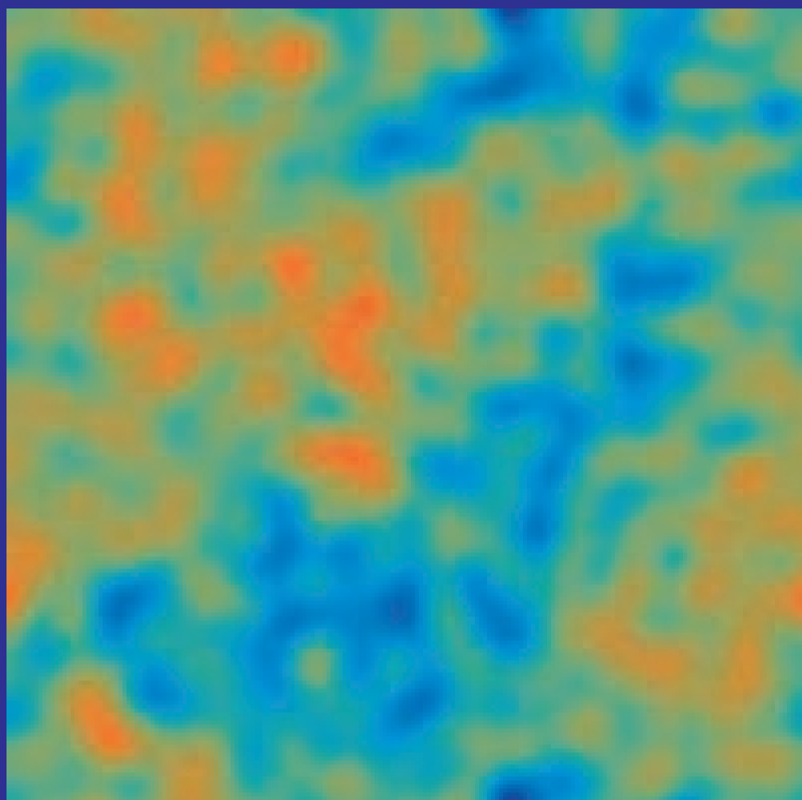
$$\Theta(\mathbf{l}) = \int d\hat{\mathbf{n}}\,\Theta(\hat{\mathbf{n}})e^{-il\cdot\hat{\mathbf{n}}}$$
$$= \tilde{\Theta}(\mathbf{l}) - \int \frac{d^2\mathbf{l}_1}{(2\pi)^2}(\mathbf{l}-\mathbf{l}_1)\cdot\mathbf{l}_1\,\tilde{\Theta}(\mathbf{l}_1)\phi(\mathbf{l}-\mathbf{l}_1)$$

- Consider fixed lens and Gaussian random CMB realizations: each pair is an estimator of the lens at $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2$ (Hu 2001):

$$\langle\Theta(\mathbf{l})\Theta'(\mathbf{l}')\rangle_{\rm CMB} \approx \left[\tilde{C}^{\Theta\Theta}_{l_1}(\mathbf{L}\cdot\mathbf{l}_1) + \tilde{C}^{\Theta\Theta}_{l_2}(\mathbf{L}\cdot\mathbf{l}_2)\right]\phi(\mathbf{L}) \quad (\mathbf{l} \neq -\mathbf{l}')$$

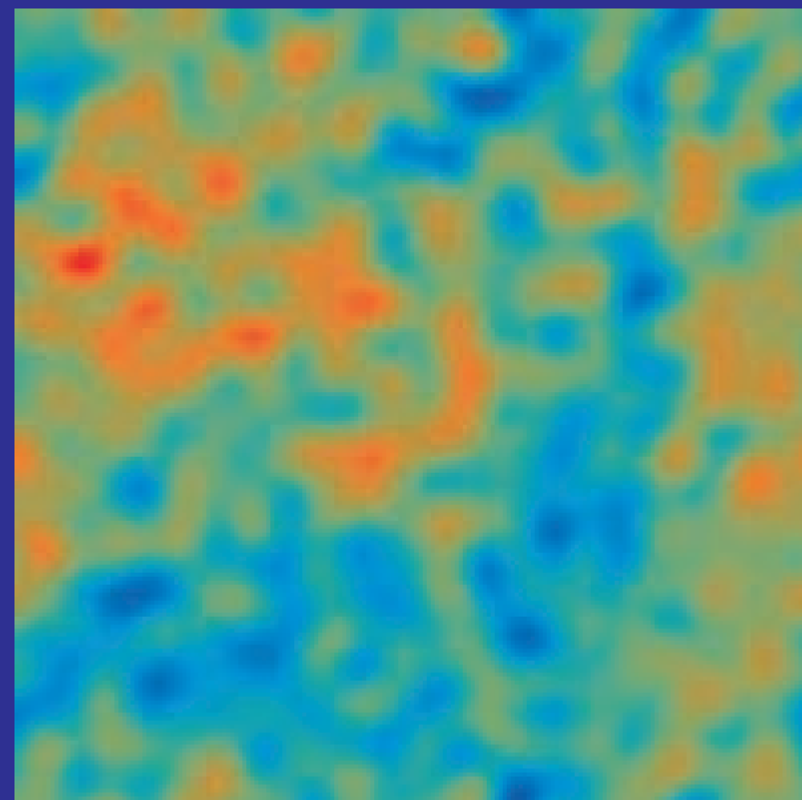
Quadratic Reconstruction

- Matched filter (minimum variance) averaging over pairs of multipole moments
- Real space: divergence of a temperature-weighted gradient



original

potential map (1000sq. deg)



reconstructed

1.5' beam; 27μK-arcmin noise

Hu (2001)

Reconstruction from the CMB

- Generalize to polarization: each quadratic pair of fields estimates the lensing potential (Hu & Okamoto 2002)

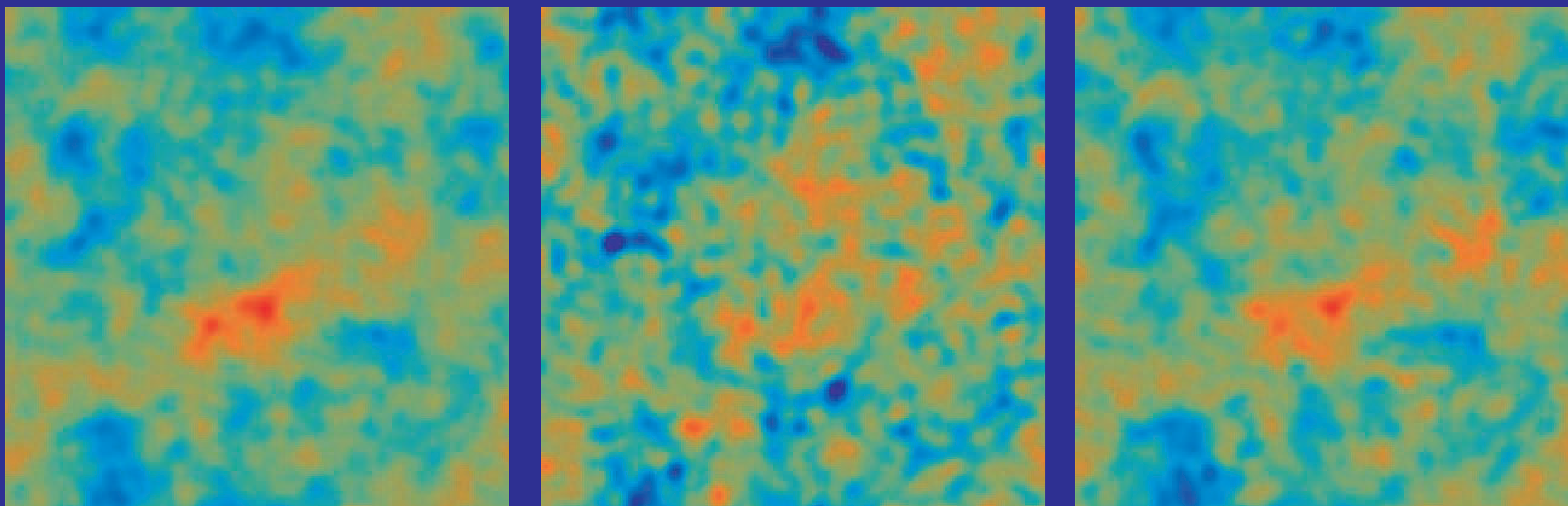
$$\langle x(\mathbf{l}) x'(\mathbf{l}') \rangle_{\rm CMB} = f_\alpha(\mathbf{l}, \mathbf{l}') \phi(\mathbf{l} + \mathbf{l}') \,,$$

  where $x \in$ temperature, polarization fields and f_α is a fixed weight that reflects geometry

- Each pair forms a noisy estimate of the potential or projected mass - just like a pair of galaxy shears
- Minimum variance weight all pairs to form an estimator of the lensing mass
- Generalize to inhomogeneous noise, cut sky and maximum likelihood by iterating the quadratic estimator (Seljak & Hirata 2002)

High Signal-to-Noise B-modes

- Cosmic variance of CMB fields sets ultimate limit for *T*,*E*
- *B*-polarization allows mapping to finer scales and in principle is not limited by cosmic variance of *E* (Hirata & Seljak 2003)



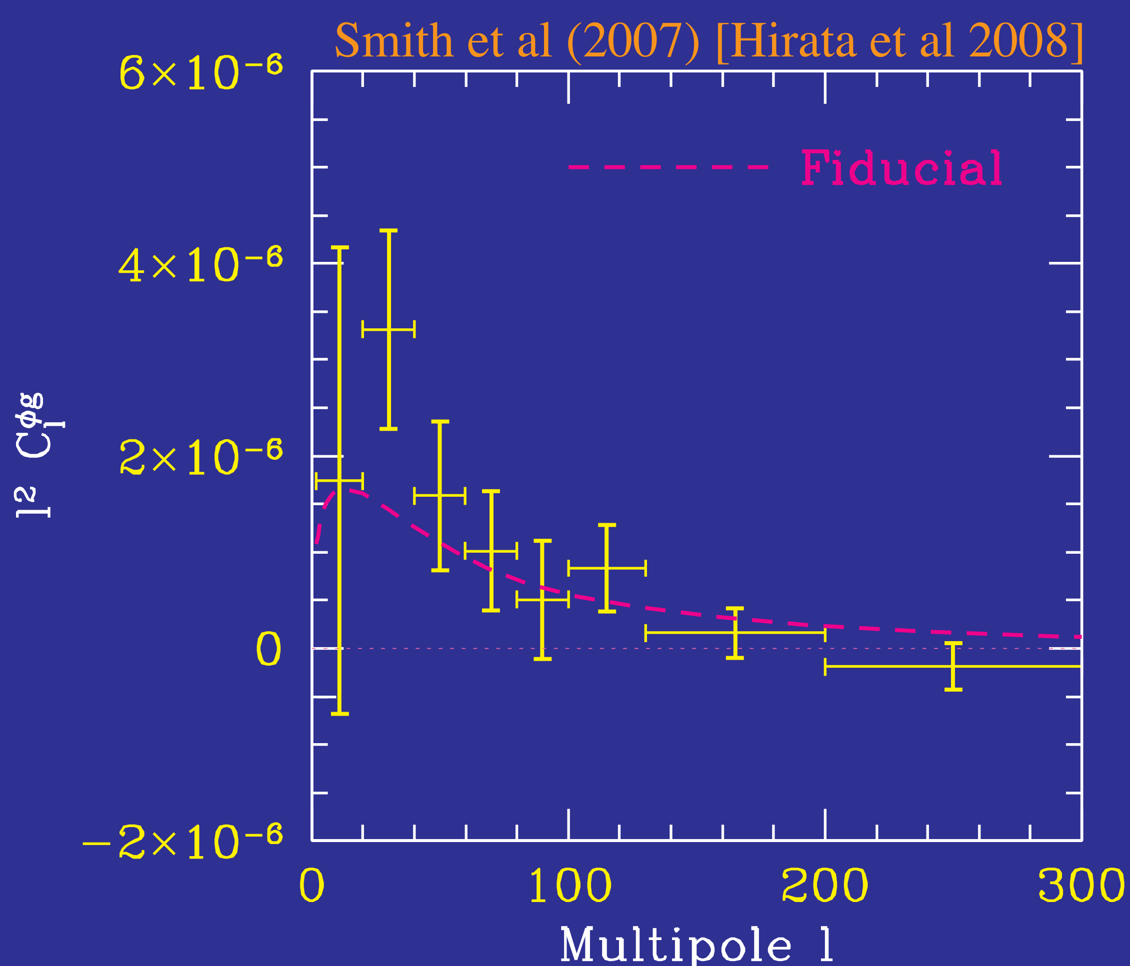
mass temp. reconstruction EB pol. reconstruction

100 sq. deg; 4' beam; 1μK-arcmin

Hu & Okamoto (2001)

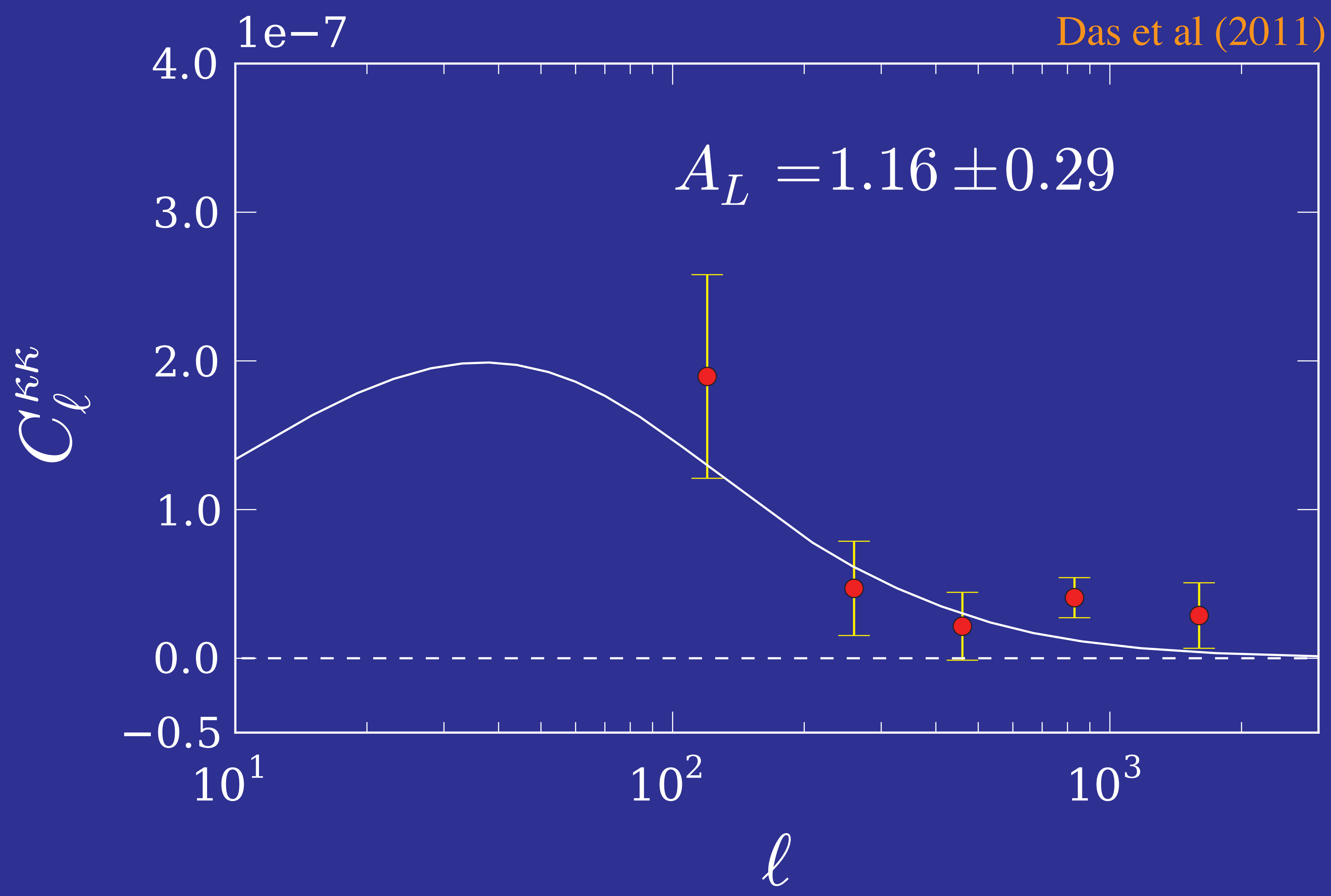
Lensing-Galaxy Correlation

- ~3σ+ joint detection of WMAP lensing reconstruction with large scale structure (galaxies)
- Consistent with ΛCDM



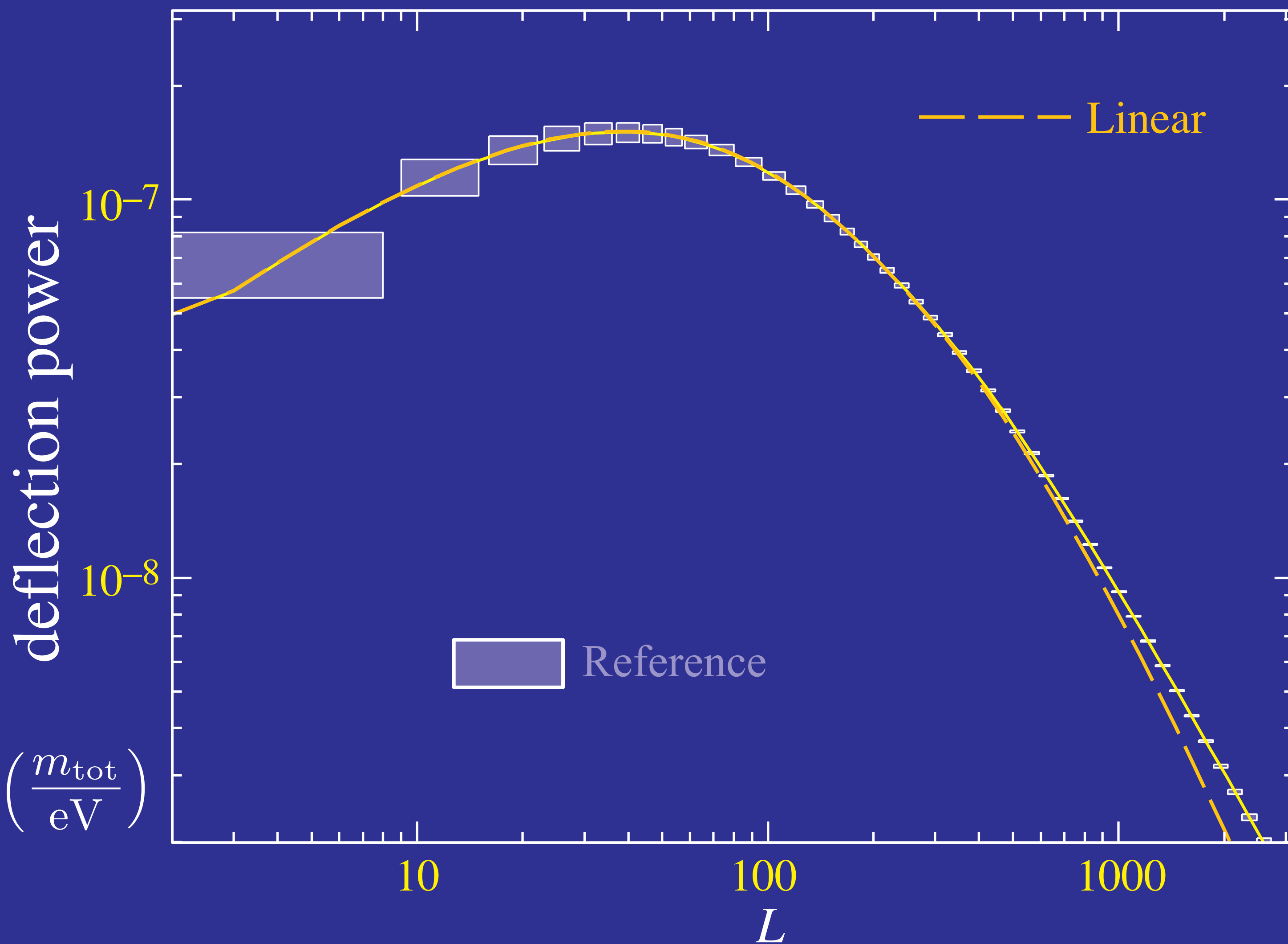
ACT Lensing Reconstruction

- ACT power spectrum of convergence reconstruction



Matter Power Spectrum

- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 h/Mpc

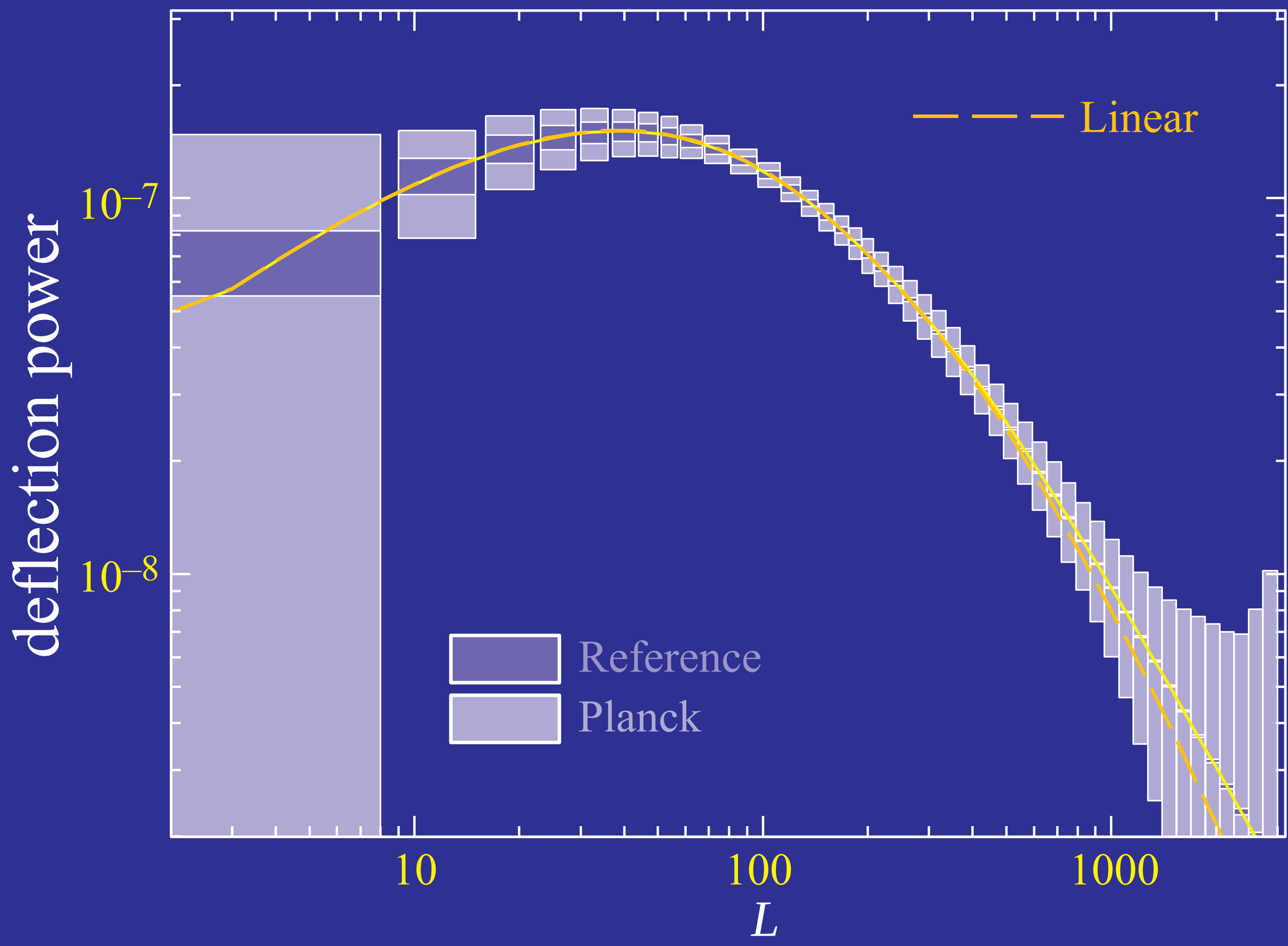


$$\frac{\Delta P}{P} \approx -0.6 \left(\frac{m_{\rm tot}}{\rm eV} \right)$$

Hu & Okamoto (2001)

Matter Power Spectrum

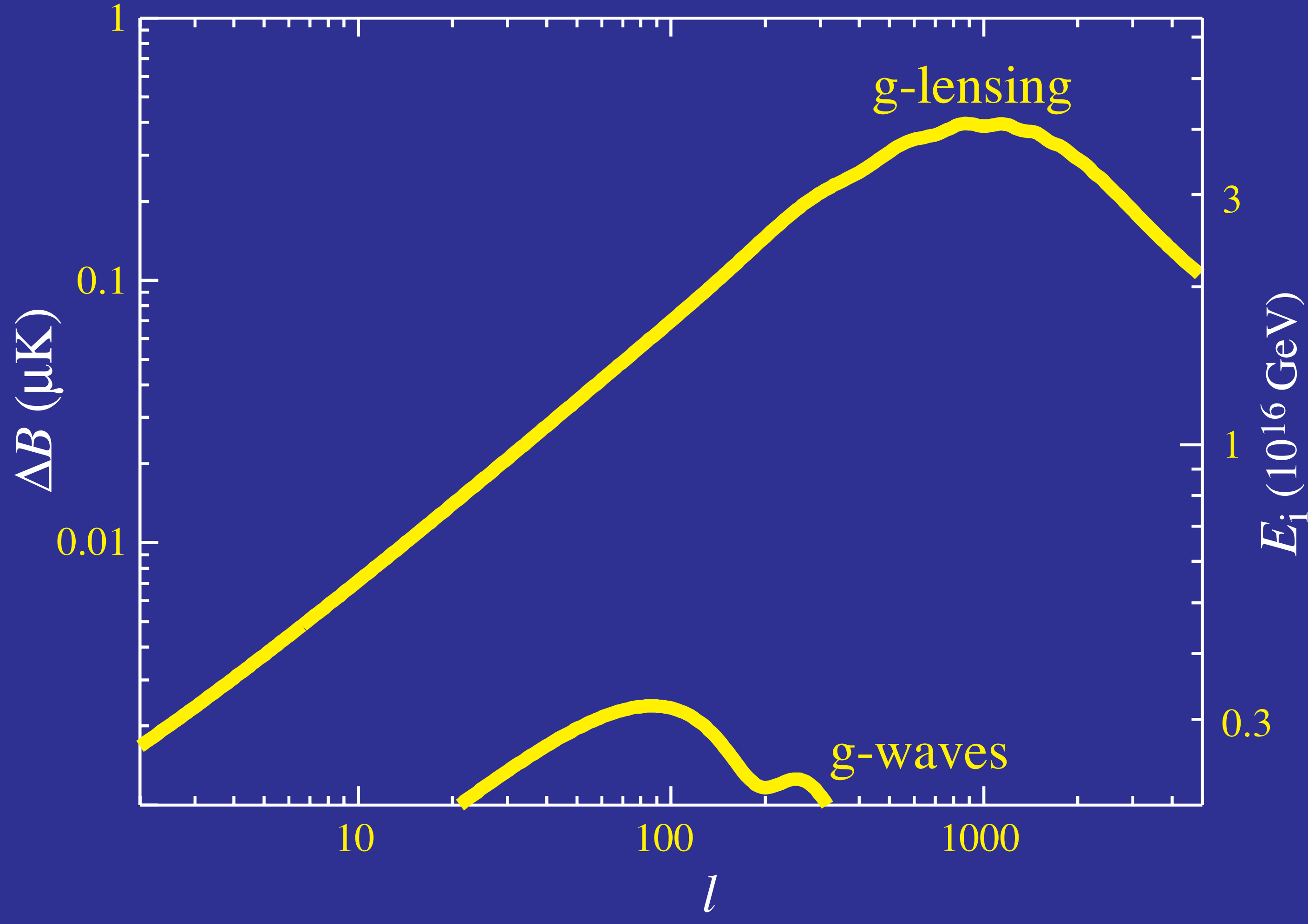
- Measuring projected matter power spectrum to cosmic variance limit across whole linear regime 0.002< k < 0.2 *h*/Mpc



Hu & Okamoto (2001)

De-Lensing the Polarization

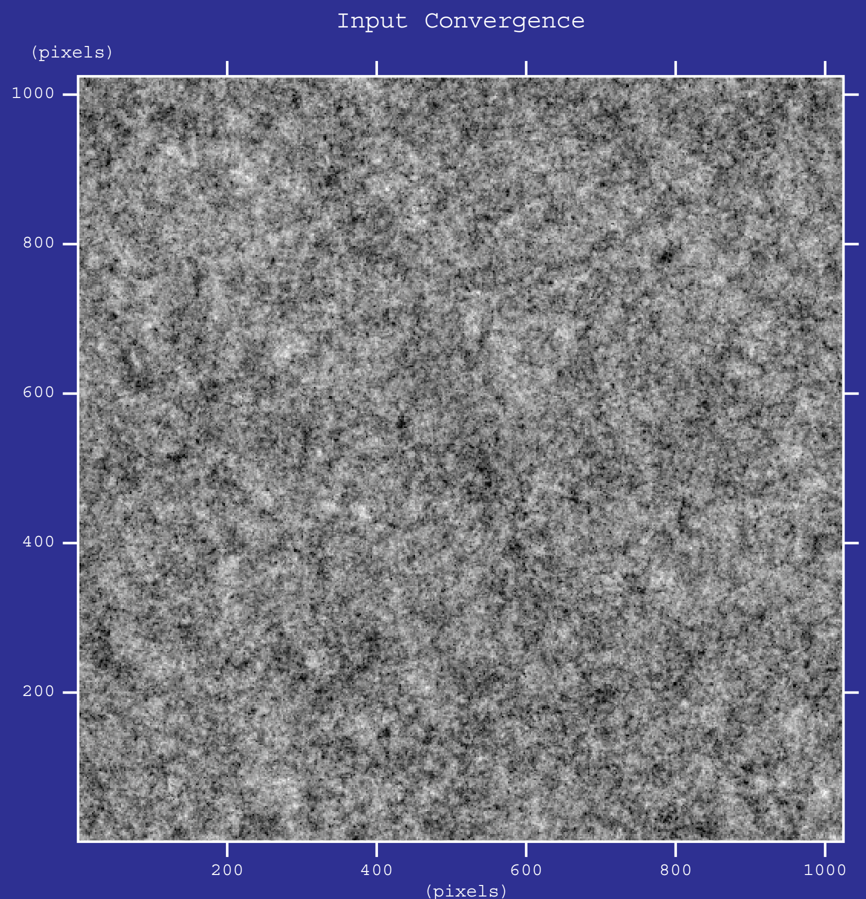
- Gravitational lensing contamination of B-modes from gravitational waves cleaned to E_i~0.3 x 10^{16} GeV
  Hu & Okamoto (2002); Knox & Song (2002); Cooray, Kedsen, Kamionkowski (2002)
- Potentially further with maximum likelihood Hirata & Seljak (2004)



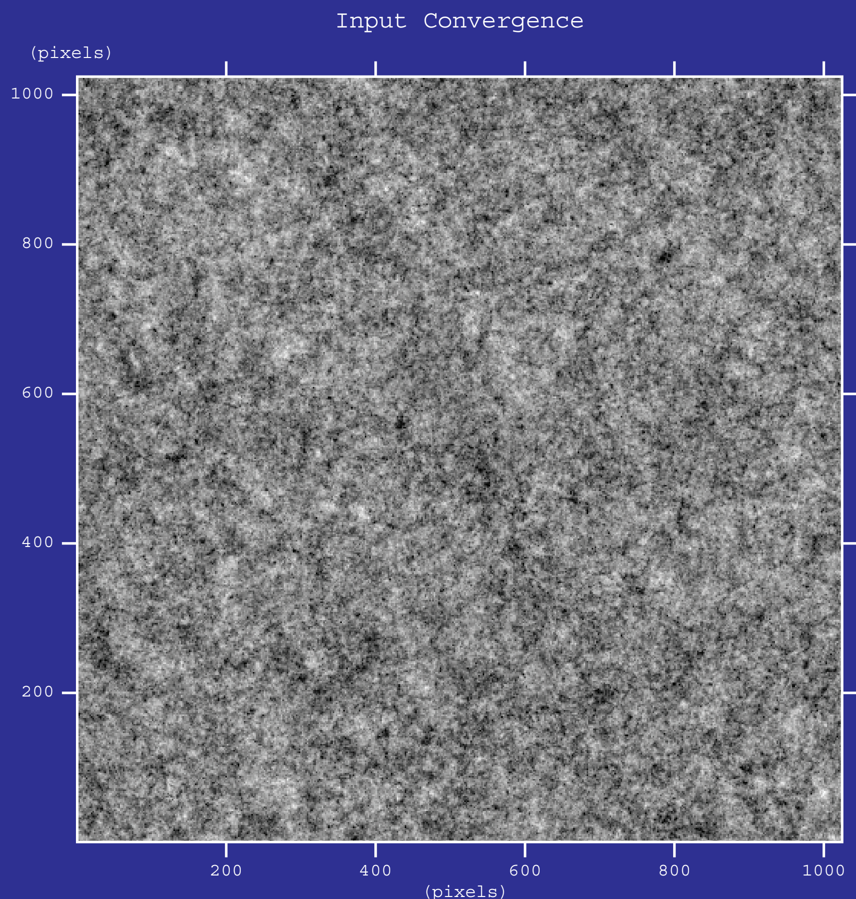
Reconstruction in the Halo Regime

- Reconstruction techniques noisy but nearly unbiased *if* gradients from lensed image and other contaminates filtered out

(Hu, DeDeo, Vale 2007)



Input κ

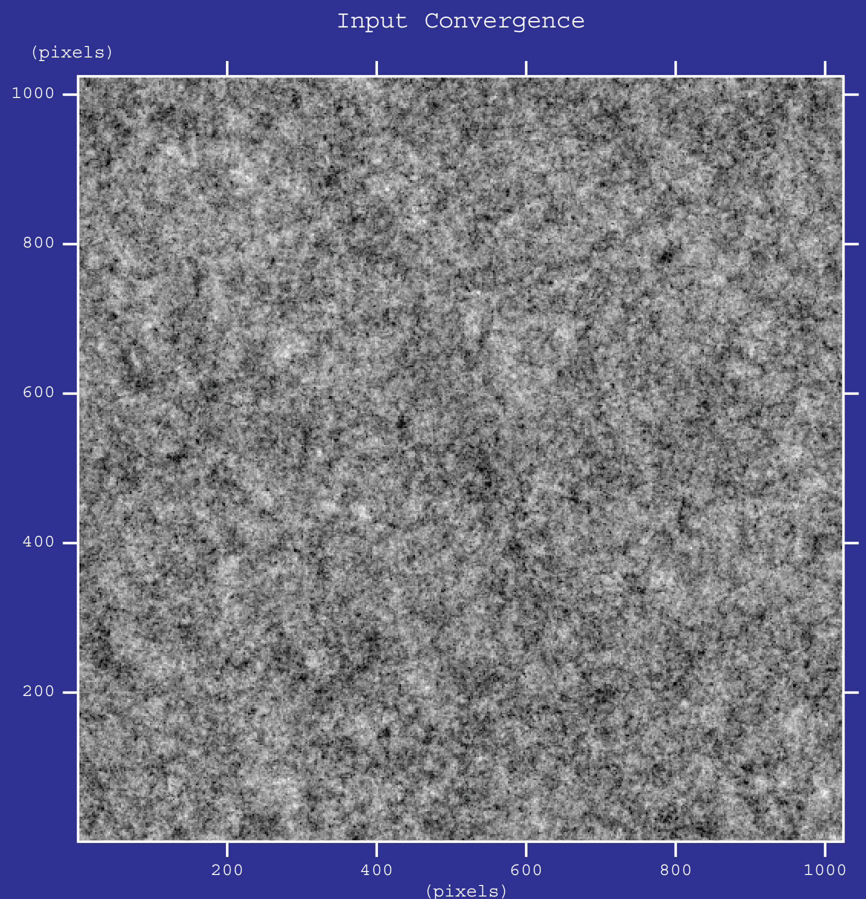


Vale (2007, unpublished); Zahn (in prep)

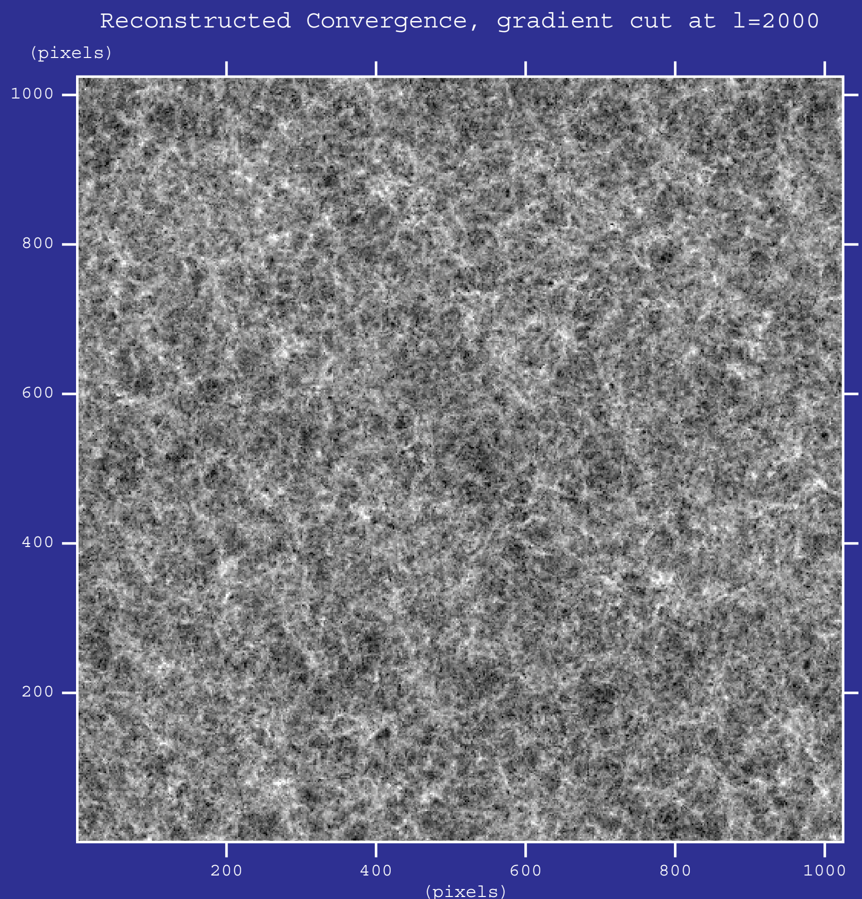
Reconstruction in the Halo Regime

- Reconstruction techniques noisy but nearly unbiased *if* gradients from lensed image and other contaminates filtered out

(Hu, DeDeo, Vale 2007)



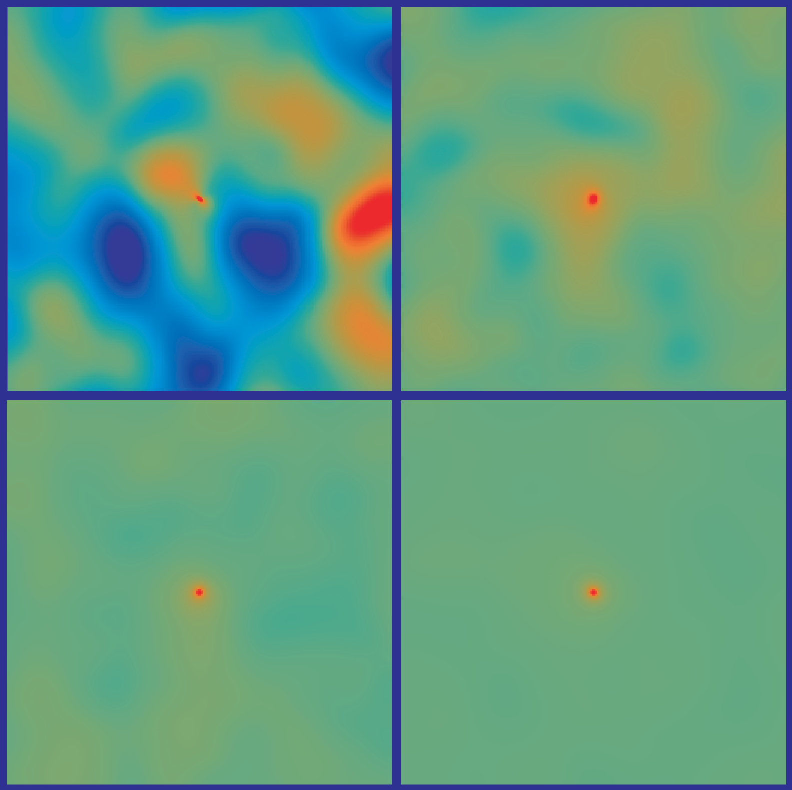
Input κ

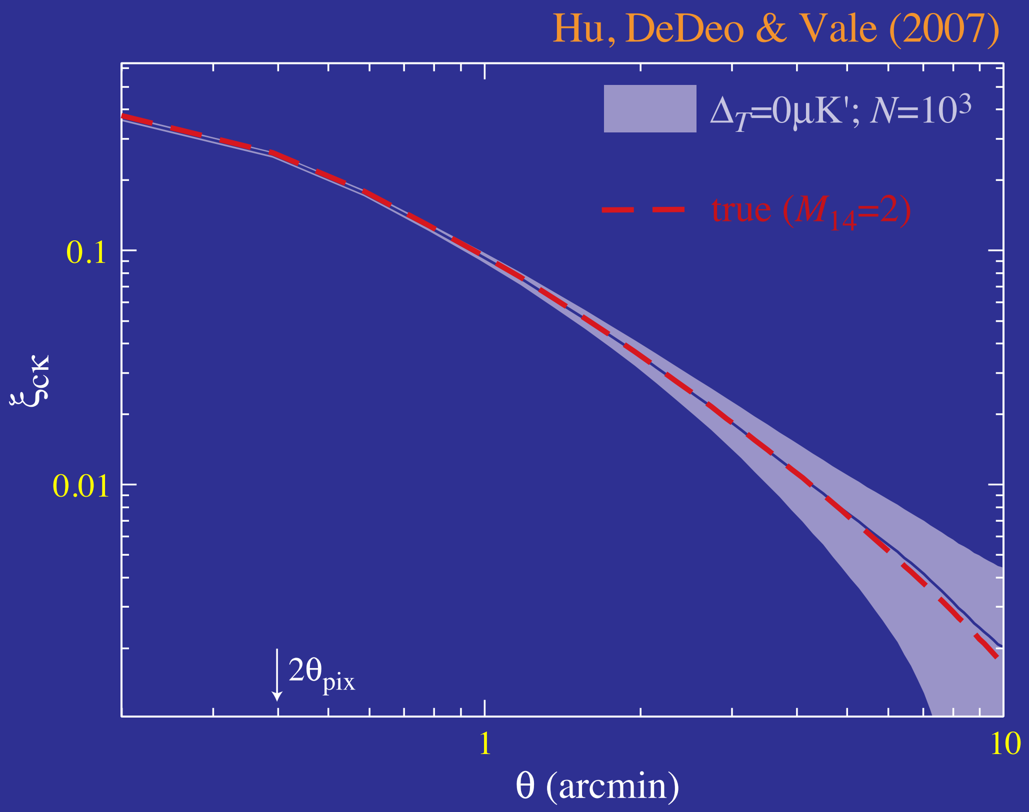


Vale (2007, unpublished); Zahn (in prep)

Cluster Lensing

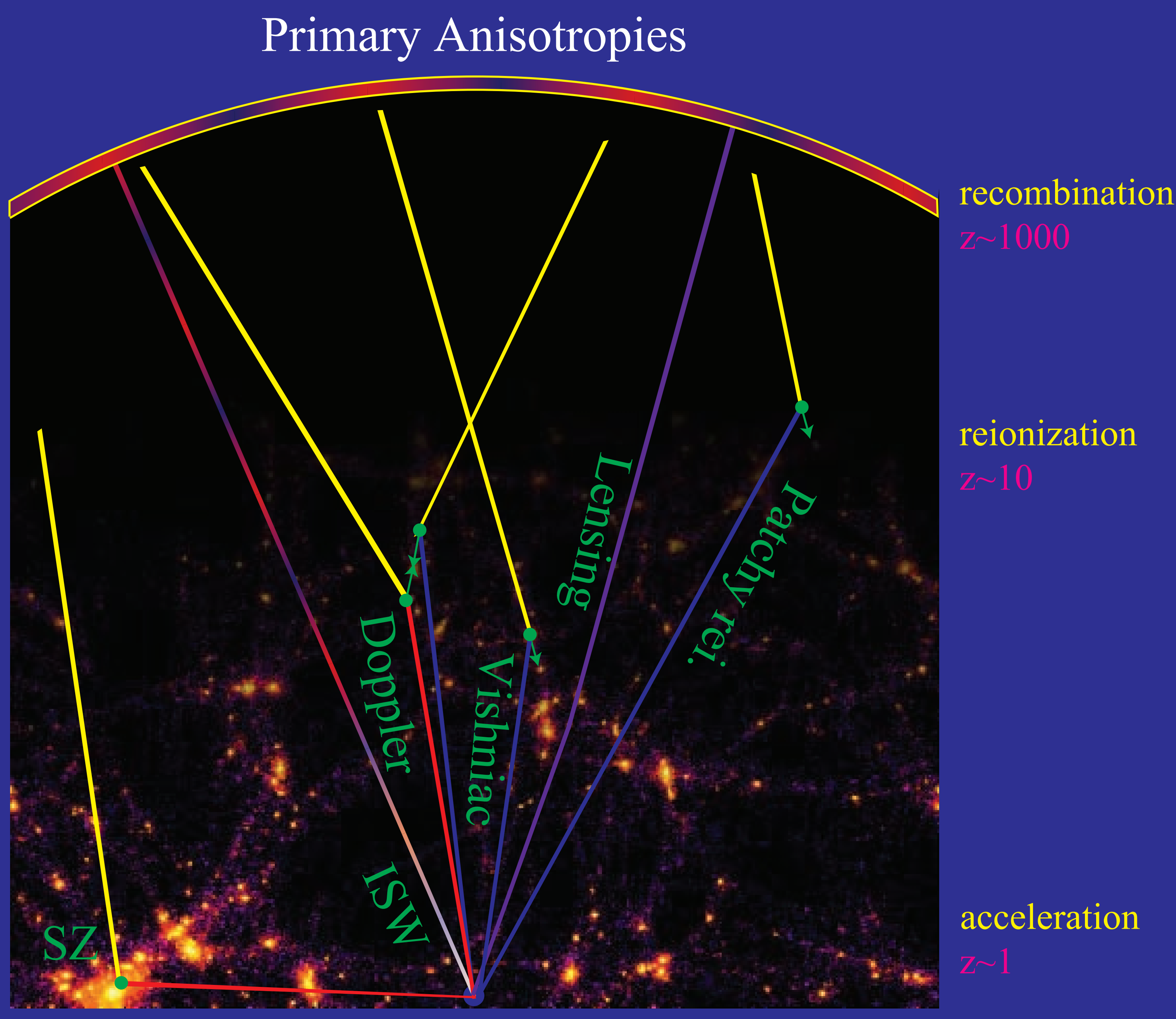
- CMB lensing reconstruction measures cluster lensing statistically through average profiles or the cluster-mass correlation function



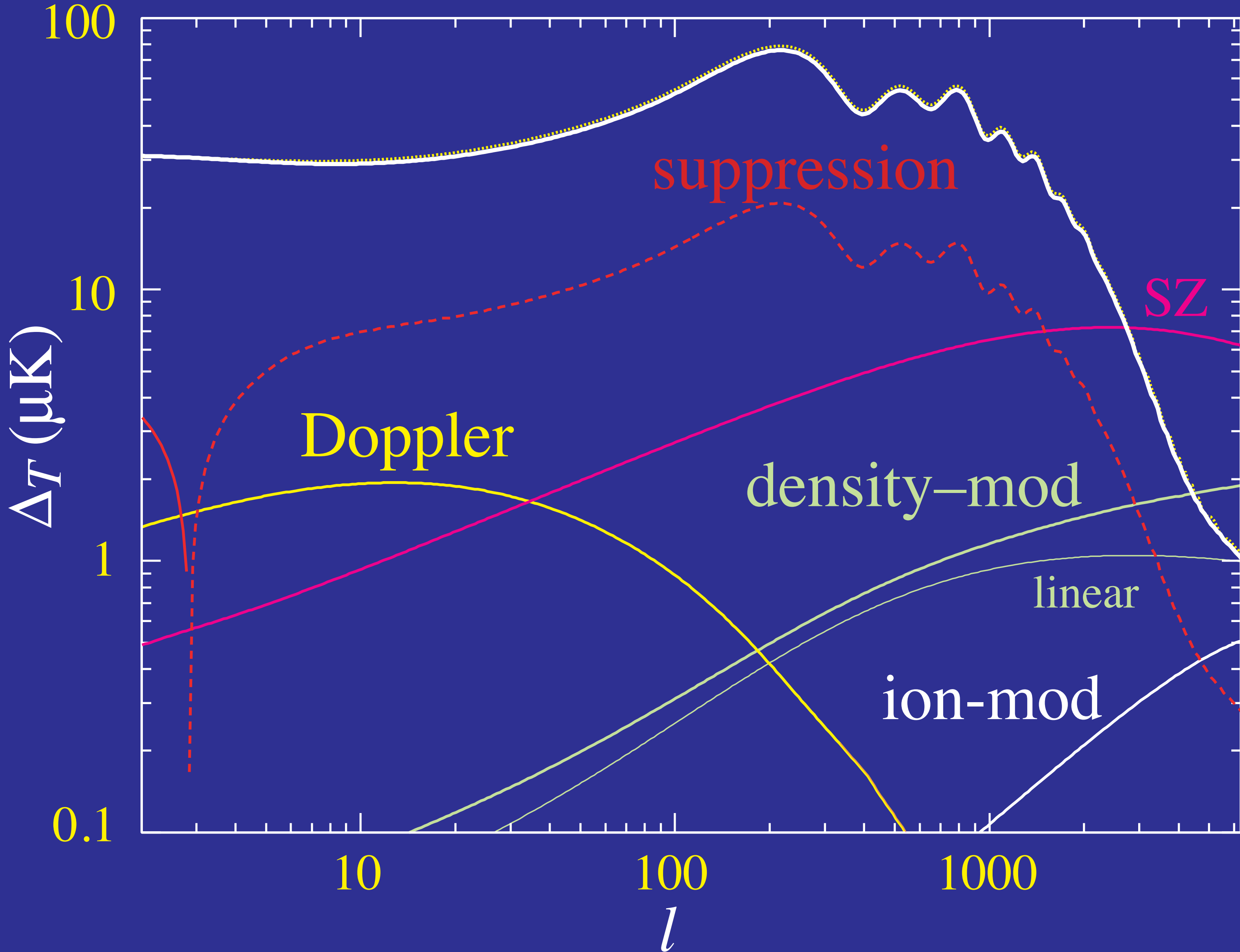


Integrated Sachs-Wolfe Effect

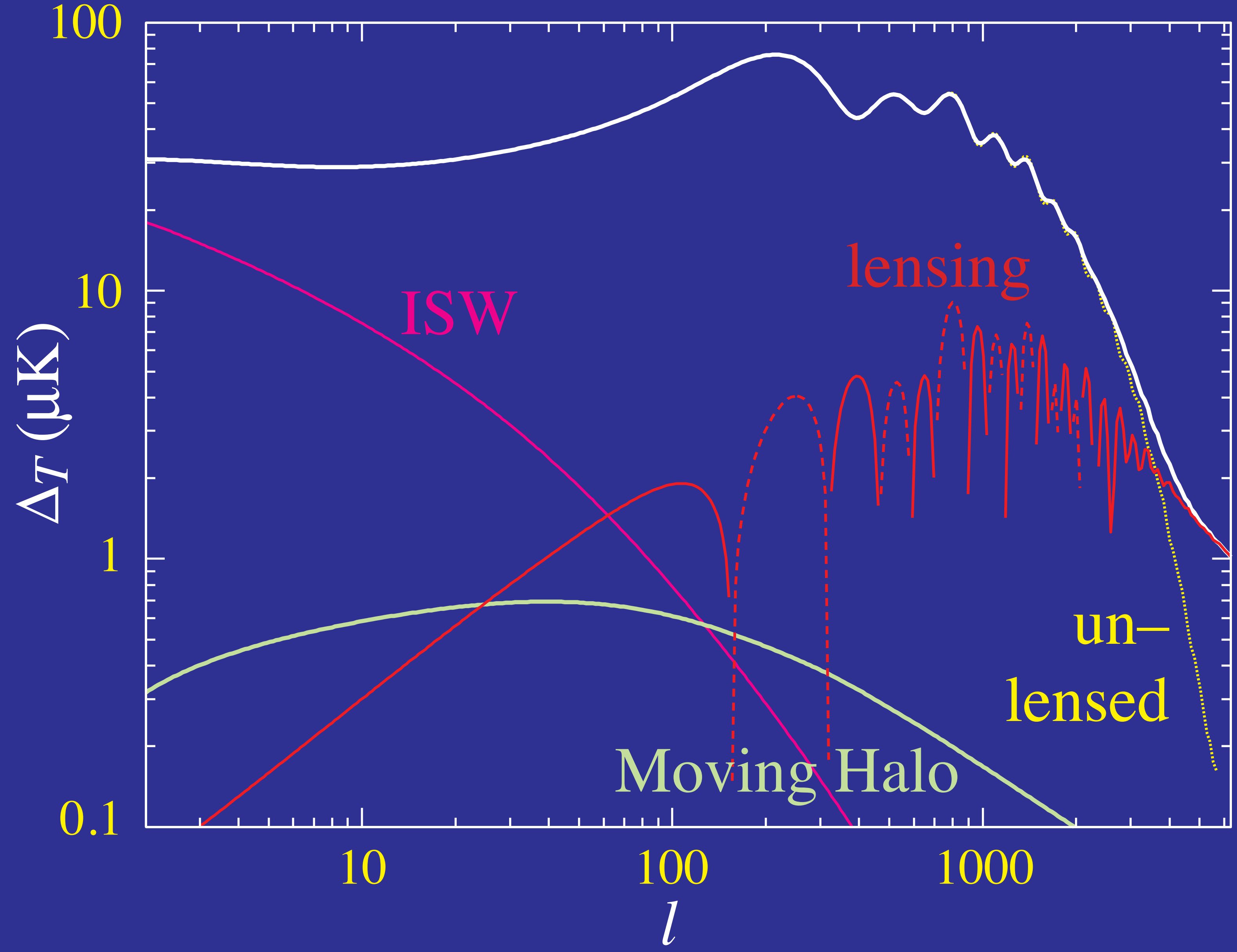
Physics of Secondary Anisotropies



Scattering Secondaries

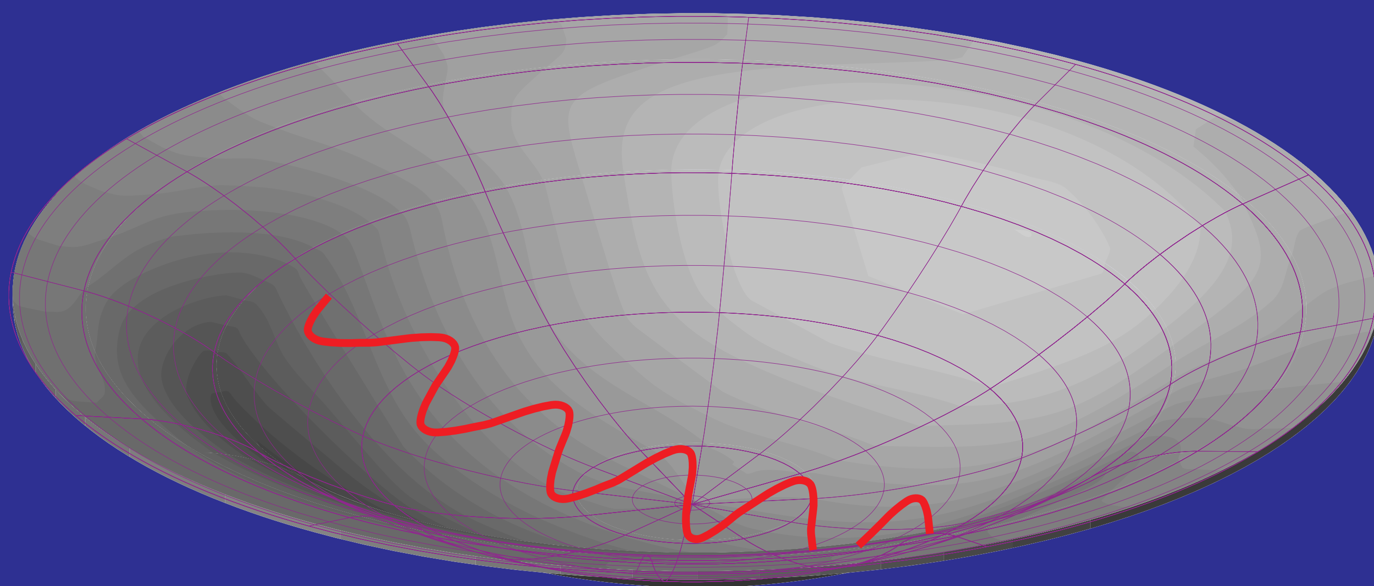


Gravitational Secondaries



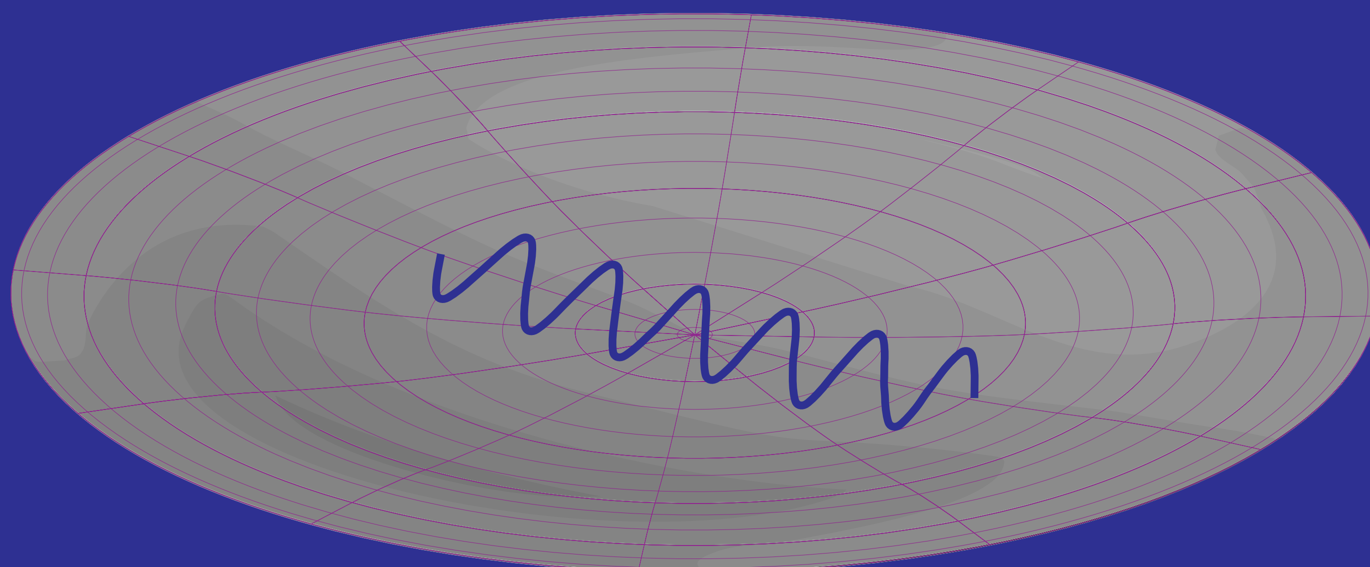
ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



ISW Effect

- Gravitational blueshift on infall does not cancel redshift on climbing out
- Contraction of spatial metric doubles the effect: $\Delta T/T = 2\Delta\Phi$
- Effect from potential hills and wells cancel on small scales



Smooth Energy Density & Potential Decay

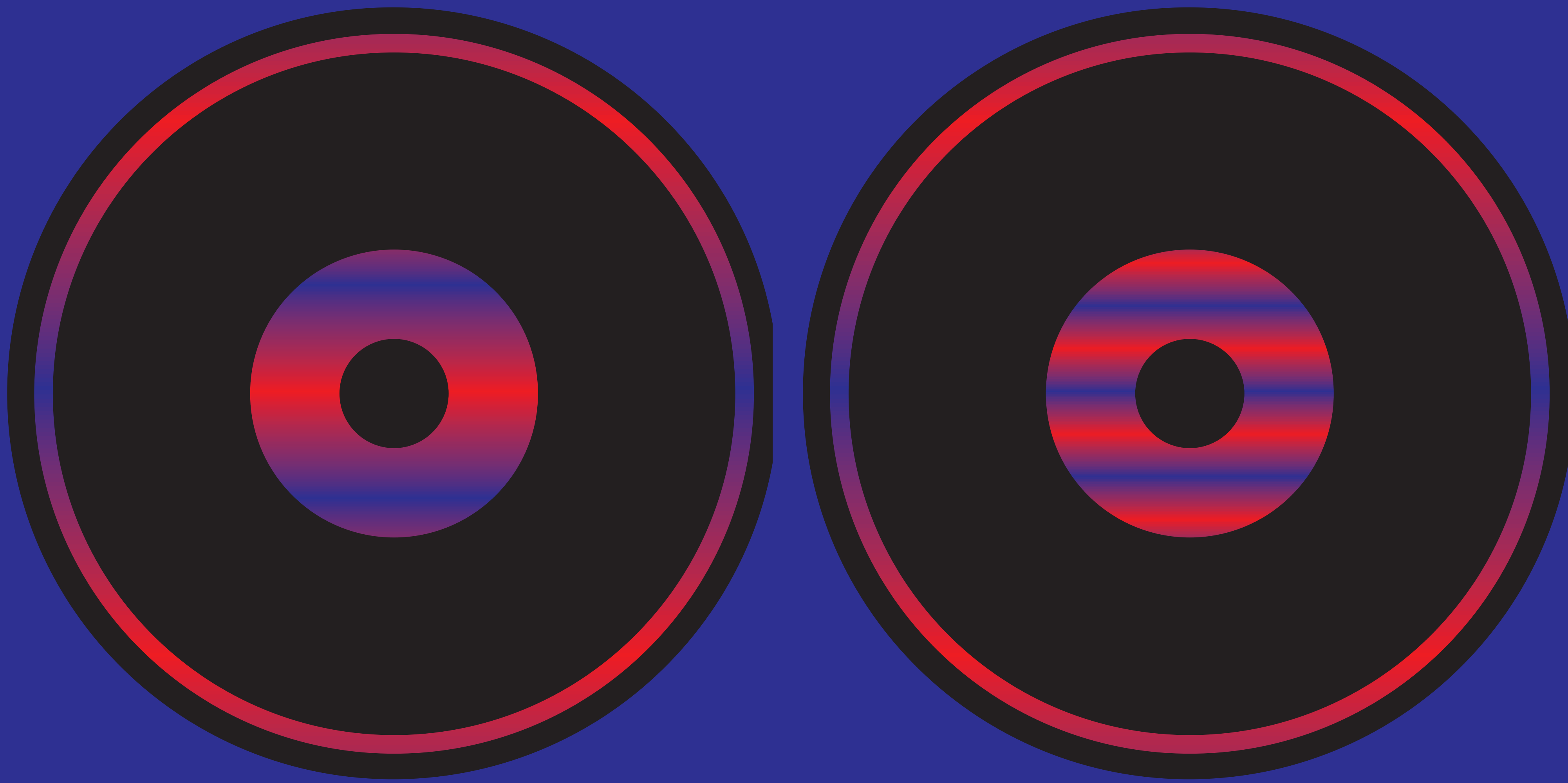
- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)

Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)
- A smooth component contributes
  density ρ to the expansion
  but not
  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion

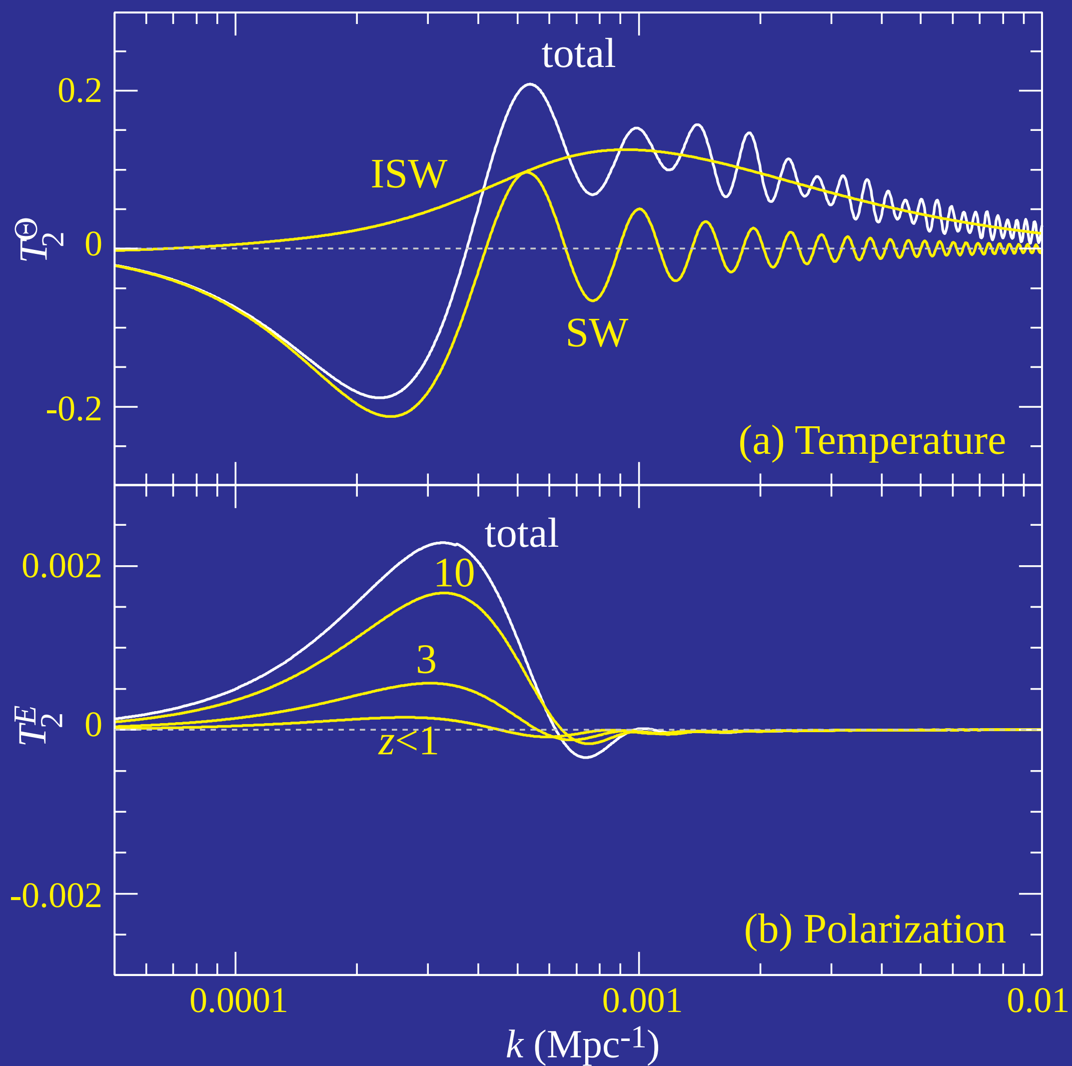
ISW Spatial Modes

- ISW effect comes from nearby acceleration regime
- Shorter wavelengths project onto same angle
- Broad source kernel: Limber cancellation out to quadrupole



Quadrupole Origins

- Transfer function for the quadrupole



Gordon & Hu (2004)

Smooth Energy Density & Potential Decay

- Regardless of the equation of state an energy component that clusters preserves an approximately constant gravitational potential (formally Bardeen curvature ζ)
- A smooth component contributes

  density ρ to the expansion

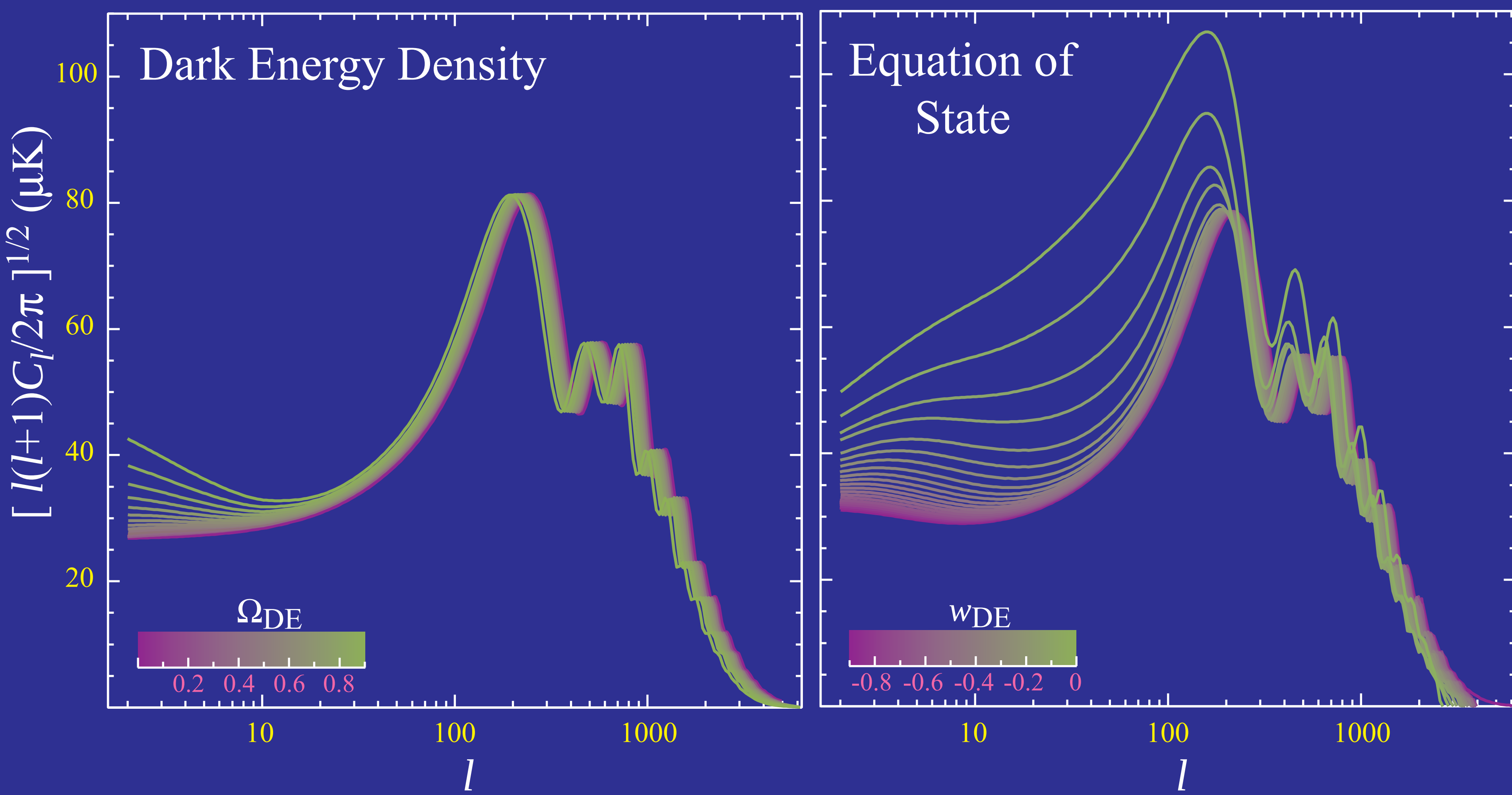
  but not

  density fluctuation $\delta\rho$ to the Poisson equation
- Imbalance causes potential to decay once smooth component dominates the expansion
- Scalar field dark energy (quintessence) is smooth out to the horizon scale (sound speed c_s=1)
- Potential decay measures the clustering properties and hence the particle properties of the dark energy

ISW & Dark Energy

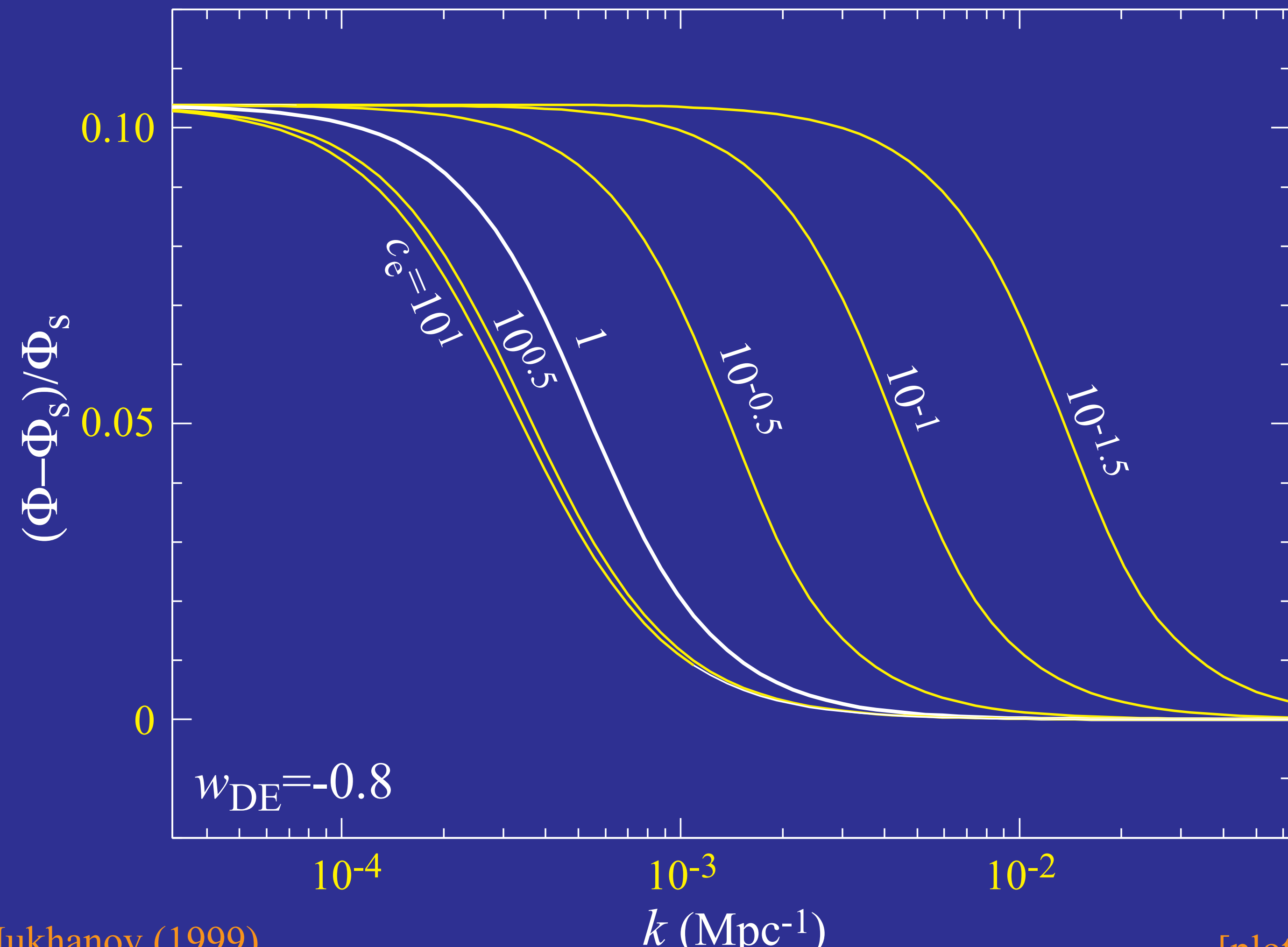
Dark Energy

- Peaks measure distance to recombination
- ISW effect constrains dynamics of acceleration



Dark Energy Sound Speed

- Smooth and clustered regimes separated by sound horizon
- Covariant definition: $c_e^2 = \delta p/\delta\rho$ where momentum flux vanishes
- For scalar field dark energy uniquely defined by kinetic term

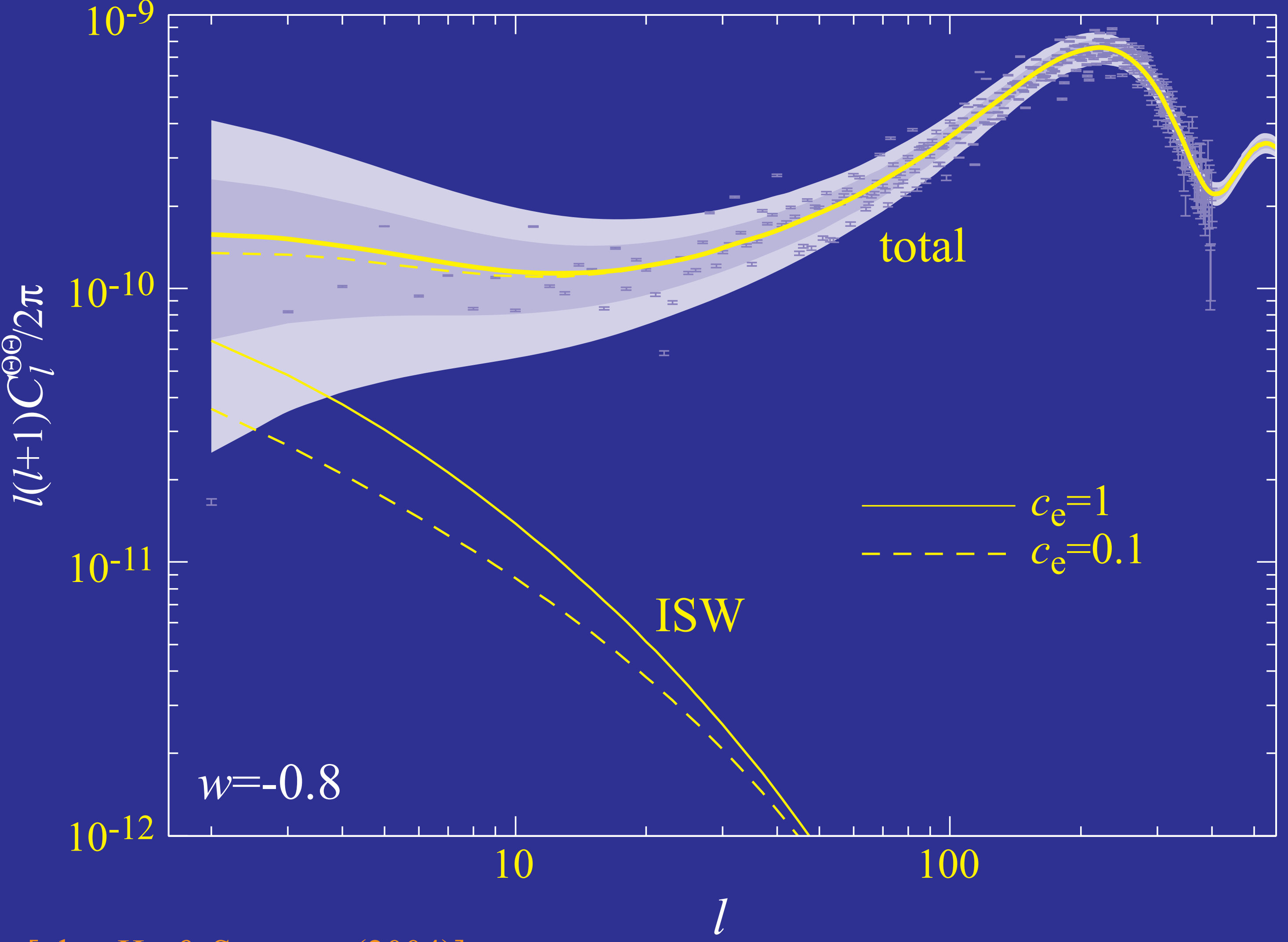


Hu (1998)
Garriga & Mukhanov (1999)

[plot: Hu & Scranton (2004)]

Dark Energy Clustering

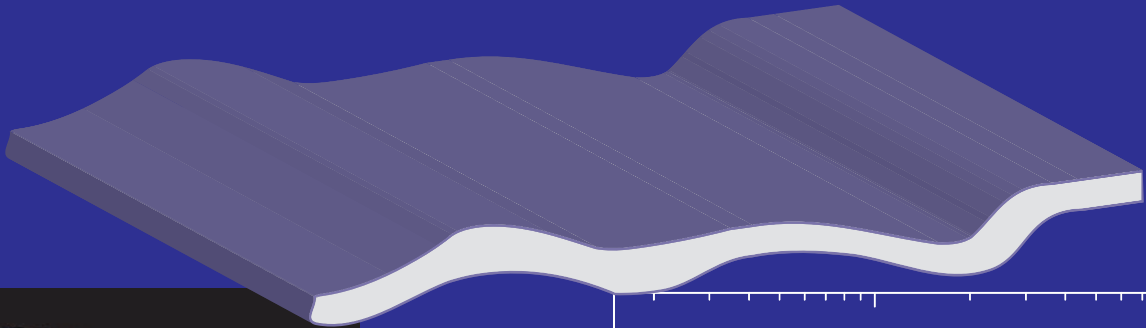
- ISW effect intrinsically sensitive to dark energy smoothness
- Large angle contributions reduced if clustered

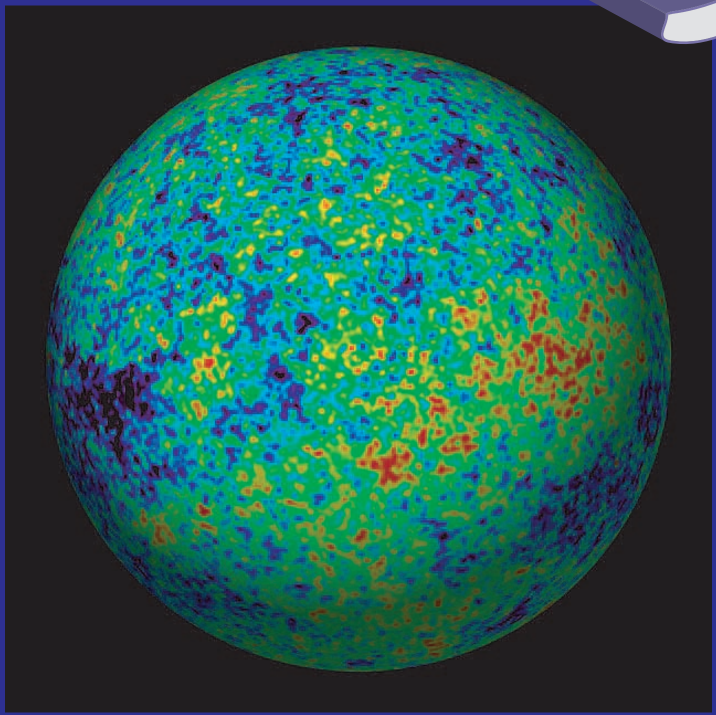


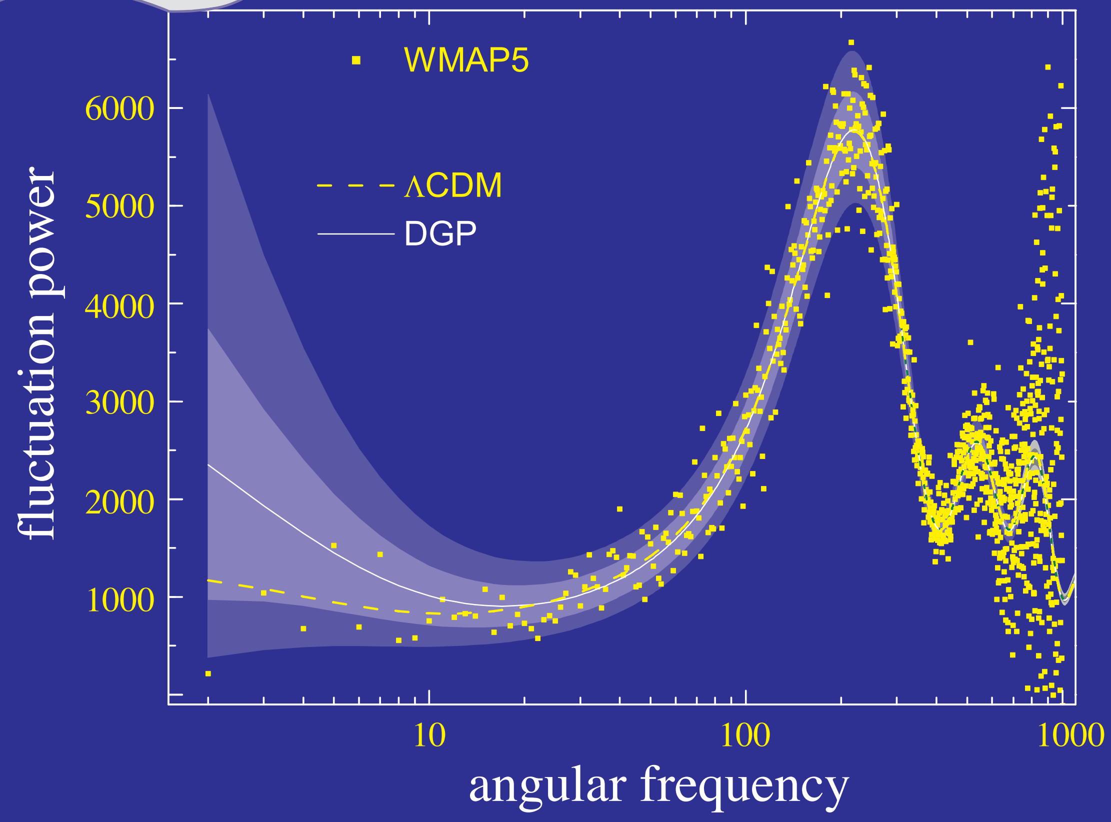
Hu (1998); [plot: Hu & Scranton (2004)]

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background







Summary

- Large-scale structure lenses the CMB causing smoothing of temperature power spectrum and creation of B modes
- Information on cosmic acceleration, neutrinos encapsulated in PCs
- 5σ detection from SPT
- Quadratic estimators reconstructs lenses associated with large scale structure, halos in principle allowing precision tests
- First results from ACT and SPT
- Differential gravitational redshifts from evolving structure causes integrated Sachs-Wolfe (ISW) effect
- Appears on large angles and contributes to quadrupole comparably to primary
- Tests the microphysics of acceleration: clustering of dark energy, modified gravity, dark matter interactions